Internet Appendix. Simulation

This Internet Appendix presents the simulations that compare various FMP construction methods. The purpose of our simulation is to study the magnitude of biases and the other statistical properties in finite samples. Jegadeesh et al. (2019) study the finite sample properties of the risk premium estimation for traded factors. However, the nontraded factors are possibly associated with a higher estimation error in factor loadings that elicit a larger finite sample EIV bias (Kleibergen, 2009). Therefore, it is necessary to reexamine these properties for nontraded factors.

Our simulation parameters match attributes of the real data. We use individual stock returns data from February 1964 to March 2016 covering 626 months and 10,833 stocks. For instance, most stocks do not have data spanning the entire sample period; hence, our simulated stocks have data only in the same periods as their corresponding stocks in the real data.[1]

The simulation procedure is described below:

Step 1: Regress excess stock returns on factors and obtain estimated betas (**β**) and residuals (**ε**) for each stock. **β** is a N×K matrix, and **ε** is a N×T matrix, where N is the number of stocks, and T is the number of time periods. Because the existing periods for most stocks are less than T, the matrix **ε** is not a balanced panel. Thus, we assign the value to be missing if one stock does not have return data for this corresponding period.

Step 2: In each simulation, we create a $T \times 1$ vector **S** by randomly selecting T numbers with replacement from 1 to T, where T is the maximum month number. Then we create simulated factors by rearranging observed factors that match the randomly chosen observation number in the vector **S**. Finally, we augment the simulated factors by adding prespecified true premia λ_0 set equal to the observed average risk premia from Chen, Roll, and Ross (1986) and Chen and Kan (2003).

Step 3: Generate simulated residuals that are randomly and normally distributed with the mean and the variance equal to the sample mean and the variance of the observed actual residuals for the stock.

[1] We also conduct simulations in which all simulated stocks have data in every period (unreported.) This leads to less overall bias, because the number of months in the first-pass time-series regressions and the number of stocks in second-pass cross-sectional regressions are much larger. However, the overall conclusions, which compare the FMP methods, remain the same.

Step 4: Construct simulated returns for each stock in each month as estimated betas, multiplied by simulated factors, plus simulated residuals.

Step 5: Apply the methods in Section 2 to construct FMPs using simulated returns and simulated factors.

Step 6: After constructing the FMPs, we reapply various Fama-Macbeth two-pass methods (the OLS, the IV, and the Stein methods with individual stocks) to estimate the risk premia using simulated returns and the FMPs, thereby obtaining simulated estimates of the FMP-based risk premia.

Replicate steps 1 to 6 1,000 times. Then we calculate the mean difference between the simulated estimates of the FMP-based risk premia and the true simulated risk premia (which is the ex ante bias). This mean difference is the predicted bias of each FMP method.[2]

Simulation Results: Bias

Table A.I presents the simulation results. Following Chen, Roll, and Ross (1986), we use four macro factors as risk factors: changes in the consumption growth rate (CG), changes in the CPI (CPI), changes in industrial production (IP), and changes in the unemployment rate (UE). We assume the beta is constant for each stock. In the first stage, we apply the univariate method to construct each FMP, and then, in the second stage, we apply the multivariate approach to estimate the risk premium.

Consistent with our expectations, the IV approach resolves the EIV bias in a two-pass regression and produces a nearly unbiased risk premia. The alpha in the IV estimation is 0.0055%, which is close to zero. The differences between the estimated risk premia and the true risk premia are minimal. For instance, the estimated risk premium for consumption growth is just 0.2% larger than its true risk premium.

That the OLS method produces the estimated risk premia that are much smaller than the true risk premia is consistent with our conjecture that the OLS is subject to downward bias caused by measurement errors in estimated betas. The bias ranges from 28.7% for consumption growth to 50% for the unemployment rate. The mispricing part (i.e., the intercept term) is 0.1412%, which

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[2] In unreported results, we also compute an ex post bias as the differences between estimated risk premia and the sample mean of the corresponding factor realizations in that particular simulation. We find the difference between ex post bias and ex ante bias is minimal, and, thus, Table A.II offers ex ante bias only.

is much larger than that produced using the IV method. The alpha from the sorting-by-beta method (OLS_SB) is even larger than that produced using the OLS method, and the risk premia bias is large as well. For example, the bias for consumption growth is 31%. The bias for the Lehmann and Modest (1988) (LM) method is as large as that for the OLS method. Furthermore, the mispricing in the LM method is 0.20%, which is even larger than the mispricing produced by the OLS method. Stein's method yields less bias than do the OLS and LM methods, but the bias in the Stein method is still large at over 20%.

Panel B in Table II reports risk premia estimated using FMPs as factors. As reported in this panel, risk premia of FMPs are nearly unbiased using the IV approach, but OLS-based risk premia have large biases. Risk premium bias for the sorting-by-beta method is relatively large, but smaller than that of the OLS method. More importantly, the results in Panel B indicate that the FMPs constructed by IV method generate the risk premium that has the same magnitude as that generated by the original risk factors. This finding is consistent with that of Shanken (1992), who shows that the mean of the traded factors (an FMP is a traded factor) is the risk premium.

We also consider the time-series approach. Specifically, we present the time-series approach using the Lamont (2001) portfolios comprising the candidate assets. This approach is constructed by regressing the risk factors on a series of basis portfolio returns and then calculating the predicted values as FMPs. Then we estimate the risk premia for the FMPs constructed by the time-series approach (TS_FMPs). The time series works well only when the correlations between the risk factors and the candidate assets are high. Our simulation in Panel B of Table II shows that the average bias for the time-series approach is larger than 40%. This is because the correlations between the macro factors and the candidate assets are low.

Simulation Results: RMSE

We also calculate the RMSE for the simulations. The error is simply the difference between the true risk premium and the risk premium estimated for each simulated replication, of which there are 1,000. We call this the "ex ante" RMSE. We also calculate the difference between the estimated risk premium in each replication and the sample mean of the corresponding risk factor realizations and then compute its RMSE across 1,000 replications. This is the "ex post" RMSE. Panel A of Table A.II reports these RMSEs. The ex ante and ex post RMSEs are quite similar, so we focus on the former.

The ex ante RMSEs with the IV method are uniformly smaller than are with the OLS method; that is, the IV method is considerably more accurate. For example, the CPI ex ante RMSE is 0.082 for the OLS method, but just 0.026 for the IV method. The Stein method produces a smaller RMSE than does the OLS method, but it is still larger than that of the IV method. The OLS method has the largest RMSE. The LM method has a marginally smaller RMSE than does the OLS method. Overall, the IV method dominates all other methods by the RMSE criterion.

Size and Power of *t*-tests

To estimate the size and power of the *t*-tests for the risk premia with the IV approach, we first consider the probability of rejecting the null hypotheses falsely, or the type I error. In this simulation, we set the true risk premium equal to zero for all factors. Then we follow the simulation procedure in Section 3.1 to estimate the risk premia and their corresponding *t*-statistic; that is, the mean estimated risk premium, divided by its corresponding standard error. We use a 5% significance level (the critical value is 1.96) and calculate the frequency of the absolute value of *t*-statistic that is larger than 1.96 in 1,000 replications. Panel B of Table A.II reports that the "size" of each macro factor is around 5% or slightly below that.

To examine the *t*-test power (the probability of rejecting the null hypothesis when the alternative hypothesis is true), we set the true premium to the mean risk premium estimated from a Fama-MacBeth two-pass regression with the OLS method. This is a relatively small risk premium (e.g., compared with that obtained with the IV method), thus implying a more conservative threshold for power. Panel B in Table A.II presents the results. The frequency of rejecting an incorrect null hypothesis is 70.9% for consumption growth and a bit higher for the other three macro factors. Overall, the size and power tests indicate that a normal *t*-statistic delivers effective inferences about the macro factor premia.

Notably, all simulations are multivariate, except for the IV* simulation. For the IV* simulation, we use a univariate regression to create FMPs because of the condition $\hat{\beta}_{IV}{}'\hat{\beta}_{EV} > 0$. To estimate the risk premia, we use a multivariate regression. That is, we create FMPs for each factor independently. The second step is to test risk premia for FMPs, where we use a multivariate regression to estimate the risk premia of all FMPs.

Table A.I. Simulation: Biases of Mispricing and Risk Premia of Factor-Mimicking Portfolios

This table reports biases in the estimated risk premium using Monte Carlo simulations for factor-mimicking portfolios (FMPs). We simulate the stock returns and factors following the description above. We create FMPs following the methods described in Section II. With these FMPs, we run cross-sectional regressions to estimate the risk premia. The first row shows the true risk premium for four macro variables when "Alpha" (mispricing) is set to be zero. The table's two panels report estimated risk premia along with their mean biases across 1,000 replications, expressed as a percentage of the true value. Panel A presents the risk premia and the biases using the instrumental variables (IV) method, an (IV*) IV method that only retains stocks whose betas in the even and odd subsamples have the same signs, an OLS regression, sorting-by-beta (OLS-SB), the Lehmann and Modest (1988) (LM) method, and the Stein (1956) method. Panel B presents the risk premia of FMPs computed using the estimated time-series coefficients in Panel A as a proxy for FMPs. We list IV, OLS, and sorting-by-beta regressions as an example of the cross-sectional approaches. In Panel B, we also tests the risk premia for the time-series FMPs constructed by Lamont (2001). The sample period is January 1964 to March 2016. The macro factors include unexpected consumption growth (CG), unexpected changes in the CPI (CPI), unexpected changes in industrial production (IP), and unexpected changes in the unemployment rate (UE).

	Alpha	CG	CPI	IP	UE
True risk premium	0	0.2	-0.2	1.2	0.3
		A. Cross-Sectional Methods			
IV	0.0055	0.2004	-0.2009	1.2043	0.3039
		0.2%	0.4%	0.4%	1.3%
IV*	-0.0003	0.2072	-0.2011	1.1732	0.3035
		3.60%	0.55%	-2.23%	1.17%
OLS	0.1412	0.1426	-0.1147	0.6124	0.1499
		-28.7%	-42.7%	-49.0%	-50.0%
OLS-SB	-0.0569	0.1379	-0.1774	1.1362	0.2685
		-31.05%	-11.30%	-5.32%	-10.50%
LM	0.2000	0.1367	-0.1150	0.6738	0.1555
		-31.7%	-42.5%	-43.9%	-48.2%
Stein	0.0533	0.153	-0.1506	0.7485	0.2441
		-23.5%	-24.7%	-37.6%	-18.6%

		B. FMPs Used to Estimate Risk Premia			
IV	0.0002	0.2068	-0.1942	1.1896	0.2948
		3.40%	-2.90%	-0.87%	-1.73%
OLS	0.0010	0.1351	-0.1353	0.9272	0.2429
		-32.45%	-32.35%	-22.73%	-19.03%
Time series	-5.0599	0.0861	-0.0803	0.7098	0.1624
		-56.95%	-59.85%	-40.85%	-45.87%
OLS-SB	-0.0128	0.1852	-0.1516	1.1448	0.2781
		-7.40%	-24.20%	-4.60%	-7.30%

Table A.II. Simulation: RMSE, Size, and Power *t*-test

Panel A reports the root-mean-square error (RMSE.) The ex ante RMSE measures the mean difference between the estimated risk premium and the true risk premium. The ex post RMSE measures the difference between the estimated risk premium and the risk factor's realization. RMSEs are computed across 1,000 replications in each simulation. Panel B shows the size and the power of *t*-statistics for the IV method. Size is based on a 1.96 critical value (a 5% significance level.). It measures the probability of improperly rejecting a true null hypothesis (simulated here as truly zero risk premia.) Power is the probability of rejecting a false null hypotheses; in this case, the alternative (true) hypothesis consists of risk premia obtained from the OLS method, which are generally smaller than that of the other methods.

A. RMSE

		CG	CPI	IP	UE
OLS	Ex ante	0.0700	0.0820	0.5906	0.1509
	Ex post	0.0690	0.0821	0.5901	0.1509
IV	Ex ante	0.0549	0.0260	0.1318	0.0224
	Ex post	0.0520	0.0239	0.1287	0.0217
LM	Ex ante	0.0850	0.0793	0.5312	0.1480
	Ex post	0.0812	0.0791	0.5400	0.1474
Stein	Ex ante	0.0622	0.0473	0.4577	0.0585
	Ex post	0.0592	0.0456	0.4553	0.0581

B. Size and Power of t*-test in IV Approach*

	CG	CPI	IP	UE
Size (%)	4.9	5.2	4.3	3.8
Power (%)	70.9	86.0	83.1	71.2