# A New Method for Factor-Mimicking Portfolio Construction

Kuntara Pukthuanthong
University of Missouri, USA

Richard Roll
California Institute of Technology, USA

Junbo Wang
Louisiana State University, USA

Tengfei Zhang
Cambridge Judge Business School, University of Cambridge, UK

**Abstract**

Literature has applied various Factor-Mimicking Portfolios methods to test asst pricing model with non-tradable factors. We extensively compare these methods in testing asset pricing models for bonds and stock markets and find that all these methods cannot demonstrate the association between several macroeconomic factors and the expected bond and stock returns. By revising an existing method, the results can be reversed. Specifically, using the adjusted method, consumption growth, inflation, and unemployment command stock premiums, while consumption growth and industrial output command corporate bond prices.

(*JEL* G10, G12, G11)

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One of the most critical concerns in asset pricing is whether or not varying average returns across assets reflect risk. Cross-sectional changes in asset returns are connected to firm factors like size, book-to-market ratio, momentum, investment, and profitability (Fama and French, 1993, 2016, 2018; Hou, Xue, and Zhang, 2015; Smith and Timmermann, 2021). Nonetheless, it has to be established whether they represent risk exposures.

Theoretically based macroeconomics variables (e.g., consumption growth; see Parker and Julliard, 2005) and other nontradable factors (e.g., intermediary financial leverage from Adrian, Etula, and Muir, 2014; labor market tightness from Kuehn, Simutin, and Wang 2017) reflect economic risks and should thus also explain cross-sectional expected returns. However, observed changes in these parameters involve measurement errors, resulting in weak asset return predictions. The prior literature (Huberman, Kandel, and Stambaugh 1987; Breeden, Gibbons, and Litzenburger 1989; Ferson, Siegel, and Xu 2006; Balduzzi and Robotti 2008) has recommended the use of factor-mimicking portfolios (FMPs), which are constructed from traded assets to represent the underlying nontradable factors, to reduce factor noise. In the existing research, FMPs are used to evaluate asset pricing models (e.g., Cooper and Priestley 2011; Barillas, Robotti, and Shanken 2019; Pukthuanthong, Roll, and Subrahmanyam 2019, henceforth PRS).[1] However, applying FMPs to macro variables still fails to price cross-sectional asset returns reliably. One view is that macro variables do not affect the pricing of assets, even though this conclusion contradicts the conventional asset pricing theory. This study takes a different view. We suggest that the methods for constructing FMPs matter for asset pricing test. To this end, we compare various classical methods in asset pricing tests empirically and find that all these existing methods cannot demonstrate the association between several macroeconomic factors. However, some adjustments on an existing method can reverse the results, leading to significant risk premium estimates for macroeconomic factors.

[1] Roll and Srivastava (2018, p. 21) point out the other applications: "mimicking portfolios have many potential uses, including (though not limited to): (1) Evaluating active manager performance, (2) Substituting for a desired investment in illiquid assets, (3) Determining the true potential for improved diversification, (4) Understanding the sources of past return volatility, (5) Predicting the likely level of future return volatility."

Classical FMP theory, as provided by Huberman, Kandel, and Stambaugh (1987) and Breeden, Gibbons, and Litzenberger (1989), contend that an FMP should be designed to optimize the correlation with the underlying component (i.e., a maximum correlation portfolio). Three FMP building approaches are proposed by researchers who adhere to this notion. The first technique is the cross-sectional approach, in which Fama-Macbeth (1973)'s cross-sectional regression of asset returns on beta loadings of nontradable variables is performed for each sample period. The calculated coefficient series is utilized as the FMP returns.

The second technique is a time-series approach, in which nontradable factors are regressed on contemporaneous returns of preselected assets, and the fitted values are used as FMP returns. The third option is the sorting-by-beta method, in which stocks are sorted by factor loadings (betas), and the FMP is the long-short portfolio between the top and bottom deciles.

Although these methods exist in the literature for a long time, there is not a comprehensive comparison among them in testing asset pricing models. In this paper, we first make comparisons among these approaches in asset pricing test using classical macroeconomic factors. Unfortunately, all these methods do not imply significant risk premium estimates. On the other hand, based on the existing literature, we propose an adjusted method (4-ADJ) for constructing FMPs, and we show that the adjustment can lead to significant risk premium estimation.

The first adjustment addresses the problem when we do a multiple-factor Fama-MacBeth regression with correlated factors (see Kan, Robotti, and Shanken, 2013). In such instances, the measurement error on a non-tradable component would result in an inaccurate FMP estimate since other associated factors are included in the regression. To address this problem, we follow Kan, Robotti, and Shanken (2013) by doing a univariate Fama-MacBeth regression of asset returns on each nontradable component to determine its FMP.

The second adjustment of our method reduces the errors-in-variables (EIV) bias that plagues the estimated beta loadings in a Fama-MacBeth regression. Following Jegadeesh et al. (2019), we use the instrumental variables (IV) methodology to address this problem. Specifically, we divide the overall sample into subsamples for months with even and odd numbers and estimate betas for each subsample independently. Then, betas from the subsample of even-numbered months are used as instrumental variables for betas from odd-numbered months, and vice versa.

The third adjustment pertains to base asset selection. Our approach generates FMPs using the IV approach to find appropriate base assets. To date, the majority of the literature uses portfolios as basis assets. Still, there is no consensus over which portfolios should be chosen.[2] We advise using all individual assets in line with Jegadeesh et al. (2019), Roll and Srivastava (2018), and Harvey and Liu (2020, 2021).

Nonetheless, non-tradable factors may be associated with a subset of assets (e.g., Cong, Feng, He, and Li 2022). We provide a screening strategy for basis assets that generate the same signs between beta loadings calculated from the even and odd month samples to solve this issue. We show that this approach can select assets correlated with the factors. The first three steps describe how to generate FMPs for each non-tradable factor of interest.

In the last adjustment, the risk premiums are calculated by applying a multifactor IV Fama-MacBeth regression to the FMPs generated for each factor. It is a two-step procedure in which FMPs are created in the first phase, and Fama-MacBeth regression is applied to FMPs in the second. Researchers have used the two-stage approach to reduce measurement error for nontradable factors in small samples (e.g., Conner, Korajczyk, and Uhlaner 2015).

We compare our adjusted method to other traditional methods by analyzing various macro factors using the criteria established by Pukthuanthong, Roll, and Subrahmanyam (2019). Specifically, we

[2] Lamont (2001) proposes economic tracking portfolios using 13 basis assets, including eight industry-sorted stock portfolios, four bond portfolios, and a stock market return. Vassalou (2003) uses six equity portfolios sorted by size and book-to-market ratio, the term spread, and the default spread. Kroencke et al. (2013) use equity portfolios sorted by size, book-to-market ratio, and a momentum portfolio. Bianchi, Guidolin, and Ravazzolo (2017) use six size and book-to-market-sorted portfolios, plus the default and term spreads. Barillas and Shanken (2017) use 15 traded factors as basis assets. Maio (2018) uses the excess market return, the value spread, the term spread, and the S&P 500 price-to-earnings ratio. Lehmann and Modest (1988) use size, the dividend yield, and variance-sorted portfolios, and Cooper and Priestley (2011) use 40 portfolios sorted by size, book-to-market ratio, momentum, and asset growth. Pukthuanthong, Roll, and Subrahmanyam (2019) use 50 portfolios sorted by size, book-to-market ratio, momentum, investment, and operating profitability. Roll and Srivastava (2018) use eight exchange-traded fund (ETF) portfolios. Lönn and Schotman (2018) even use 900 portfolios from the Kenneth French data library.

investigate (1) if the FMPs of macro factors are connected with the systematic risk of returns and (2) whether the FMPs of factors command a risk premium.

Pukthuanthong, Roll, and Subrahmanyam (2019) argue that a genuine risk factor must be associated with systematic risk (represented by the covariance matrix of returns) in criteria 1. Our adjusted method yields FMPs that are correlated with the covariance of stock returns. However, for other techniques (such as time-series methods), FMPs lack a consistent correlation with systemic risk. Regarding criteria 2, the monthly risk premium for the FMPs of consumption growth constructed with the adjusted approach is )0.136% and is significant. Other methods produce negative or insignificant risk premiums. We also find that the FMPs for the unemployment rate and the Consumer Price Index (CPI) contain significant risk premia when they are constructed with our method, but not when obtained via other approaches.

Using corporate bond returns as basis assets, we also generate FMPs from various methodologies and use these FMPs to estimate the risk premiums of several macro and bonds variables. Our approach is the only FMP construction method that meets both requirements. We find that consumer growth, industrial output, bond market factors, and default spread are related to significant and positive risk premiums.

In addition to the empirical results, we also examine all methods in simulations. For the classical methods, all existing methods produce a bias in estimated risk premiums. Our adjusted method can control for the bias and produce a reasonable statistic power.

Our research is based on a large body of research that creates and analyzes the performance of FMPs. Surprisingly, given the amount and significance of the literature, no study provides a complete overview and compares various methods. It is hard to list all articles that use FMP. Thus we discuss two recently published research. Bessembinder, Cooper, and Zhang (2019) employ the time-series technique to estimate benchmark returns and evaluate aberrant stock returns after corporate events, following a long line of works that use this approach (see our footnote 2). Engel et al. (2020) developed factor-mimicking portfolios dubbed "climate change hedged portfolios" with both time-series and cross-sectional methods. We add to the literature on asset pricing by thoroughly comparing different approaches in FMP construction and examine the asset pricing models for equity and bond markets.

Our paper adds to the existing literature on constructing FMPs. Classical papers, such as Huberman, Kandel, and Stambaugh (1987) and Breeden, Gibbons, and Litzenberger (1989), propose to use the time-series, cross-sectional or sorting-by-beta approaches to construct FMPs. Instead of mimicking portfolios for factors, Roll and Srivastava (2018) create portfolios that mimic individual stock performances. Fama and French (2020) construct factor-mimicking portfolios for characteristics using a cross-sectional methodology and show that these characteristic-mimicking portfolios have more explanatory power over the average return than characteristic-sorting-based factors (such as SMB and HML). We suggested a new FMPs based on the 4-ADJ methodology, and compare it with other existing methods.

Our research also joins a new subfield of the literature that proposes novel FMP construction techniques. Giglio and Xiu (2021) suggest a three-step approach to estimate the FMP of factors to tackle the problem of missing variables in risk premium calculation. Giglio, Xiu, and Zhang (2022) expand on Giglio and Xiu (2021) and present a novel technique for selecting test assets. Based on the standard Fama-Macbeth cross-sectional method, our research proposes an alternate way for FMP construction and asset selection.

**1. Classical Approaches toward FMP Construction**

The classical approach to constructing FMPs involves maximizing an FMP's correlation with the nontradable factor. Huberman, Kandel, and Stambaugh (1987), Breeden, Gibbons, and Litzenberger (1989), and Roll and Srivastava (2018) built the theoretical framework for maximum correlation portfolio construction.

Let $N$ denote the number of candidate assets to be included in FMPs, $T$ the number of periods, $K$ the number of factors, $\boldsymbol{R}$ the $T \times N$ matrix of asset returns, $\boldsymbol{f}$ the $T \times K$ matrix of true risk factor realization of $T$ periods, $\boldsymbol{\varepsilon}_f$ the $T \times K$ measurement error, and $\tilde{\boldsymbol{f}} = \boldsymbol{f} + \boldsymbol{\varepsilon}_f$ the $T \times K$ vector of the observed nontraded factor. To simplify the analysis without loss of generality, assume that the measurement error is uncorrelated with the asset returns and the factors; that is, $cov(\boldsymbol{\varepsilon}_f, \boldsymbol{R}) = 0$ and $cov(\boldsymbol{\varepsilon}_f, \boldsymbol{f}) = 0$.

FMPs are the weighted returns of $N$ candidate assets with the weights $\boldsymbol{w}$, where $\boldsymbol{w}$ is an $N \times K$ vector. Then FMPs for the $k$th factor (denoted by $\boldsymbol{FMP_k}$) are determined by multiplying asset returns and their weights, $\boldsymbol{Rw_k}$, where $\boldsymbol{w_k}$ is the $k$th column of the weight matrix, $\boldsymbol{w}$.

For a given value of the loading of the true risk factor $k$ $(f_k)$ on its mimicking portfolio, $\beta_{\mathrm{k,m}}$, Breeden, Gibbons, and Litzenberger (1989) show that

$$\beta_{\mathrm{k,m}} = corr(f_k, FMP_k)\frac{\sqrt{Var(FMP_k)}}{\sqrt{Var(f_k)}} . \tag{1}$$

From Equation (1), the variance of the FMPs $(FMP_k)$ is reciprocal to the correlation between $FMP_k$ and $f_k$, because the variance for factor $k$ $(var(f_k))$ is constant.

Therefore, maximizing the correlation between FMPs and their observed factor is equivalent to minimizing an FMP's variance. Specifically, we want to select the weight for the following minimization problem:

$$\boldsymbol{Min_{w_k}} \;\; \boldsymbol{w_k}'\boldsymbol{V}\boldsymbol{w_k} + 2(\boldsymbol{\beta_{k,m}} - \boldsymbol{w_k}'\boldsymbol{\beta})\,\boldsymbol{\lambda} , \tag{2}$$

where $\boldsymbol{V}$ is the covariance matrix of candidate asset returns, and $\boldsymbol{\lambda}$ is the $K \times 1$ vector of the Lagrange multiplier. $\boldsymbol{\beta}$ is the $N \times K$ loadings matrix for asset returns on risk factors. $\boldsymbol{\beta_{k,m}}$ is the $1 \times K$ vector, which reflects the loading of the mimicking portfolio on the $k$th factor ($\beta_{k,m}$). Typically, $\boldsymbol{\beta_{k,m}}$ is set to be a vector of $\beta_{k,m}$ for the $k$th factor and zero otherwise.

Solving the first-order condition of expression (2) and setting $\beta_{k,m}$ equal to one for all k, we find that the optimal weight is

$$\boldsymbol{w} = \boldsymbol{V}^{-1}\boldsymbol{\beta}\left[\boldsymbol{\beta}'\boldsymbol{V}^{-1}\boldsymbol{\beta}\right]^{-1} . \tag{3}$$

Then the return of the maximum correlation portfolio can be computed as

$$\boldsymbol{w}'\boldsymbol{R} = \left[\boldsymbol{\beta}'\boldsymbol{V}^{-1}\boldsymbol{\beta}\right]^{-1}\boldsymbol{\beta}'\,\boldsymbol{V}^{-1}\boldsymbol{R}. \tag{4}$$

Following Equation (4), three methods can be used to construct FMPs.

### 1.1 Time-series method

In this approach, we estimate the FMP return by regressing nontradable factors, $\tilde{\boldsymbol{f}}$, on returns of the test assets:

$$\tilde{\boldsymbol{f}} = a + b\boldsymbol{R} + \boldsymbol{u}. \tag{5}$$

The time-series approach is used widely in the literature to construct factor-mimicking portfolios of macroeconomic factors (e.g., Vassalou 2003; Petkova 2006; Barillas et al. 2019; Lönn and Schotman 2018; Maio 2018).

### 1.2 Cross-sectional method

We estimate the coefficients for regressing the realized return on its factor loadings across test assets (i.e., a Fama-MacBeth regression). Specifically,

$$\boldsymbol{R} = \alpha + \gamma\boldsymbol{\beta} + \boldsymbol{v}, \tag{6}$$

where the estimated coefficient $\gamma$, using the generalized least squares (GLS) method with weighting matrix $\mathbf{V}$, is the FMP return in Equation (4).

The matrix $\mathbf{V}$ is more challenging to estimate for many test assets. A noteworthy scenario, in this case, is to assess the coefficient for the regression in Equation (6) using ordinary least squares (OLS). Another particular scenario is Lehmann and Modest (hereafter LM 1988). They suggest using a diagonal matrix, which only contains the variances of the residuals from regression (6), to replace $\boldsymbol{V}^{-\mathbf{1}}$. Cooper and Priestley (2011), Pukthuanthong, Roll, and Subrahmanyam (2019), and Roll and Srivastava (2018) apply this approach.

Stein (1956) and James and Stein (1961) provide a shrinking technique to reduce root-mean-square errors (RMSEs). Their contraction beta may be expressed as

$$\widehat{\boldsymbol{\beta}}_{\boldsymbol{Stein}} = \left(1 - \frac{(N-3)}{\|\widehat{\beta}_{\sigma}^{*}\|}\right)\widehat{\boldsymbol{\beta}}_{\boldsymbol{new}} + \bar{\widehat{\beta}},$$

where $\widehat{\boldsymbol{\beta}}_{\boldsymbol{\sigma}}^{*} = [\hat{\beta}_{\sigma}^{1}, \hat{\beta}_{\sigma}^{2}, \ldots, \hat{\beta}_{\sigma}^{N}]$, and $\hat{\beta}_{\sigma}^{i} = \frac{\hat{\beta}^{i} - \bar{\bar{\beta}}}{\sigma^{i}}$, in which $\sigma^{i}$ is the standard error of $\hat{\beta}^{i}$. Also, $\|\widehat{\boldsymbol{\beta}}_{\boldsymbol{\sigma}}^{*}\| = \sum_{i=1}^{N} (\hat{\beta}_{\sigma}^{i})^{2}$, $\bar{\bar{\beta}} = \frac{1}{N} \sum_{i=1}^{N} \hat{\beta}^{i}$ or the mean of $\hat{\beta}^{i}$, and $\widehat{\boldsymbol{\beta}}_{\boldsymbol{new}} = [\hat{\beta}^{1} - \bar{\bar{\beta}}, \hat{\beta}^{2} - \bar{\bar{\beta}}, \ldots, \hat{\beta}^{N} - \bar{\bar{\beta}}]$.

Then the Stein-adjusted estimate is

$$\boldsymbol{\lambda_t} = (\boldsymbol{B}_{\boldsymbol{Stein}}^{'} \boldsymbol{B}_{\boldsymbol{Stein}})^{-\boldsymbol{1}} \boldsymbol{B}_{\boldsymbol{Stein}}^{'} \boldsymbol{R_t}, \tag{12}$$

where $\boldsymbol{B}_{\boldsymbol{Stein}}$ is the $N \times (K+1)$ matrix containing all the Stein shrinkage betas $\widehat{\boldsymbol{\beta}}_{\boldsymbol{Stein}}$, augmented by a vector of one. Stein's method can reduce the OLS estimator's mean-square error by reducing the standard error. Hence, the FMP constructed by this approach, $\boldsymbol{\lambda_t}$, will be less volatile.

### 1.3 Sorting-by-beta method

Suppose that we divide *N* assets into 10 groups by their factor loadings. Assume that assets 1 through $M = \frac{N}{10}$ are in the group with the smallest beta, ⋯, and assets $9M + 1$ to $10M$ are in the group with the largest beta. We can write the FMP return as,

$$\frac{1}{M} \left( \sum_{i=9M+1}^{10M} E\left(R^{i}\right) - \sum_{i=1}^{M} E\left(R^{i}\right) \right). \tag{7}$$

The FMP return is the difference between the average expected returns of the high- and low-beta groups. Similar to factors constructed by sorting characteristics, this long-short portfolio return in Equation (7) reflects the risk premium of the nontradable factor. Herskovic, Moreira, and Muir (2019) apply this approach.

## 2. Our adjusted method

There are four adjusted methods in our method.

(1) The FMP can be determined using a cross-sectional return on the loadings of multiple parameters. However, suppose various factors correlate, and these variables include measurement errors. The estimate of FMPs using a multifactor regression might be troublesome in this instance (see Kan, Robotti, and Shanken 2013). Specifically, a measurement error on a non-tradable component would result in an

inaccurate FMP estimate for that factor when the associated factors are included in the regression.[3] In other words, it does not accurately represent the genuine factor. We follow Kan, Robotti, and Shanken (2013)'s approach to tackling this problem. Specifically, we suggest utilizing univariate Fama-MacBeth regression to develop an FMP for each component.[4]

(2) The Fama-MacBeth regression is known to be subject to an EIV bias when testing asset pricing models. We recommend using the IV method following Jegadeesh et al. (2019). Specifically, we partition the overall sample into subsamples for months with odd and even numbers. We conduct separate time-series regressions for the odd- and even-numbered month subsamples to estimate the odd- and even-numbered month betas for each asset.

With the odd-numbered month betas as IV (instrumental variable) betas and even-numbered month betas as EV (evaluation variable) betas, we construct the matrices for the betas of all assets: $\widehat{\boldsymbol{B}}_{\boldsymbol{IV}}$ and $\widehat{\boldsymbol{B}}_{\boldsymbol{EV}}$, where $\boldsymbol{B}$ is the $N \times (K+1)$ matrix containing all the betas augmented by a vector of one, that is, $\boldsymbol{B} = [\boldsymbol{1}, \boldsymbol{\beta}]$, where $\boldsymbol{1}$ is an $N \times 1$ vector of ones.

Then we estimate a cross-sectional IV regression. That is, for each even-numbered month $t$, we run a cross-sectional regression, and the FMP at time $t$ can be written as

$$\widehat{\boldsymbol{\gamma}}_{\boldsymbol{t}} = (\widehat{\boldsymbol{B}}_{\boldsymbol{IV}}{}' \boldsymbol{V}^{-1} \widehat{\boldsymbol{B}}_{\boldsymbol{EV}})^{-1} \widehat{\boldsymbol{B}}_{\boldsymbol{IV}}{}' \boldsymbol{V}^{-1} \boldsymbol{R}_{\boldsymbol{t}}. \tag{9}$$

Here, $\boldsymbol{R}_{\boldsymbol{t}}$ is the excess return for even-numbered months and is an $N \times 1$ vector, and $\widehat{\boldsymbol{\gamma}}_{\boldsymbol{t}}$ is a $(K+1) \times 1$ matrix containing all estimated FMPs augmented by mispricing in the first column.[5] Correspondingly, for each odd-numbered month, we take the betas in the even-numbered month subsample as the IVs and estimate the same regression to obtain the FMP.

(3) Following Jegadeesh et al. (2019), Roll and Srivastava (2018), and Harvey and Liu (2021). we propose using a large number of assets (e.g., individual stocks or bonds) to construct FMPs for nontradable

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[3] Internet Appendices A.1 and A.2 depict a two-factor Fama-MacBeth regression scenario and demonstrate that the generated FMP is a linear mixture of the genuine factor and the other associated factors.

[4] Internet Appendix A.2 explains the logic for the single-factor technique.

[5] We use the OLS-IV approach, not the GLS-IV approach, because Roll and Ross (1994) find that only an OLS approach correctly and economically interprets the coefficient. In addition, the covariance matrix of individual assets is not invertible when the number of assets is larger than the number of time periods.

factors. Ideally, all assets in the return space should be included; however, doing so can create issues if not all of these assets are correlated with the factor (e.g., Cong, Feng, He, and Li 2022). Therefore, selecting assets correlated with the factor is necessary (so the factor loadings of the selected assets are not just pure noise). We propose choosing only assets in which $\hat{\beta}_{IV}'\hat{\beta}_{EV} > 0$ (where $\hat{\beta}_{IV}$ and $\hat{\beta}_{EV}$ are estimated betas in IV and EV samples) or retain only assets in which the IV and EV betas have the same signs.[6]

The proposed asset selection method can also resolve the "weak IV problem." The IV approach employs the beta loading of the factor in a subsample (from the even-numbered month subsample) as the IV for the beta of the factor in a different subsample (from odd-numbered months and vice versa). Cautious readers might question whether the estimation error in the beta estimation could be severe, which amounts to the weak IV problem in statistics. The weak IV problem arises when IV and EV are not correlated. By choosing the assets with $\hat{\beta}_{IV}'\hat{\beta}_{EV} > 0$ (which imposes a positive correlation), we can mitigate this problem.[7]

(4) We propose a two-stage technique in which the first stage is the construction of FMPs for factors and the second stage is calculating the risk premium using the FMPs produced in the first stage. Following Conner, Korajczyk, and Uhlaner (2015), the two-stage technique addresses the problem of measurement error. In section 6, we show through simulations that the method can correct the EIV bias and has good statistical power to detect the risk premium of the nontradable factors.

## 3. Data

This section describes the factors and data used in this paper. The descriptive statistics for these variables are shown in Table 1.

[Table 1 about here]

### 3.1 Stock return data

[6] Internet Appendix A.3 shows that this approach can select assets correlated with factors.

[7] A similar weak IV issue arises if the sample period ($T$) is limited. In this case, the estimated betas could be uninformative and/or noisy in a statistical sense, for example, if the researcher were to have yearly data for their proposed risk factor only. The data set created by the Federal Reserve Bank of St. Louis Research (FRED) is at a monthly frequency; thus, the weak IV issue can be mitigated.

CRSP provides monthly individual stock returns. The statistics span from January 1964 to March 2016 (627 months). We exclude equities having a price of less than \$1 or a market capitalization of less than \$6 million by the existing literature. We also remove equities with less than 60 consecutive months of positive performance. After these exclusions, 10,833 stocks remain in our sample; an average month has 2,850 stocks, and the total number of observations is 1,784,351. The mean return of individual stocks above a risk-free rate (1-month Treasury-bill rate) is 1.012% per month. In contrast, the median return is just 0.44% per month, demonstrating a significant positive skewness in stock returns.

### 3.2 Factor data

We retrieve macroeconomic indicators from the Federal Reserve Bank of St. Louis Research (FRED) website: (1) the increase in per capita consumption (Personal consumption expenditures (PCE in FRED code) divided by population (POPTHM)), (2) the change in the consumer price index (CPIAUCSL), (3) the change in industrial production (INDPRO), and (4) the change in the unemployment rate (UNCH) (UNRATE).

Following Chen, Roll, and Ross (1986), we employ the innovations in these macroeconomic variables as factors. We use residuals from a first-order vector autoregression (VAR) to quantify innovations. Results using the initial differences as innovations are reliable. Nevertheless, as explained by Boguth and Kuehn (2013), residuals from a VAR provide a more conservative specification for risk exposures. The trading variables are from the website of Kenneth R. French (excess market return, small-minus-large market capitalization, and high-versus-low book/market portfolio returns).

We acquire 25 portfolios, including size and book-to-market, ten industry portfolios from Kenneth French's website, and four bond portfolio returns, including 1-, 5-, and 10-year Treasury-bond and Moody's seasoned Baa corporate bond portfolios, from FRED since time-series FMPs requires portfolios as testing assets. We also examine other consumption-related factors, including the CAY factor (the ratio of consumption to aggregate wealth, proposed by Lettau and Ludvigson 2001) and the consumption volatility factor presented by Boguth and Kuehn (2013). These factors are available on Martin Lettau and Oliver Boguth’s websites.

### 3.3 Corporate bond-return and factor data

We use Trade Reporting and Compliance Engine (TRACE) transaction data for corporate bonds from August 2002 through June 2017. TRACE provides intraday trading prices, trading volumes, sell and purchase indications, and others for corporate bonds. We adhere to Bai, Bali, and Wen (2019) 's data screening and return estimate methodology. Monthly corporate bond returns are determined by dividing the average quoted price at the end of the current month, accumulated interest, and coupon payment for a month by the average quoted price at the end of the preceding month or the beginning of the current month. The excess return of a bond is the difference between its calculated total return and the risk-free rate, which is represented by the 1-month Treasury bill rate. Our final sample consists of 331,728 observations, and the average cross-sectional excess return is 0.389 percent, similar to Bai, Bali, and Wen (2019). Only bonds with at least 30 consecutive monthly returns are included in our final selection of 6,421.

We first evaluate four non-tradable macroeconomic indicators regarding bond variables before adding the default spread, term spread, and corporate bond market return. The default spread is the difference in return between Moody's BAA-rated and AAA-rated long-term corporate bonds. Spread refers to the difference in yield between 10- and 1-year Treasury bonds. These time series are sourced from Professor Robert Shiller’s website. The monthly market return for corporate bonds is the evenly weighted average of the sample's corporate bond returns.

## 4. Applying FMPs in a Test of the Asset Pricing Model

This section applies FMPs constructed by various methods to test whether the observed factors are risk related and whether they can price assets. We focus on the FMPs of the four macro factors and augment this analysis with traded factors (the FF three factors) to allow for comparison.[8] We examine the two criteria suggested by Pukthuanthong, Roll, and Subrahmanyam (2019) (PRS criteria):

(1) FMPs of macro factors should be correlated with the systematic risk of returns;

(2) FMPs of factors should command the risk premium.

**4.1 First criterion: Correlation of FMPs with the systematic risk of returns**

If the FMP of an observed factor represents a risk factor, it should be related to the systematic risk of returns. Following Pukthuanthong, Roll, and Subrahmanyam (2019), we test whether the FMP is related to the cross-sectional covariance of asset returns. Specifically, we first apply the asymptotic approach of Connor and Korajczyk (1988) (CK) to extract ten principal components from the equities return series. The principal components of the covariance matrix of returns represent the systematic part of the asset returns. We then compute canonical correlations between 10 CK principal components and the factor candidates and test the significance of these canonical correlations.

To illustrate, we first collect a set of $N$ equities for the factor candidates. The test assets should be from different industries and have enough heterogeneity to detect the underlying risk premium associated with factors. Second, we use the CK approach to extract $L$ principal components (PCs) from the return series. With $T$ time-series units up to time $t$, we compute the $T \times T$ matrix $\boldsymbol{\Omega_t} = \frac{\boldsymbol{1}}{\boldsymbol{T}}\ \boldsymbol{RR'}$, where $\mathbf{R}$ is the return vector. CK demonstrate for large $N$, analyzing the eigenvectors of $\boldsymbol{\Omega_t}$is asymptotically equivalent to factor analysis. The first $L$ eigenvectors of $\boldsymbol{\Omega_t}$ form the factor estimates. The cutoff point for $L < N$ is chosen so that the PCs explain at least 90% of the cumulative variance. Third, we collect a set of *K-factor* candidates. Our study includes four macroeconomic factors (CG, CPI, IP, and UE) and three Fama and French factors (MKT, SMB, and HML).

Finally, from the second step above, we compute the canonical correlation between the factor candidates and the corresponding eigenvectors. First, we use the $L$ eigenvectors from step 2 and the $K$ factor

[8] We also test for robustness by using alternative traded factors, such as the Carhart four factors, the FF five factors, and the FF six factors, and find these results are robust to these specifications. In some cases, we must drop the market factor to achieve robust results because of the strong correlation between the FMP of consumption growth and the market factors. Theoretically speaking, consumption growth and the market factor should present a similar risk.

candidates from step 3 and calculate the covariance matrix over a sample period *t*, $\mathbf{V_t}$ **(L+K x L+K)**. From the covariance matrix $\mathbf{V_t}$ in each period, we break out a submatrix, the cross-covariance matrix, denoted by $\mathbf{C_t}$ having *K* rows and *L* columns. The entry in the i$^{th}$ row and j$^{th}$column being the covariance between factor candidate i and eigenvector j. We need to break out the covariance submatrix of the factor candidates, $\mathbf{V_{f,t}}$ ($K \times K$) and the covariance submatrix of the real eigenvectors, $\mathbf{V_{e,t}}$ ($L \times L$). We then can find two weighting column vectors, $\lambda_t$ and $\kappa_t$ on the factor candidates and eigenventures, respectively ($\lambda_t$ has *K* rows, $\kappa_t$ has *L* rows), that maximize the correlation between the two weighted vectors. The covariance between the weighted averages of factor candidates and eigenvectors is $\lambda_t' C_t \kappa_t$, and their correlation is

$$\rho = \frac{\lambda_t' C_t \kappa_t}{\sqrt{\lambda_t' V_{f,t} \lambda_t \kappa_t' V_{e,t} \kappa_t}}. \tag{13}$$

We maximize the correlation across all choices of $\lambda_t$and $\kappa_t$. The maximum exists when the weight is $\lambda_t = V_{f,t}^{-1/2} h_t$, where $h_t$ is the eigenvector corresponding to the maximum eigenvalue in the matrix $V_{f,t}^{-1/2} C_t V_{e,t}^{-1} C_t' V_{f,t}^{-1/2}$. $\kappa_t$ is proportional to $h_t$.

We maximize the correlation again, subject to the constraint that the new vectors are orthogonal to the old one, and so on. As a result, there are min(L,K) pairs of orthogonal canonical variables sorted from the highest correlation to the smallest. We transform each correlation into a variable asymptotically distributed as Chi-Square under the null hypothesis that the actual correlation is zero. Chi-Square provides a method of testing whether the factor candidates as a group are conditionally related (on date t) to the covariance matrix of actual returns). Also, by examining the relative sizes of the weightings in λt, we can understand which factor candidates are more related to real return covariances. The intuition behind the canonical correlation approach is that the proper underlying drivers of actual returns are undoubtedly changes in perceptions about macroeconomic variables. The factor candidates and the eigenvectors need not be isomorphic to a particular macro variable. Instead, each candidate or eigenvector is some linear combination of all the pertinent macro variables, the well-known "rotation" problem in principal components or factor analysis. PRS criteria assert that some linear combinations of the factor candidates are strongly related to different linear combinations of the eigenvectors representing the actual factors. Canonical correlation is intended for precisely this application. Any factor candidate that does not display a significant (canonical) correlation with its associated best linear combination of eigenvectors can be rejected as a viable factor. It is not significantly associated with the covariance matrix of actual asset returns.

We compute asymptotic PCs that represent the covariance matrix. We split the overall sample into five subsamples with ten years each for the first four subperiods and 13 years in the last subperiod to consider nonstationary.[9] For each subsample, we use CK to extract 10 PCs. We retain the first 10 PCs, which account for nearly 90% of the cumulative eigenvalues or the total volatility in the covariance matrix, implying they capture most of the stock variations.

Next, we proceed to estimate the canonical correlations. We have several factor candidates and, thus several pairs of canonical variates. We take the following steps to derive the significance levels of each factor candidate reported in the first row of Table 2. First, for each of the seven canonical pairs, the eigenvector weights for the 10 CK PCs are taken, and the weighted average CK PC or the canonical variate for the 10 CK PCs that produced the canonical correlation for this particular pair is constructed.[10] Then a regression using each CK PC canonical variate as the dependent variable and the actual candidate factor values as independent variables are run over the sample months in each subperiod. The square root of the $R$-squared from the regression is the canonical correlation. After proper normalization, the coefficients for the regressions are equal to the eigenvector's weighting elements for the candidate factors. The $t$-statistic from the regression then gives the significance level of each candidate factor. With the seven pairs of canonical variates in each subperiod, and a canonical correlation for each one, we have a total of 35 regressions. The first row presents the mean $t$-statistic of all canonical correlations. The second row shows the mean $t$-statistic across cases when the canonical correlation is statistically significant. The third row shows the average number of significant canonical correlations across subperiods.

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[9] These five subsamples are 1964–1973, 1974–1983, 1984–1993, 1994–2003, and 2004–2016.
[10] There are min $(L, K)$ possible pairs. In our application, $L = 10$ and $K = 7$.

A risk factor must satisfy two conditions: (1) the FMP is significantly related to any canonical variate in all decades, or it has a mean *t*-statistic exceeding the one-tailed 2.5% cutoff based on the chi-squared value, and (2) in each sub-period, if the FMP has an average number of significant *t*-statistics exceeding 1.75 (the bottom row of each panel).[11] We examine this criterion for the FMPs constructed using various methods. Table 2 presents the results.

[Table 2 about here]

Notably, the four underlying macro factors do not pass, whereas the three FF factors pass this criterion. FMPs constructed by all the cross-sectional and sorted beta methods satisfy this criterion. For FMPs built by the time-series method, IP and the unemployment rate do not pass this criterion.

### 4.2 Second criterion: Risk premium estimation using FMPs

This section evaluates the degree to which different FMP construction approaches provide precise risk premium estimates. To prevent the EIV bias of beta loadings in the risk premium estimate step, we use the IV approach developed by Jegadeesh et al. (2019) to FMPs created using various techniques. Following the methodology of Jegadeesh et al. (2019), we additionally winsorize the most extreme estimated coefficients from Fama-MacBeth cross-sectional regressions at the 1.5% and 98.5% levels.[12]

Recent research indicates that non-tradeable factors may be subject to high measurement errors. Kleibergen (2009) demonstrates that when measurement error predominates, the t-ratio distribution under the null hypothesis might differ from a regular normal distribution; hence, the usual Fama-Macbeth test cannot be employed. In addition, the FMP's construction error impacts the asset price test. We examine these concerns in Section 6 and estimate the upper bound of measurement error for macro factors. We

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[11] Pukthuanthong, Roll, and Subrahmanyam (2019) require an average number of significant decade *t*-statistics exceeding 2.5 from 10 canonical variates (one-fourth of the total number of canonical variates). We have seven factor candidates; thus, 1.75 come from using the same proportion as theirs. The reason to choose this value comes from Pukthuanthong, Roll, and Subrahmanyam (2019): "This is a conservative threshold to ensure we do not miss a true factor at our necessary condition stage. We focus on the significant canonical correlations, rather than all canonical correlations, because insignificant CCs imply that none of the factors matter, so using them would be over-fitting."

[12] For the robust check, we also apply the classical Fama-MacBeth OLS regression. We find similar results, though the estimated risk premiums are smaller.

demonstrate via simulations that we can apply usual statistical inferences to the macro factors even when the measurement error is at its upper bound and there is a construction error in the FMP.

**4.2.1 Cross-sectional approaches.**

Panel A of Table 3 presents the risk premium estimates derived from 4-ADJ FMPs and FMPs derived from various cross-sectional methodologies (OLS, LM, and Stein methods).

[Table 3 about here]

The findings demonstrate that risk premiums assessed by various methodologies often vary significantly in quantity and significance. When adjusting for the Fama-French three factors, we find that the risk premium for consumption growth is -0.057 (t-value = -1.110) when using the OLS approach but 0.136 (t-value = 2.070) when using the 4-ADJ method. The consumption growth risk premiums for the LM and Stein techniques are less than those for the 4-ADJ approach and are likewise negligible. With the OLS and Stein techniques, the estimated risk premium for industrial output is negative, contrary to the theoretical prediction and in stark contrast to the 4-ADJ method predictions. Using the 4-ADJ technique, we establish a negative risk premium for the unemployment rate. Negative unemployment beta equities are riskier since their returns decline during high unemployment. Therefore, in an economy with a high employment rate, equities with a positive unemployment beta mitigate risk. Moreover, the CPI has a substantial negative risk premium for all cross-sectional techniques. The risk premiums for the three Fama and French variables, whose signs and significance levels are used to estimate risk premiums, do not differ much among estimation techniques.

In conclusion, when the 4-ADJ approach is used with controls, CG, CPI, and unemployment rate are related to substantial risk premiums. Table 3 also includes risk premium estimates that do not account for the three Fama-French factors. Nonetheless, these conclusions may be subject to model misspecification without Fama-French variables. As shown by Giglio and Xiu (2021), Kan, Robotti, and Shanken (2013), and Jiang, Kan, and Zhan (2015), the test for macro factors may be deceptive when there is misspecification. Consequently, the results with control findings are more reliable. We present two robustness tests in Section 4.4 to address the misspecification problem.

#### 4.2.2 Time-series approach.

We calculate the risk premium using Lamont (2001)-constructed FMPs. Eight industry-sorted stock portfolios, four bond portfolios, and a stock market return are utilized to produce FMPs.[13] The predicted risk premium for consumption growth is negative and statistically significant compared to the IV and OLS techniques, contradicting the idea that a positive premium should be related to consumption growth. In addition, the estimated risk premiums for all macro parameters, except consumption growth, are small.

#### 4.2.3 Sorting-by-beta approaches.

We estimate betas using time-series multivariate regressions and then sort equities into ten deciles based on the estimated beta. FMPs are derived from the differential in equally weighted portfolio returns between the top and lowest deciles of stocks (high-minus-low). FMPs are used to estimate the risk premiums shown in Table 3, panel A. The projected risk premium is only significant for CPI when we include the three FF factors. CG and IP are only significant when FF variables are excluded. As previously noted, the model without FF factors is susceptible to model misspecification.

### 4.3 Weak IV issue and stock selection in the 4_ADJ approach

As noted in Section 2.3, if the returns of certain stocks have minimal correlation with macro factors, the correlation between endogenous variables (EV betas) and instrumental variables (IV betas) may be relatively low, known as the IV weakness problem. To address this problem, we only choose stocks whose EV betas match the sign of their respective IV betas. On average, 54% of the stocks are used for each factor, and only 6.5% are utilized for all factors. For example, 6,319 of the 10,833 stocks are used to develop FMPs for consumption growth. The stocks used for the CPI, industrial production, and unemployment rate are 6,240, 4,288, and 6,501.

Table 4 displays the correlation between EV and IV betas for the whole sample (including all stocks) and our adjusted sample (with stock selection).

[13] The eight value-weighted industry portfolios sorted by SIC code include basic industries, capital goods, construction, consumer goods, energy, finance, transportation, and utilities. The four bond portfolios include long-term (10-year) government bonds, intermediate-term (5-year) government bonds, one-year government bonds, and low-grade corporate bonds. The stock market portfolio is the market factor from Fama and French (1993).

[Table 4 about here]

When generating FMPs using total and adjusted samples, we first analyze the correlations between the EV and IV betas (panel A of Table 4). We can observe that the correlations between the EV and IV betas for macro variables are pretty low for all stocks. Even more unfortunate, three of the four have negative absolute values. The IVs seem to be relatively weak in this instance. Using the sample altered to include only companies with EV and IV betas with the same sign enhances the correlation considerably. (For example, the correlation between the EV and IV betas for the FMP of consumption growth rises to 0.477)

Panel B of Table 4 highlights the advantages of mitigating the weak instrument issue on estimating risk premiums following the construction of FMPs. Risk premiums are calculated using the Fama-MacBeth regression with instrumental variables method. However, we employ two distinct FMPs: FMPs with the adjusted sample (FMP adj) and FMPs with the complete sample (FMP full). We derive EV and IV beta using the IV technique by regressing excess stock returns on FMPs in odd and even months. The association between EV and IV betas is seen in Panel B. For FMP full, the signs of the correlations between the EV and IV betas for industrial output (IP) and unemployment rate (UE) are negative. The negative connection demonstrates the significance of asset selection during the FMP construction phase to the risk premium estimate phase. Given the negative correlations between EV and IV betas, calculating the risk premium using the IV method is unreliable in the absence of an asset selection technique in the FMP building process. As seen in panel B, the sample selection method (FMP adj) increases the IV correlation.

### 4.4 Robustness

The risk premium for consumption growth is significant when its associated FMP is used. Nonetheless, the estimated risk premium can be biased (Lewellen, Nagel, and Shanken 2011; Giglio and Xiu 2021), and the t-ratio can be inaccurate (Kan, Robotti, and Shanken 2013; Jiang, Kan, and Zhan 2015) when the model is misspecified. To resolve this problem, we account for the three Fama-French factors in Table 3. In addition to controlling for different combinations of the six Fama-French factors, the findings are consistent in most specifications. In addition, we estimate the risk premiums for our 4-ADJ FMP using the three-pass approach of Giglio and Xiu (2021). In the asset pricing test, the design of the three-pass technique accounts for model misspecification. Internet Appendix C details the findings. Consistent with our primary results in Table 3, we find that the risk premiums for consumption growth and unemployment rate are substantial.

Lettau and Ludvigson (2001) develop a conditional consumer capital asset pricing model (CAPM) that can explain average stock returns in the cross-section with the consumption-to-wealth ratio (CAY). Table 5 displays the anticipated risk premiums when the CAY component is included. In all but two specifications, the CAY factor is always tiny and linked with a negative risk premium. For the 4-ADJ FMP, the consumption growth component has a considerably positive risk premium. This result accords with our results shown in Table 3. Lettau and Ludvigson (2001) incorporate a CAY-consumption growth interaction term; its risk premium is minimal. CG remains significant at 5% for the 4-ADJ technique and 10% for the OLS and sorting-by-beta procedures. CG is positive and significant at the 1% level for the 4-ADJ approach but either negligible or negative for the other techniques when the three FF factors are included.

[Table 5 about here]

Boguth and Kuehn (2013) find that consumption volatility, a proxy for macroeconomic uncertainty, is a source of risk and has a negative risk premium. We also add consumption volatility as a control variable in an unreported table. We find that the risk premium for consumption growth is still significant. Therefore, using 4-ADJ FMPs for consumption growth, we obtain results confirming that consumption growth is a substantial risk factor that can explain the cross-sectional stock returns conditional on other consumption-related factors.

## 5. Using FMPs to Test Risk Premiums in the Corporate Bond Market

Bond yields are influenced by firm fundamentals and the economic cycle (Ludvigson and Ng 2009). Fama and French (1993) suggest two traded bond factors: default spreads and term spreads. Even after adjusting for bond characteristics such as length and rating, Gebhardt, Hvidkjaer, and Swaminathan (2005) show that the default spread predicts cross-sectional bond returns considerably. Bai, Bali, and Wen (2019) discover that value-at-risk and bond ratings influence the default and term spread. According to Bessembinder et al. (2008), atypical bond returns may be explained by a bond market return, unexpected gross domestic product (GDP) growth, and unexpected inflation. Following these publications, we assess the FMPs for four macroeconomic variables, including a broad bond market return (MKT B), the default spread (DS), and a term spread (TS).

We use many FMP building techniques, including 4-ADJ, OLS, LM, Stein, time series, and sorting by beta. Following the criteria established by Pukthuanthong, Roll, and Subrahmanyam (2019), we first examine the relationship between FMPs and the covariance matrix of individual corporate bond returns. Our bond sample data spans around 15 years, from July 2002 to July 2017. Because not all bonds contain complete data, we divided the sample period into three equal subperiods. Table 6 displays the findings in Panel A. All FMPs constructed by methods other than the time-series technique satisfy this requirement. Only consumption growth, CPI, IP, MKT, and DS pass the time-series process. Except for IP, none of the macro factors pass without FMP construction.

Panel B of Table 6 displays the results of measuring the risk premium for corporate bond returns like that used for stock returns. Consumption growth and industrial output carry substantial risk premiums for 4-ADJ FMPs. The risk premiums associated with time-series Stein, LM, OLS, time-series, and sorting-by-beta FMPs are negligible.

[Table 6 about here]

## 6. Simulation of the risk-premium estimation by using FMPs

### 6.1. Simulation procedure

This appendix presents simulations that compare various FMP construction methods. The purpose is to study the magnitude of biases in risk-premium estimations due to the measurement error of beta loadings, as well as other statistical properties in finite samples. The finite-sample properties of risk-premium estimation for traded factors are studied by Jegadeesh et al. (2019). However, the nontradable factors are possibly associated with higher estimation errors in factor loadings, which elicit larger finite sample errors-in-variables bias (Kleibergen 2009). Therefore, it is necessary to reexamine these properties for nontradable factors.

Our simulation parameters match the attributes of the real data. We use individual stock-returns data from February 1964 to March 2016, covering 626 months and 10,833 stocks. For instance, most stocks do not have data spanning the entire sample period; hence, our simulated stocks have data only in the same periods as their corresponding stocks in the real data.[14]

The simulation procedure is as follows.

Step 1: Regress excess stock returns on factors and obtain estimated betas ($\boldsymbol{\beta}$) and residuals ($\boldsymbol{\varepsilon}$) for each stock. $\boldsymbol{\beta}$ is an $N$ by $K$ matrix, and $\boldsymbol{\varepsilon}$ is an $N$ by $T$ matrix, where $N$ is the number of stocks, and $T$ is the number of periods. Since the existing periods for most stocks are less than $T$, the matrix $\boldsymbol{\varepsilon}$ is not a balanced panel, in which we define the value as missing if one stock does not have return data on the corresponding period.

Step 2: In each simulation, create a $T \times 1$ vector $\mathbf{S}$ by randomly selecting $T$ numbers with replacement from 1 to $T$, where $T$ is the maximum month number. Then create simulated factors by rearranging observed factors to match the randomly chosen observation number in the vector $\mathbf{S}$. Finally, augment the simulated factors by adding prespecified true premiums $\lambda_0$ set equal to the observed average risk premiums from Chen, Roll, and Ross (1986) and Chen and Kan (2003).

[14] We also conduct simulations in which all simulated stocks have data in every period (unreported). This leads to less overall bias, because the number of months in first-pass time-series regressions and the number of stocks in second-pass cross-sectional regressions are much larger. However, the overall conclusions comparing FMP methods remain the same.

Step 3: Generate simulated residuals that are randomly selected and normally distributed, for which the mean and variance are equal to the sample mean and variance of the observed actual residuals for the stock.

Step 4: Construct simulated returns for each stock in each month as estimated betas multiplied by simulated factors plus simulated residuals.

Step 5: Apply the methods in Section 2 of the paper to construct FMPs using simulated returns and simulated factors.

Step 6: After construction of the FMP, reapply various Fama-Macbeth two-pass methods (OLS, 4_ADJ, and Stein with individual stocks) to estimate risk premiums using simulated returns and the FMPs, thereby obtaining simulated estimates of FMP-based risk premiums.

Notably, all simulations are multivariate, except 4_ADJ. For the 4_ADJ, we use a univariate regression to create FMP due to the condition of $\hat{\beta}_{IV}{}'\hat{\beta}_{EV} > 0$. To estimate risk premiums, we use multivariate regression. That is, we create FMPs for each factor independently. The second step is to test risk premiums for the FMP, where we use multivariate regression to estimate risk premiums of all FMPs.

Repeat these six steps 1,000 times. Then calculate the mean difference between simulated estimates of FMP-based risk premiums and the true simulated risk premiums (which is the ex-ante bias). This mean difference is the predicted bias of each FMP method.[15]

**6.2. Simulation results: Bias**

Table 7 presents the simulation results. Following Chen, Roll, and Ross (1986), we use four macro factors as risk factors: changes in the consumption growth rate (CG), changes in the Consumer Price Index (CPI), changes in industrial production (IP), and changes in the unemployment rate (UE). We assume that beta is

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[15] In unreported results, we also compute an ex-post bias as the differences between estimated risk premiums and the sample mean of the corresponding factor realizations in that particular simulation. We find the difference between ex-post bias and ex-ante bias is minimal; thus we provide only ex-ante bias in Table 2 in the paper.

constant for each stock. In the first stage, we apply the univariate method to construct each FMP; then, in the second stage, we apply a multivariate approach to estimate the risk premium.

Consistent with our expectations, the 4_ADJ approach resolves the EIV bias in a two-pass regression and produces nearly unbiased risk premiums. The alpha in the 4_ADJ estimation is 0.0055%, which is close to 0. The differences between the estimated risk premiums and true risk premiums are minimal. For instance, the estimated risk premium for consumption growth is just 0.2% larger than its true risk premium.

[Table 7 around here]

The OLS method produces the estimated risk premiums that are much smaller than the true risk premiums, which is consistent with our conjecture that the OLS is subject to downward bias caused by measurement errors in estimated betas. The bias ranges from 28.7% for consumption growth to 50% for the unemployment rate. The mispricing part (intercept term) is 0.1412%, which is much larger than for 4_ADJ. The alpha from the sorting-by-beta method is even larger than OLS, and the risk premium bias is large as well. For example, the bias for consumption growth is 31%. The biases for the Lehmann and Modest (LM) method is as large as that for OLS. Furthermore, the mispricing in the LM method is 32%, even larger than the mispricing of OLS. Stein's method yields less bias than the OLS and LM methods, but the biases in Stein's method are still large, over 20%.

We also consider the time-series approach. Specifically, we present the time series approach using Lamont's (2001) portfolios as basis assets, which is constructed by regressing risk factors on a series of basis portfolio returns, and then calculating predicted values as FMPs. Then we estimate the risk premiums for the FMPs constructed by the time-series approach (TS_FMPs). Our simulation in Panel B of Table 7 shows that the average bias for the time-series approach is larger than 40%.

**6.3 Simulation results: Root-mean-square errors**

We also calculate the root-mean-square errors (RMSEs) for the simulations. The error is simply the difference between the true risk premium and the risk premium estimated for each simulated replication, of which there are 1,000. We call this the ex-ante RMSE. We also calculate the difference between the estimated risk premium in each replication and the sample mean of the corresponding risk-factor realizations and then compute its root mean square across 1,000 replications. This is the ex-post RMSE. These RMSEs are reported in Panel A of Table 8. Since the ex-ante and ex-post RMSEs are quite similar, we focus on the former.

The ex-ante RMSEs with 4_ADJ are uniformly smaller than with OLS; i.e., 4_ADJ is considerably more accurate. For example, the CPI ex-ante RMSE is 0.082 for OLS, but just 0.026 for 4_ADJ. Stein has smaller RMSE than OLS, but it is still larger than that of 4_ADJ. OLS has the largest RMSE. LM has marginally smaller RMSE than OLS. Overall, the 4_ADJ method dominates all other methods by the RMSE criterion.

[Table 8 about here]

### 6.4. The size and power of *t*-tests

To estimate the size and power of *t*-tests for risk premiums with the 4_ADJ approach, we first consider the probability of rejecting the null hypotheses falsely (i.e., the type-I error). In the simulations, we set the true risk premium equal to 0 for all factors. Then we follow the simulation procedure to estimate risk premiums and their corresponding *t*-statistic (i.e., the mean estimated risk premium divided by its corresponding standard error). We use a 5% significance level (the critical value is 1.96) and calculate the frequency of the absolute value of the *t*-statistic that is larger than 1.96 in 1,000 replications. Panel B of Table 8 reports that the size of each macro factor is around 5% or slightly below.

To examine the *t*-test power (the probability of rejecting the null hypothesis when the alternative hypothesis is true), we set the true premium to the mean-risk premium estimated from a Fama-Macbeth two-pass regression with OLS. This is a relatively small risk premium (e.g., compared to that obtained with 4_ADJ), thus implying a more conservative threshold for power. Panel B in Table 8 presents the results. The frequency of rejecting an incorrect null hypothesis that the risk premium is 70.9% for consumption growth and a bit higher for the other three macro factors. Overall, size and power tests indicate that a normal *t*-statistic delivers effective inferences about macro factor premiums.

## 7. Conclusion

How to generate FMPs as the traded versions of nontradable variables to use them to estimate risk premiums is the subject of considerable research. Our study examines several existing FMP methods and propose a new adjusted method. The simulation results demonstrate that our technique can yield trustworthy statistical judgments.

Empirically, we evaluate the macro variables with our approach based on Pukthuanthong, Roll, and Subrahmanyam's criteria (2019). We find that our FMPs are risk related. Consumption growth, CPI, and unemployment rate have significant risk premiums for the stock market, while consumption growth and industrial production have significant risk premiums for the corporate bond market when we apply our method to construct FMPs for four classical macroeconomic factors. Other methodologies in the literature have not been able to get similar results. Our strategy generally explores new ground in evaluating asset pricing models for nontradable variables via factor-mimicking portfolios.

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**Table 1. Descriptive statistics**

This table reports the summary statistics for the stock and bond returns, as well as macro variables and traded factors. The summary statistics include the number of observations, mean, median, standard deviation, and (1st, 5th, 25th, 75th, 95th, and 99th) percentiles. Panel A reports the statistics for excess stock returns and their explanatory variables. For stock returns, we have 10,833 stocks in total and 626 months of data. Stock returns are over a risk-free rate (1-month Treasury bill rate). The macro variables include the consumption growth rate (CG), the consumer price index (CPI), industrial production (IP), the unemployment rate (UE), and the consumption-to-wealth ratio (CAY). The traded factors include the excess stock market return (MKT), a small-minus-big size portfolio (SMB), and a high-minus-low book-to-market portfolio (HML). Panel B lists the statistics for corporate bond returns and related macro variables and factors. We have 6,421 bonds and 179 months of data for corporate bond returns. The bond returns are over a risk-free rate (1-month Treasury bill rate). In addition to the four macro variables (CG, CPI, IP, and UE), the bond market excess return (MKT_bond), default spread (DS), and term spread (TS) are taken as factors. MKT_B is the equally weighted return of all corporate bond returns in our sample over the risk-free rate. DS is a default spread, measured by the return difference between Moody's long-term corporate BAA-rated and AAA-rated bonds. TS is a term spread measured by the return difference between 10-year Treasury bonds and 1-year Treasury bonds. Section 3 describes the sources of these data in detail.

*A. Statistics for stock returns and their explanatory variables*

| | *N* | Mean | Median | SD | 1st | 25th | 75th | 99th |
|---|---|---|---|---|---|---|---|---|
| Stock return | 1,784,351 | 1.012 | 0.440 | 13.341 | –31.764 | –5.194 | 6.209 | 42.179 |
| CG | 626 | 0.470 | 0.437 | 0.545 | -1.091 | -0.314 | 0.166 | 0.780 |
| CPI | 626 | 0.327 | 0.283 | 0.323 | -0.494 | -0.095 | 0.162 | 0.485 |
| IP | 626 | 0.206 | 0.258 | 0.742 | -2.333 | -0.887 | -0.172 | 0.635 |
| UE | 626 | 6.105 | 5.800 | 1.637 | 3.400 | 3.800 | 5.000 | 7.200 |
| MKT | 626 | 0.490 | 0.785 | 4.466 | –11.804 | –2.100 | 3.450 | 11.178 |
| SMB | 626 | 0.229 | 0.130 | 3.108 | –6.695 | –1.520 | 2.050 | 8.435 |
| HML | 626 | 0.349 | 0.310 | 2.819 | –8.097 | –1.160 | 1.710 | 7.930 |
| CAY | 626 | –0.002 | –0.002 | 0.021 | –0.046 | –0.015 | 0.015 | 0.034 |

*B. Statistics for bond returns and its explanatory variables*

| | *N* | Mean | Median | SD | 1st | 25th | 75th | 99th |
|---|---|---|---|---|---|---|---|---|
| Bond return | 331,728 | 0.389 | 0.236 | 2.544 | –5.764 | –0.349 | 1.089 | 7.248 |
| CG | 179 | 0.257 | 0.276 | 0.368 | -1.132 | -0.256 | 0.107 | 0.478 |
| CPI | 179 | 0.171 | 0.186 | 0.317 | -0.849 | -0.325 | 0.040 | 0.321 |
| IP | 179 | 0.066 | 0.134 | 0.694 | -2.763 | -0.863 | -0.221 | 0.452 |
| UE | 179 | 6.427 | 5.800 | 1.749 | 4.329 | 4.500 | 5.000 | 7.875 |
| MKT_bond | 179 | 0.380 | 0.390 | 1.875 | –6.031 | –0.367 | 1.094 | 7.276 |
| DS | 179 | 1.093 | 0.960 | 0.469 | 0.579 | 0.853 | 1.218 | 3.090 |
| TS | 179 | 1.805 | 1.870 | 1.003 | –0.374 | 1.235 | 2.590 | 3.368 |

**Table 2. Correlation with the systematic risk of equity returns**

This table reports the canonical correlations between FMPs and the principal components (PCs) of the covariance matrix of individual stocks. The factor candidates include FMPs constructed using the 4-ADJ, OLS, Lehmann and Modest (1988), Stein (1956), and Lamont (2001) methods. As a comparison, we also include the canonical correlation results for the original risk factors. The PCs of the stock returns are extracted, as explained in Pukthuanthong, Roll, and Subrahmanyam (2019), using the Connor and Korajczyk (1988) method. We summarize the significance levels of factor candidates. The following procedure is implemented to derive the significance levels of each factor candidate: First, for each canonical pair, we take the eigenvector weights for the 10 PCs and constrain the weighted average PC (which is the canonical variate for the 10 PCs that produce the canonical correlation for this particular pair). Then a regression using each CK PC canonical variate as the dependent variable and the candidate factor realizations as independent variables is run over the sample months, 120 months for the first four decades and 156 for the last subperiod of 13 years. The square root of the $R$-squared from this regression (not the adjusted $R$-squared) is the canonical correlation. The regression coefficients are equal (after proper normalization) to the eigenvector's factors for the candidate factor. The $t$-statistics from the regression then give the significance level of each candidate factor. Each decade has seven pairs of canonical variates and a canonical correlation for each; thus, we have a total of 35 regressions (7 regressions per subperiod). The first row (Avg $t$) presents the mean $t$-statistic over all canonical correlations. The second row (Avg $t$ sig. CC) reports the mean $t$-statistic when the canonical correlation is statistically significant. The third row (Avg # sig. CC) reports the average number of significant canonical correlations over the five subperiods. The critical rejection levels for the $t$-statistics are 1.65 (10%), 1.96 (5%), and 2.59 (1%). We assume that an FMP or factor is related to the risk of asset returns if it (1) is significantly related to any canonical variate in all decades or has a mean $t$-statistic in the second row that exceeds the one-tailed, 2.5% cutoff based on the chi-squared value, and (2) has an average number of significant canonical variate pairs exceeding 1.75 (the third row of each panel).

| | FMP | | | | Equity factors | | |
|---|---|---|---|---|---|---|---|
| | CG | CPI | IP | UE | MKT | SMB | HML |
| | | | | Original | | | |
| Avg $t$ | 1.14 | 1.15 | 1.10 | 1.01 | 10.66 | 6.70 | 3.31 |
| Avg $t$ (sig. CC) | 1.25 | 1.37 | 1.04 | 1.14 | 22.80 | 14.19 | 6.87 |
| Avg # sig. CC | 1.40 | 1.40 | 1.40 | 0.60 | 2.80 | 2.80 | 2.60 |
| | | | | FMP_4-ADJ | | | |
| Avg $t$ | 2.43 | 2.57 | 1.89 | 2.22 | 9.71 | 5.00 | 3.30 |
| Avg $t$ (sig. CC) | 3.00 | 3.27 | 2.10 | 2.57 | 13.41 | 6.49 | 4.25 |
| Avg # sig. CC | 3.00 | 3.20 | 2.20 | 2.80 | 3.40 | 3.80 | 3.20 |
| | | | | FMP_OLS | | | |
| Avg $t$ | 2.50 | 2.53 | 2.05 | 2.66 | 10.07 | 5.45 | 3.42 |
| Avg $t$ (sig. CC) | 3.01 | 3.12 | 2.41 | 3.39 | 13.80 | 7.18 | 4.46 |
| Avg # sig. CC | 2.80 | 3.00 | 2.80 | 3.20 | 3.20 | 4.00 | 4.20 |
| | | | | FMP_LM | | | |
| Avg $t$ | 2.28 | 2.52 | 2.07 | 1.93 | 3.35 | 3.77 | 3.03 |
| Avg $t$ (sig. CC) | 2.72 | 3.13 | 2.28 | 2.30 | 4.21 | 4.79 | 3.97 |
| Avg # sig. CC | 3.20 | 3.20 | 3.20 | 1.80 | 3.00 | 4.40 | 3.20 |
| | | | | FMP_Stein | | | |
| Avg $t$ | 2.50 | 2.53 | 2.05 | 2.66 | 10.07 | 5.45 | 3.42 |
| Avg $t$ (sig. CC) | 3.01 | 3.12 | 2.41 | 3.39 | 13.80 | 7.18 | 4.46 |
| Avg # sig. CC | 3.40 | 3.40 | 3.60 | 3.00 | 3.00 | 4.00 | 3.00 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | FMP_time series | | | | |
| Avg *t* | 1.39 | 1.57 | 1.37 | 1.40 | 9.79 | 6.74 | 3.25 |
| Avg *t* (sig. CC) | 2.01 | 2.09 | 1.63 | 1.70 | 19.45 | 13.17 | 6.09 |
| Avg # sig. CC | 2.00 | 1.80 | 1.80 | 1.80 | 3.40 | 3.20 | 3.60 |
| | | | FMP_SB | | | | |
| Avg *t* | 2.01 | 2.80 | 2.16 | 2.15 | 6.54 | 5.42 | 5.23 |
| Avg *t* (sig. CC) | 2.21 | 3.20 | 2.38 | 2.36 | 8.07 | 6.68 | 6.46 |
| Avg # sig. CC | 2.40 | 4.00 | 3.20 | 3.20 | 3.40 | 3.60 | 3.40 |

**Table 3. Risk premiums in the equity market using FMPs**

Panel A reports the risk premium estimates after FMPs are constructed. 4-ADJ, OLLS, LM, Stein, time-series, and sorting-by-beta approaches construct FMPs included in this table. See Section 1 for details. For all FMP approaches, we apply a multivariate IV regression (Jegadeesh et al. 2019) to estimate the risk premium. The sample period is January 1964 to March 2016. To be included, an individual stock must have at least 60 contiguous monthly returns in CRSP. The risk factors include four macroeconomic variables, the consumption growth rate (CG), unexpected CPI changes (CPI), unexpected changes in industrial production (IP), and unexpected changes in the unemployment rate (UE). MKT is the excess market return (the value-weighted return of all CRSP firms in the United States); SMB is the Fama-French small-minus-big size factor, and HML is the Fama-French high-minus-low book-to-market factor. *t-statistics* appear in parentheses. **p* < 0.1; ***p* < 0.05; ****p* < 0.01.

*A. Risk premium estimation using FMPs*

| | Intercept | CG | CPI | IP | UE | MKT | SMB | HML |
|---|---|---|---|---|---|---|---|---|
| | | | | Cross-sectional approach | | | | |
| 4-ADJ | 0.464*** | 0.164*** | –0.017 | 0.009 | –0.022** | | | |
| | (3.980) | (3.238) | (–0.920) | (0.192) | (–2.000) | | | |
| OLS | 0.617*** | 0.203*** | -0.068** | -0.089 | -0.078 | | | |
| | (4.907) | (2.843) | (-2.371) | (-0.660) | (-1.352) | | | |
| LM | 0.553*** | 0.103*** | -0.034** | -0.02 | -0.027** | | | |
| | (4.944) | (2.586) | (-2.341) | (-0.508) | (-2.291) | | | |
| Stein | 0.617*** | 0.122*** | -0.053** | -0.037 | -0.044 | | | |
| | (4.907) | (2.843) | (-2.371) | (-0.660) | (-1.352) | | | |
| 4-ADJ | 0.515*** | | | | | 0.553** | 0.281* | –0.474*** |
| | (4.805) | | | | | (2.281) | (1.816) | (–3.411) |
| OLS | 0.426*** | | | | | 0.599*** | 0.06 | -0.409*** |
| | (4.306) | | | | | (3.079) | (0.450) | (-2.806) |
| LM | 0.342*** | | | | | 0.723*** | 0.019 | -0.390*** |
| | (3.430) | | | | | (3.651) | (0.157) | (-2.958) |
| Stein | 0.426*** | | | | | 0.598*** | 0.054 | -0.405*** |
| | (4.306) | | | | | (3.079) | (0.450) | (-2.806) |
| 4-ADJ | 0.653*** | 0.136** | –0.048** | 0.094 | –0.028** | 0.545** | 0.262* | –0.487*** |
| | (5.985) | (2.070) | (–2.132) | (1.606) | (–2.134) | (2.269) | (1.835) | (–3.766) |

| | Intercept | CG | CPI | IP | UE | MKT | SMB | HML |
|---|---|---|---|---|---|---|---|---|
| | | | | Cross-sectional approach | | | | |
| OLS | 0.424*** | -0.057 | -0.039** | -0.058 | 0.006 | 0.732*** | 0.062 | -0.387*** |
| | (3.833) | (-1.110) | (-2.276) | (-0.999) | (0.440) | (3.709) | (0.506) | (-3.084) |
| LM | 0.213 | -0.232** | -0.046** | 0.072 | -0.015 | 0.923*** | 0.009 | -0.486*** |
| | (1.335) | (-1.985) | (-2.140) | (0.973) | (-0.670) | (4.068) | (0.061) | (-3.042) |
| Stein | 0.424*** | -0.281 | -0.092** | -0.204 | 0.015 | 0.736*** | 0.063 | -0.446*** |
| | (3.833) | (-1.110) | (-2.276) | (-0.999) | (0.440) | (3.709) | (0.506) | (-3.084) |
| | | | | Time-series approach | | | | |
| | 2.154** | -0.152 | 0.057 | -0.274 | 0.036 | | | |
| | (2.318) | (-1.004) | (0.540) | (-1.059) | (0.790) | | | |
| | -2.129* | | | | | 3.176** | 1.316 | -1.112 |
| | (-1.882) | | | | | (2.466) | (1.361) | (-1.383) |
| | -0.166 | -0.211** | 0.1 | -0.097 | 0.037 | 1.798** | -0.371 | -1.322*** |
| | (-0.223) | (-2.017) | (1.114) | (-0.525) | (1.466) | (2.076) | (-0.783) | (-2.577) |
| | | | | Sorting-by-beta approach | | | | |
| | 0.705*** | 0.879*** | -0.516** | -0.449* | -0.304 | | | |
| | (5.270) | (3.011) | (-2.251) | (-1.692) | (-1.065) | | | |
| | 0.549*** | | | | | 0.717** | 0.253 | -0.605** |
| | (5.382) | | | | | (2.315) | (0.832) | (-2.116) |
| | 0.637*** | 0.253 | -0.843*** | 0.107 | -0.348 | 0.683** | 0.244 | -0.664** |
| | (5.886) | (0.682) | (-3.002) | (0.320) | (-1.111) | (2.181) | (0.779) | (-2.350) |

**Table 4. Strength of the instrumental variables**

This table presents the strength of instrumental variables measured as the correlations between EV betas and IV betas for a sample with and without asset selection. Panel A shows the correlations in the stage of constructing the FMPs. The sample with "EV*IV>0" includes only stocks whose EV beta and IV beta have the same signs (FMP been built by using this sample denoted by "FMP_Adj"). "All" includes all individual stocks (FMP constructed by using this sample indicated by "FMP_full"). Panel B presents the correlations between IV and EV betas for the second stage, where we estimate the risk premiums of FMPs using the IV Fama-Macbeth regression. We offer the cases in which FMP1 and FMP2 from the first stage are used to calculate risk premiums. The sample period is January 1964 to March 2016. To be included, an individual stock must have at least 60 contiguous monthly returns in CRSP. See Table 1 for the notation of the factor candidates we use. We consider three specifications, only four macro factors, only three Fama-French factors, and combined four macro factors and FF three factors.

*A. Correlations between IV and EV betas in the first stage (constructing FMPs)*

| | Sample | CG | CPI | IP | UE | MKT | SMB | HML |
|---|---|---|---|---|---|---|---|---|
| *FMP_adj* | EV*IV>0 | 0.477 | 0.616 | 0.577 | 0.606 | 0.638 | 0.572 | 0.756 |
| *FMP_full* | All | -0.006 | 0.045 | -0.025 | -0.019 | 0.474 | 0.353 | 0.311 |

*B. Correlations between IV and EV betas in the second stage (estimating the risk premiums of FMPs)*

| | FMP | CG | CPI | IP | UE | MKT | SMB | HML |
|---|---|---|---|---|---|---|---|---|
| Macros | *FMP_adj* | 0.455 | 0.266 | 0.252 | 0.229 | | | |
| | *FMP_full* | 0.112 | 0.440 | -0.384 | -0.295 | | | |
| FF3 | *FMP_adj* | | | | | 0.343 | 0.314 | 0.358 |
| | *FMP_full* | | | | | 0.571 | 0.514 | 0.435 |
| Combined | *FMP_adj* | 0.136 | 0.172 | 0.207 | 0.160 | 0.276 | 0.279 | 0.294 |
| | *FMP_full* | 0.007 | 0.081 | -0.147 | -0.018 | 0.473 | 0.386 | 0.305 |

**Table 5. Estimated risk premiums of consumption growth and CAY using FMPs**
This table shows the estimated risk premiums for consumption growth and the CAY factor of Lettau and Ludvigson (2001), with and without controlling for the Fama-French three factors. CAY is the log ratio of aggregate consumption to aggregate wealth. FMPs are constructed for each factor using four methods (4-ADJ, OLS, time series, and sorting by beta). CG is the unexpected consumption growth rate. MKT is the excess market return; SMB is the Fama-French small-minus-big size factor; and HML is the Fama-French high-minus-low book-to-market factor. The monthly sample is from January 1964 to March 2016. *t*-statistics based on Newey-West standard errors appear in parentheses. $^*p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$.

| | Intercept | CAY | CG | CG*CAY | MKT | SMB | HML |
|---|---|---|---|---|---|---|---|
| 4-ADJ | 0.751*** | –0.171* | | | | | |
| | (4.002) | (–1.757) | | | | | |
| OLS | 0.802*** | -0.103* | | | | | |
| | (5.152) | (-1.680) | | | | | |
| Time series | 0.534* | -0.355 | | | | | |
| | (1.861) | (-0.982) | | | | | |
| Sorting by beta | 0.871*** | -0.236 | | | | | |
| | (5.102) | (-1.052) | | | | | |
| 4-ADJ | 0.566*** | | 0.125** | –0.003 | | | |
| | (3.761) | | (2.017) | (–0.022) | | | |
| OLS | 0.764*** | | 0.052 | 0.017 | | | |
| | (3.382) | | (1.179) | (0.189) | | | |
| Time series | 0.145 | | 0.073 | 0.14 | | | |
| | (0.369) | | (1.037) | (1.519) | | | |
| Sorting by beta | 0.800*** | | 0.447* | 0.353 | | | |
| | (5.444) | | (1.654) | (1.147) | | | |
| 4-ADJ | 0.498*** | –0.08 | 0.150** | –0.03 | | | |
| | (3.384) | (–0.920) | (2.391) | (–0.229) | | | |
| OLS | 0.520*** | -0.136** | 0.058* | 0.001 | | | |
| | (4.151) | (-2.139) | (1.811) | (0.013) | | | |
| Time series | 0.214 | -1.625*** | 0.004 | -0.008 | | | |
| | (0.620) | (-3.977) | (0.084) | (-0.124) | | | |
| Sorting by beta | 0.770*** | -0.575*** | 0.502** | -0.114 | | | |
| | (5.358) | (-2.798) | (2.069) | (-0.369) | | | |
| 4-ADJ | 0.584*** | 0.135 | 0.238*** | –0.197 | 0.635** | 0.361** | –0.504*** |
| | (5.452) | (1.145) | (2.692) | (–1.165) | (2.494) | (2.166) | (–3.169) |
| OLS | 0.388*** | -0.161 | -0.086** | -0.055 | 0.668*** | 0.172 | -0.389*** |
| | (3.703) | (-1.570) | (-2.054) | (-0.625) | (3.387) | (1.381) | (-3.157) |
| Times series | -1.347 | 0.049 | -0.07 | 0.168 | 2.804* | -0.741 | 0.393 |
| | (-1.089) | (0.044) | (-0.480) | (1.096) | (1.818) | (-0.690) | (0.800) |
| Sorting by beta | 0.812*** | -1.919 | -0.813 | 1.092 | 0.897 | -0.11 | -0.339 |
| | (4.344) | (-1.350) | (-0.661) | (0.879) | (0.904) | (-0.061) | (-0.416) |

**Table 6. Testing corporate bond factors using corporate bond returns**

This table examines corporate bond factors. We construct FMPs using 4-ADJ, OLS, LM, Stein, TS, and SB methods with individual corporate bond returns. Then FMPs are tested following Pukthuanthong, Roll, and Subrahmanyam (2019). Panel A presents correlation of FMPs with the systematic risk of bond returns using the same approach described for equities in Table 2. The PCs of bond returns are extracted using the Connor and Korajczyk (1988) method. Then the following procedure is implemented: First, for each canonical pair, we take the eigenvector weights for the 10 PCs and construct the weighted average PC (which is the canonical variate for the 10 PCs that produce the canonical correlation for this particular pair). Then a regression using each CK PC canonical variate as the dependent variable and the candidate factor realizations as independent variables is run over the sample months. *t*-statistics from the regression, then give the significance level of each candidate factor. Our sample period is from July 2002 to July 2017, split into three subperiods (60 months each). Each subperiod has seven pairs of canonical variates and a canonical correlation for each; thus, we have a total of 21 regressions (7 regressions per subperiod). The first row (Avg *t*) presents the mean *t*-statistic over all canonical correlations. The second row (Avg *t* sig. CC) reports the mean *t*-statistic when the canonical correlation is statistically significant. The third row presents the average number of significant canonical correlations out of 7 canonical variate pairs. The critical rejection levels for the *t*-statistics are 1.65 (10%), 1.96 (5%), and 2.59 (1%). An FMP or factor is related to the risk of asset returns if it is significantly related to any canonical variate in all decades or has a mean *t*-statistic in the second row that exceeds the one-tailed 2.5% cutoff based on the chi-squared value, and the average number of significant canonical correlation across subperiods is at least 1.75. Panel B shows risk premiums for macro factors by applying the IV Fama-Macbeth method on FMPs. See Section 1 for a description of each technique. The factors that are related to risk are highlighted in grey.

*A. Correlation with the systematic risk of equity returns*

| | CG | CPI | IP | UE | MKT | DS | TS |
|---|---|---|---|---|---|---|---|
| | | | | Original | | | |
| Avg *t* | 1.04 | 1.33 | 1.56 | 0.92 | 5.06 | 1.28 | 0.92 |
| Avg *t* (sig. CC) | 0.75 | 1.74 | 3.47 | 1.00 | 24.40 | 1.73 | 2.08 |
| Avg # sig. CC | 1.00 | 2.00 | 2.33 | 1.67 | 1.33 | 2.00 | 1.00 |
| | | | | Cross-sectional approach | | | |
| | | | | FMP_4-ADJ | | | |
| Avg *t* | 4.08 | 3.20 | 3.56 | 2.57 | 4.44 | 3.52 | 2.45 |
| Avg *t* (sig. CC) | 5.03 | 3.87 | 4.27 | 3.06 | 5.55 | 4.44 | 2.82 |
| Avg # sig. CC | 4.33 | 4.33 | 4.00 | 3.33 | 5.00 | 4.33 | 2.00 |
| | | | | FMP_OLS | | | |
| Avg *t* | 4.66 | 4.89 | 5.59 | 3.68 | 7.93 | 3.68 | 2.15 |
| Avg *t* (sig. CC) | 5.83 | 6.24 | 7.16 | 4.69 | 10.33 | 4.54 | 2.53 |
| Avg # sig. CC | 4.00 | 4.00 | 4.67 | 4.67 | 4.33 | 4.67 | 3.00 |
| | | | | FMP_Stein | | | |
| Avg *t* | 4.66 | 4.89 | 5.59 | 3.68 | 7.93 | 3.68 | 2.15 |
| Avg *t* (sig. CC) | 5.83 | 6.24 | 7.16 | 4.69 | 10.33 | 4.54 | 2.53 |
| Avg # sig. CC | 4.00 | 4.00 | 4.67 | 4.67 | 4.33 | 4.67 | 3.00 |
| | | | | FMP_LM | | | |
| Avg *t* | 3.33 | 4.06 | 4.08 | 2.97 | 8.78 | 3.71 | 2.05 |
| Avg *t* (sig. CC) | 4.05 | 5.09 | 5.19 | 3.70 | 11.41 | 4.54 | 2.32 |
| Avg # sig. CC | 3.33 | 3.67 | 4.33 | 3.67 | 3.00 | 3.67 | 3.00 |
| | | | | Time-series approach | | | |
| Avg. *t* | 1.64 | 1.33 | 1.59 | 1.42 | 1.96 | 1.57 | 1.08 |
| Avg *t* (sig. CC) | 3.37 | 1.97 | 2.70 | 2.68 | 4.64 | 3.00 | 1.38 |
| Avg # sig. CC | 2.00 | 2.00 | 2.67 | 1.67 | 2.33 | 2.00 | 1.00 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Sorting-by-beta approach | | | | | | |
| Avg. *t* | 2.85 | 4.06 | 3.23 | 2.99 | 6.69 | 2.47 | 2.20 |
| Avg *t* (sig. CC) | 3.42 | 5.13 | 4.08 | 3.75 | 8.65 | 2.95 | 2.56 |
| Avg # sig. CC | 4.33 | 3.67 | 4.00 | 3.33 | 4.33 | 3.00 | 2.67 |

*B. Estimated risk premiums using FMPs*

| | Intercept | CG | CPI | IP | UE | MKT_B | DS | TS |
|---|---|---|---|---|---|---|---|---|
| | Cross-sectional approach | | | | | | | |
| 4-ADJ | 0.892*** | 0.291*** | –0.049 | 0.586*** | –0.090 | 0.376* | 0.176** | –0.355 |
| | (3.838) | (3.082) | (–0.589) | (2.838) | (–1.174) | (1.870) | (2.275) | (–1.389) |
| OLS | 0.081 | -0.035 | 0.003 | 0.082 | -0.013 | 0.178 | 0.042 | -0.012 |
| | (1.045) | (-0.546) | (0.076) | (0.776) | (-0.551) | (1.131) | (0.599) | (-0.100) |
| LM | -1.400 | -0.291 | -0.060 | 0.023 | 0.311 | 1.161 | -0.315 | -0.286 |
| | (-1.145) | (-0.514) | (-0.132) | (0.035) | (0.734) | (0.977) | (-0.461) | (-0.271) |
| Stein | 0.081 | -0.125 | 0.005 | 0.156 | -0.052 | 0.182 | 0.107 | -0.066 |
| | (1.045) | (-0.546) | (0.076) | (0.776) | (-0.551) | (1.131) | (0.599) | (-0.100) |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Time-series approach | | | | | | | |
| Time series | -0.59** | 0.03 | 0.027 | -0.465*** | 0.057 | 1.409*** | 0.288*** | 1.284*** |
| | (-2.34) | (0.509) | (0.405) | (-3.073) | (1.488) | (3.071) | (3.243) | (3.783) |
| | Sorting by beta approach | | | | | | | |
| Sorting by beta | 0.083 | -0.837** | -0.069 | 0.292 | -0.012 | 0.325 | 1.041*** | 0.006 |
| | (0.720) | (-1.986) | (-0.208) | (1.033) | (-0.059) | (1.164) | (3.333) | (0.020) |

**Table 7. Simulation on Biases of Mispricing and Risk Premiums of FMPs**

This table reports biases in the estimated risk premiums from Monte Carlo simulations for FMPs. We simulate the stock returns and factors as described in Appendix A. We create FMPs following the methods described in Section 3 of the paper. With these FMPs, we run cross-sectional regressions again to estimate risk premiums. The first rows in the table show the true risk premiums for four macro variables when alpha (mispricing) is set to 0. The table reports estimated risk premiums along with their mean biases across 1,000 replications, expressed as a percentage of the true value. Panel A presents risk premiums and biases on the IV method that only retains stocks whose betas in even and odd subsamples have the same sign in the IV method, OLS, sorting-by-beta, Lehmann and Modest (1988), (LM), and Stein (1956). In Panel B, we present the time-series FMP methods by using Lamont (2001)'s basis assets. Panel C presents the risk premiums of FMPs computed from the estimated time-series coefficients in Panel A as a proxy for the FMPs. We list 4_ADJ, OLS, and sorting-by-beta as examples of cross-sectional approaches. The sample period is January 1964 to March 2016. To be included, ind4_ADJidual stocks must have at least 60 continuous monthly returns on CRSP. The macro factors include unexpected consumption growth (CG), unexpected changes in the CPI (CPI), unexpected changes in industrial production (IP), and unexpected changes in the unemployment rate (UE).

| | Alpha | CG | CPI | IP | UE |
|---|---|---|---|---|---|
| True risk premium | 0 | 0.2 | –0.2 | 1.2 | 0.3 |
| Panel A: Cross-sectional methods | | | | | |
| 4_ADJ | –0.0003 | 0.2072 | –0.2011 | 1.1732 | 0.3035 |
| | | 3.60% | 0.55% | –2.23% | 1.17% |
| OLS | 0.1412 | 0.1426 | –0.1147 | 0.6124 | 0.1499 |
| | | –28.7% | –42.7% | –49.0% | –50.0% |
| Sorting-by-beta | –0.0569 | 0.1379 | –0.1774 | 1.1362 | 0.2685 |
| | | –31.05% | –11.30% | –5.32% | –10.50% |
| LM | 0.2000 | 0.1367 | –0.1150 | 0.6738 | 0.1555 |
| | | –31.7% | –42.5% | –43.9% | –48.2% |
| Stein | 0.0533 | 0.153 | –0.1506 | 0.7485 | 0.2441 |
| | | –23.5% | –24.7% | –37.6% | –18.6% |
| Panel B: Time-series method | | | | | |
| Time-series | –5.0599 | 0.0861 | –0.0803 | 0.7098 | 0.1624 |
| | | –56.95% | –59.85% | –40.85% | –45.87% |

**Table 8. Simulation on RMSE, Size, and Power *t*-Test**

Panel A reports the root-mean-square error (RMSE.) The ex-ante RMSE measures the mean difference between the estimated risk premium and the true risk premium. The ex-post RMSE measures the difference between the estimated risk premium and the risk factor's realization. The RMSEs are computed across 1,000 replications in each simulation. Panel B shows the size and power of *t*-statistics for the 4_ADJ method. Size is based on the 1.96 critical value (a 5% significance level.). It measures the probability of improperly rejecting a true null hypothesis (simulated here as truly zero-risk premiums.) Power is the probability of rejecting a false null hypothesis; in this case, the alternat4_ADJe (true) hypothesis consists of risk premiums obtained from the OLS method, which are generally smaller than those of the other methods.

Panel A: RMSE

| | | CG | CPI | IP | UE |
|---|---|---|---|---|---|
| OLS | Ex-ante | 0.0700 | 0.0820 | 0.5906 | 0.1509 |
| | Ex-post | 0.0690 | 0.0821 | 0.5901 | 0.1509 |
| 4_ADJ | Ex-ante | 0.0549 | 0.0260 | 0.1318 | 0.0224 |
| | Ex-post | 0.0520 | 0.0239 | 0.1287 | 0.0217 |
| LM | Ex-ante | 0.0850 | 0.0793 | 0.5312 | 0.1480 |
| | Ex-post | 0.0812 | 0.0791 | 0.5400 | 0.1474 |
| Stein | Ex-ante | 0.0622 | 0.0473 | 0.4577 | 0.0585 |
| | Ex-post | 0.0592 | 0.0456 | 0.4553 | 0.0581 |

Panel B: Size and power of *t*-test in the 4_ADJ approach

| | CG | CPI | IP | UE |
|---|---|---|---|---|
| Size | 4.9% | 5.2% | 4.3% | 3.8% |
| Power | 70.9% | 86.0% | 83.1% | 71.2% |

## Internet Appendix

### Appendix A. Measurement Error Issues and the Four-Procedure Method

### A.1. Measurement Error and FMP Construction

This subsection examines the simple case in which the expected asset returns depend on a single nontradable factor. We follow the notation used in Section 1, so that $\tilde{f}$ denotes the observed nontradable factor; $f$ the genuine factor; and $\varepsilon_f$ the measurement error. To simplify the analysis without loss of generality, recall that the measurement error is uncorrelated with the asset returns and factors; that is, $cov(\varepsilon_f, \boldsymbol{R}) = 0$ and $cov(\varepsilon_f, f) = 0$. Moreover, assume that the actual asset pricing model is $\boldsymbol{R} = \boldsymbol{\alpha} + \boldsymbol{\beta} f + \boldsymbol{\varepsilon}$.

The FMP is constructed by the two-pass Fama-MacBeth method. For a sample of length $T$, let $R_t^i$ be the return on asset $i$ at time $t$ and $\tilde{f}_t$ be the observed factor at time $t$. Define $\boldsymbol{R}^i = [R_T^i, \cdots, R_T^i]'$, and $\tilde{\boldsymbol{f}} = [\tilde{f}_1, \cdots, \tilde{f}_T]'$. The first pass estimates beta by running the following time-series regression for each asset:

$$\boldsymbol{R}^i = \alpha^i + \beta^i \tilde{\boldsymbol{f}} + \boldsymbol{\varepsilon}^i, \tag{A1}$$

where $\alpha^i$ and $\beta^i$ are regression coefficients, and $\boldsymbol{\varepsilon}^i = [\varepsilon_T^i, \cdots, \varepsilon_T^i]'$ is the regression residual. The second pass is a cross-sectional regression at each time point $t$ (for $t \in \{1,2,\cdots,T\}$). Define $\boldsymbol{R_t} = [R_t^1, \cdots, R_t^N]$ and $\boldsymbol{\beta} = [\beta^{\boldsymbol{1}}, \cdots, \beta^{\boldsymbol{N}}]$. Then the regression can be written as

$$\boldsymbol{R_t}' = a_t + \lambda_t \widehat{\boldsymbol{\beta}}' + \boldsymbol{\eta_t}. \tag{A2}$$

Here, we use $\widehat{\boldsymbol{\beta}}$ to represent the estimated value of the factor loadings $\boldsymbol{\beta}$, and the residual is $\boldsymbol{\eta_t} = [\eta_t^1, \cdots, \eta_t^N]$. The estimated coefficient, $\lambda_t$, is the return for the FMP at time $t$.

The critical difference between Equations (A1) and (A2) and the Fama-MacBeth regression with the traded factors is that the nontradable factor, $\tilde{f}$, contains significant measurement errors.

### A.2. FMP Contamination and Resolution

Assume that there are two factors, $\tilde{f}_1$ and $\tilde{f}_2$. (Two-factor assumptions apply in the remaining proposition of Appendix A, and not one proposition changes if the true model contains more than two factors.) Factors 1 and 2 have measurement errors denoted by $\varepsilon_{f_1}$ and $\varepsilon_{f_2}$. Moreover, assume that the two factors and the measurement errors are correlated (i.e., $cov(f_1, f_2) \neq 0$ and

$cov(\varepsilon_{f_1}, \varepsilon_{f_2}) \neq 0$). Employing the same vector notation used in Section 2.1, we can write the time-series regression model as

$$\boldsymbol{R^i} = \alpha^i + \beta_1^i \boldsymbol{f}_1 + \beta_2^i \boldsymbol{f}_2 + \boldsymbol{\varepsilon}^i. \tag{A3}$$

If factor 1 (2) has a measurement error, we must replace $f_1$ ($f_2$) by $\tilde{f}_1$ ($\tilde{f}_2$), its observed version, before being able to run regression (A3). Then with the estimated beta coefficients, we compute the following cross-sectional regression:

$$\boldsymbol{R}_t{}' = \alpha_t + \lambda_{1t}\widehat{\boldsymbol{\beta}}_1{}' + \lambda_{2t}\widehat{\boldsymbol{\beta}}_2{}' + \boldsymbol{\eta}_t. \tag{A4}$$

If the true factors and/or measurement errors are correlated, $\tilde{f}_1$ and $\tilde{f}_2$ are correlated. Kan, Robotti, and Shanken (2013) formally analyze this case. Below, we will present one simplified case to illustrate the intuition behind FMP contamination. In this scenario, we assume only factor 1 contains a measurement error, whereas factor 2 does not.

**Proposition A1**: In the first-pass regression (A3), we assume that factor 1's measurement error is uncorrelated with asset returns, regression residuals, and both factors; factor 2's measurement error is zero; the regression residuals are also uncorrelated with both factors, and the betas are uncorrelated with the cross-sectional regression errors.

(A) When sample size $T$ converges to infinity, we have

$$\hat{\beta}_1^i \to \frac{var(f_1)var(f_2) - cov(f_1,f_2)^2}{DET_1}\beta_1^i \equiv B_1^i, \text{ and } \hat{\beta}_2^i \to \beta_2^i + \frac{var(\varepsilon_{f_1})(\beta_1^i cov(f_1,f_2) + \beta_2^i var(f_2))}{DET_1} \equiv B_2^i, \tag{A5}$$

where $DET_1 = \left(var(f_1) + var\left(\varepsilon_{f_1}\right)\right) var(f_2) - cov(f_1, f_2)^2$.

(B) In the second-pass regression (A4), when both $T$ and $N$ (the number of test assets) converge to infinity, then

$$\hat{\lambda}_{1t} \to w_1\gamma_{1t} + w_2\gamma_{2t}, \tag{A6}$$

where

$$w_1 = \frac{1}{DET_2}\frac{var(f_1)var(f_2) - cov(f_1,f_2)^2}{DET_1}\left(\overline{var}\left(B_2^i\right)\overline{var}\left(\beta_1^i\right) - \overline{cov}\left(\beta_1^i, B_2^i\right)\right),,$$

$$w_2 = \frac{1}{DET_2}\frac{var(f_1)var(f_2) - cov(f_1,f_2)^2}{DET_1}\left(\overline{var}\left(B_2^i\right)\overline{cov}\left(\beta_1^i, \beta_2^i\right) - \overline{cov}\left(\beta_1^i, B_2^i\right)\overline{cov}\left(\beta_2^i, B_2^i\right)\right),$$

$$DET_2 = \overline{var}\left(B_1^i\right)\overline{var}\left(B_2^i\right) - \overline{cov}\left(B_1^i, B_2^i\right)^2,$$

$$\gamma_{1t} = f_{1t} - E(f_1) + \gamma_1,$$
$$\gamma_{2t} = f_{2t} - E(f_2) + \gamma_2.$$

Here, $\overline{var}$ and $\overline{cov}$ are the cross-sectional variance and covariance, respectively. The upper bar distinguishes them from their time-series companions. Moreover, $\gamma_1$ and $\gamma_2$ are the underlying risk premiums belonging to factors 1 and 2.

(C) If $cov(f_1, f_2) = 0$, then $w_2 = 0$.

See Internet Appendix D for the proof.

From Equation (A5) in Proposition A1 (A), the estimated beta for factor 2 is a function of factor 1's beta. Moreover, when we employ the estimated betas in the cross-sectional regression, Equation (A6) in Proposition A1 (B) shows that the constructed FMP for factor 1 is also affected by factor 2. That is, the FMP is a false representation of factor 1. In conclusion, we cannot construct a pristine FMP using the multivariate Fama-MacBeth method. As discussed in the main text, we propose using a one-factor Fama-MacBeth regression to construct FMPs. We rely on the following assumption to show how this method resolves the FMP contamination problem.

**Assumption:** When the number of assets is large enough, the cross-sectional correlation of factor loadings of two uncorrelated factors is small. That is, if factors 1 and 2 (denoted by $f_1^*$ and $f_2^*$) are uncorrelated, the factor loadings (denoted by $\beta_1^{i*}$ and $\beta_2^{i*}$ for asset *i*) satisfy

$$\overline{cov}(\beta_1^{i*}, \beta_2^{i*}) \to 0 \qquad \text{(A7)}$$

when the number of test assets, *N*, converges to infinity.

In Equation (A7), $\beta_1^{i*}$ and $\beta_2^{i*}$ represent the sensitivity of asset *i* to two uncorrelated factors, $f_1^*$ and $f_2^*$, respectively. A(7) implies that the sensitivity of a typical asset to factor 1 is not correlated with the sensitivity of that asset to factor 2. Suppose consumption growth ($f_1^*$) and HML ($f_2^*$) are uncorrelated factors for stocks. This assumption, which is only true about uncorrelated factors, implies that cyclical firms are not necessarily more likely to be either value or growth firms. If consumption growth and HML are correlated, cyclical firms could be related to value or growth firms. We orthogonalize the factors (both macro factors and Fama-French factors), estimate the factor loadings for all individual stocks, and calculate the cross-sectional correlations of factor loadings for any two orthogonal factors. The average absolute correlation is only 8%, and the maximum absolute correlation is 20%. This low correlation between factor loadings seems to be consistent with the assumption.[16]

Assume that the true model is

$$\boldsymbol{R^i} = \alpha^i + \beta_1^{i*}\boldsymbol{f_1^*} + \beta_2^{i*}\boldsymbol{f_2^*} + \boldsymbol{\varepsilon}^i,$$

where the two factors are uncorrelated. When we run the first-pass regression using one factor only, that is, we run the following regression,

$$\boldsymbol{R^i} = \alpha^i + \beta_1^{i*}\boldsymbol{f_1^*} + \boldsymbol{\varepsilon}^i,$$

the factor loading is the same as running the regression from the underlying factor model. In the second pass, the true model should be

$$\boldsymbol{R_t}' = \alpha_t + \lambda_{1t}\widehat{\boldsymbol{\beta}}_1^{*\prime} + \lambda_{2t}\widehat{\boldsymbol{\beta}}_{\boldsymbol{2}}^{*\prime} + \boldsymbol{\nu_t},$$

where $\lambda_{1t} = f_{1t}^* - E(f_1^*) + \gamma_1^* = f_{1t} - E(f_1) + \gamma_1$ and$\lambda_{2t} = f_{2t}^* - E(f_2^*) + \gamma_2^*$.

When Equation (A7) is imposed on the uncorrelated factors $\boldsymbol{f_1^*}$ and $\boldsymbol{f_2^*}$, we run a one-factor Fama-MacBeth regression as follows:

$$\boldsymbol{R_t}' = \alpha_t + \lambda_{1t}\widehat{\boldsymbol{\beta}}_1^{*\prime} + \boldsymbol{\eta_t}, \qquad \text{(A8)}$$

then Proposition A2 shows that the estimated coefficient in this regression depends on $f_1$ only.

**Proposition A2**: Assume Equation (A7) holds. Suppose the measurement error in factor 1 is uncorrelated with asset returns, regression residuals, and both factors. In that case, the regression residuals are also uncorrelated with both factors, the betas are uncorrelated with the cross-sectional regression errors, and the estimated coefficient from the model (A8):

$$\hat{\lambda}_{1t} \to c[f_{1t} - E(f_1) + \gamma_1],$$

where $c = [var(f_1) + var(\varepsilon_{f_1})]/var(f_1)$, as sample period *T* and the number of test assets, *N*, converge to infinity.

See Internet Appendix D for proof.

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[16] The concept underlying Equation (A7) is implicit to the sorting method widely applied in finance literature. For example, sorting stocks on the consumption beta implicitly assumes that the average loadings of other uncorrelated factors are the same among high or low groups. Otherwise, if a high consumption beta is associated with a lower beta on HML when HML is uncorrelated with consumption growth, the high consumption-beta group has a low beta in HML. Then the consumption risk premium may just be a value premium. This will lead to a spurious conclusion that consumption growth requires a risk premium.

## A.3. Rationale for Stock Selection

Let $\hat{\beta}_{IV} = \beta + \varepsilon_{IV}$ and $\hat{\beta}_{EV} = \beta + \varepsilon_{EV}$, where $\beta$ is the true factor loading, and $\varepsilon_{IV}$ and $\varepsilon_{EV}$ are the estimation errors in the IV and EV samples. Following Jegadeesh et al. (2019), we assume that the estimation errors in the IV and EV samples are uncorrelated, $E(\hat{\beta}_{IV}{}'\hat{\beta}_{EV}) = E\big((\beta + \varepsilon_{IV})'(\beta + \varepsilon_{EV})\big) = E(\beta^2)$. If the asset return is correlated with the factor, and $\beta$ is not zero, the expected value of the multiplication of the IV and EV betas should be positive.

**Appendix B: Practical Steps of our Approach**

This appendix presents a step-by-step guide to the 4-PROC approach with an example. Suppose we want to create FMPs for a nontradable factor (e.g., consumption growth). In this example, we would take the following steps:

1. Separate the asset return sample into even- and odd-numbered months.
2. For each asset, run a time-series regression of even-numbered month returns on the nontradable factor (only one factor) to get even-numbered month betas. $\boldsymbol{\beta}_{even} = [\beta_{1,even}; \cdots; \beta_{N,even}]$ represents an $N \times 1$ vector of beta loadings in the even-numbered month sample for all assets ($N$ is the number of assets). Then regress the odd-numbered month returns on the nontradable factors to get an odd-numbered month beta. $\boldsymbol{\beta}_{odd} = [\beta_{1,odd}; \cdots; \beta_{N,odd}]$ represents an $N \times 1$ vector of beta loadings in the odd-numbered month sample for all assets.
3. Select all assets when odd- and even-numbered month betas have the same sign, that is, select asset $i$ if $\beta_{i,even} \times \beta_{i,odd} > 0$ . $\boldsymbol{\beta}_{even,sub}$ and $\boldsymbol{\beta}_{odd,sub}$ denote the even- and odd-numbered month betas for all assets in this selected subsample.
4. Apply the IV cross-sectional regression following Jegadeesh et al. (2019): for an even-numbered month, we regress the cross-sectional selected asset returns selected from step 3 on their even-numbered month beta ($\boldsymbol{\beta}_{even,sub}$), instrumented by the odd-numbered month beta ($\boldsymbol{\beta}_{odd,sub}$). For an odd-numbered month, we regress the cross-sectional selected asset returns on $\boldsymbol{\beta}_{odd,sub}$, instrumented by $\boldsymbol{\beta}_{even,sub}$. Note that this step is a single-factor regression (beta is estimated from a single-factor time-series regression). Given T months, we obtain T estimated coefficients. The estimated coefficients are FMPs.

The above steps construct the FMP for one nontradable factor. Apply the same steps for all remaining nontradable factors one by one (e.g., employment growth, inflation, industrial production).

Now, we have FMPs for several nontradable factors, for example, FMP_CG, FMP_CPI, FMP_IP, and FMP_UE. We further estimate the risk premiums of these FMPs by taking the following steps:

1. Separate the asset return sample into even- and odd-numbered months.

2. Run time-series regressions for each asset. Regress the excess returns on all FMPs (FMP_CG, FMP_CPI, FMP_IP, and FMP_UE) in the odd- and even-numbered month subsamples, which are the *multiple-factor* regressions. $\boldsymbol{B}_{even}$ and $\boldsymbol{B}_{odd}$ (both $N \times K$) represent the betas of all $K$ factors for all assets.

3. There is no asset selection.

4. Apply a multifactor IV cross-sectional regression following Jegadeesh et al. (2019): for an even-numbered month, we regress the cross-sectional returns of all assets on $\boldsymbol{B}_{even}$, instrumented by $\boldsymbol{B}_{odd}$. For an odd-numbered month, we regress the cross-sectional asset returns on $\boldsymbol{B}_{odd}$, instrumented by $\boldsymbol{B}_{even}$. Note that this step is a multiple-factor model as well. Suppose we have $T$ months, then we have $T$ estimated coefficients.

5. The average of the estimated coefficients is the risk premium, and the *t*-statistic is the ratio of the average over the standard deviation of estimated risk premiums (which is the Fama-Macbeth standard error).

## Appendix C: Robustness Tests

### Using Giglio and Xiu's (2021) Three-Pass Method to Resolve Model Misspecification

To resolve the model misspecification issue, especially the omitted variable problem, in the risk premium estimation, Giglio and Xiu (2021) provide a three-pass method. Their method can be described as follows. First, extract four principal components from the covariance matrix of 202 portfolio returns. Second, run a Fama-MacBeth regression of cross-sectional individual stock returns on the first four principal components to estimate their risk premiums (denoted by $\widehat{\boldsymbol{\gamma}}_p$). Third, regress each factor on these principal components. The estimated coefficients are denoted by $\widehat{\boldsymbol{\delta}}_p$. The risk premium of the factor is computed as $\widehat{\boldsymbol{\gamma}}_p{}'\widehat{\boldsymbol{\delta}}_p$.

We apply their methodology to the four-procedure FMPs of the four macro factors. Table E1 presents the result. The monthly premiums of consumption growth (CG) and unemployment (UE) are 0.09% and 0.01%, which are significant. The $R$-squared's of consumption growth and unemployment rates are also high, indicating that consumption growth and unemployment survive the omitted variable problems.

**Table C1. Risk premium estimation using Giglio and Xiu's (2021) method**

This table presents the risk premium estimation by applying Giglio and Xiu's (2021) three-pass method to four-procedure FMPs. The risk factors include the consumption growth rate (CG), unexpected CPI changes (CPI), unexpected changes in industrial production (IP), and unexpected changes in the unemployment rate (UE). The first row displays the estimated risk premium for the four-procedure FMP of each underlying factor. The second row shows the corresponding $R$-squared in the third-pass regression in the Giglio and Xiu's method, which captures the explanatory power of the first four principal components on FMP.

| | CG | CPI | IP | UE |
|---|---|---|---|---|
| Risk premium | 0.090** | -0.016 | -0.013 | 0.010* |
| $t$-ratio | (1.96) | (-1.14) | (0.93) | (-1.67) |
| $R^2$ | .621 | .207 | .0347 | .169 |

## Appendix D. Proofs of the Propositions

***Proof of Proposition A1***

(1) In the first-pass regression, when $T$ converges to infinity, we have

$$\hat{\beta}_1^i \to \frac{1}{DET_1}\left(var(f_2)cov\left(\tilde{f}_1, R^i\right) - cov(f_1, f_2)cov\left(f_2, R^i\right)\right).$$

Use $\boldsymbol{R^i} = \boldsymbol{\alpha^i} + \boldsymbol{\beta_1^i f_1} + \boldsymbol{\beta_2^i f_2} + \boldsymbol{\varepsilon^i}$ to replace $\boldsymbol{R^i}$ and note that $\varepsilon_{f_1}$ is uncorrelated with the factors and returns, resulting in

$$\hat{\beta}_1^i \to \frac{1}{DET_1}\beta_1^i(var(f_1)var(f_2) - cov(f_1, f_2)^2).$$

Similarly,

$$\hat{\beta}_2^i \to \frac{1}{DET_1}\left((var(f_1) + var\left(\varepsilon_{f_1}\right))cov\left(f_2, R^i\right) - cov(f_1, f_2)cov\left(\tilde{f}_1, R^i\right)\right)$$

$$\to \beta_2^i + \frac{1}{DET_1}\left(var\left(\varepsilon_{f_1}\right)(\beta_2^1 cov(f_1, f_2) + \beta_2^i var(f_2))\right).$$

(2) In the second pass, regress the returns on $B_1^i$ and $B_2^i$. Following Shanken (1992), we can write the true model as

$$\boldsymbol{R}_t' = (f_{1t} - E(f_1) + \gamma_1)\boldsymbol{\beta_1}' + (f_{2t} - E(f_2) + \gamma_2)\boldsymbol{\beta_2}' + \boldsymbol{\eta_t}.$$

The estimated coefficient for factor 1 when both $N$ and $T$ go to infinity converges to

$$\hat{\lambda}_{1t} \to \frac{1}{DET_2}\frac{var(f_1)var(f_2) - cov(f_1,f_2)^2}{DET_1}(\overline{var}\left(B_2^i\right)\overline{cov}\left(\beta_1^i, R^i\right) - \overline{cov}(\beta_1^i, B_2^i)\overline{cov}(B_2^i, R^i))$$

$$\to \frac{1}{DET_2}\frac{var(f_1)var(f_2) - cov(f_1,f_2)^2}{DET_1}\left(\left(\overline{var}\left(B_2^i\right)\overline{var}\left(\beta_1^i\right) - \overline{cov}\left(\beta_1^i, B_2^i\right)^2\right)\gamma_{1t} +\right.$$
$$\left.(\overline{var}\left(B_2^i\right)\overline{cov}\left(\beta_1^i, \beta_2^i\right) - \overline{cov}(\beta_1^i, B_2^i)\overline{cov}(\beta_2^i, B_2^i))\gamma_{2t}\right).$$

(3) When $cov(f_1, f_2) = 0$, $B_2^i = \beta_2^i\left(1 + \frac{var(\varepsilon_{f_1})var(f_2)}{DET_1}\right)$, then

$$\overline{var}\left(B_2^i\right)\overline{cov}\left(\beta_1^i, \beta_2^i\right) - \overline{cov}\left(\beta_1^i, B_2^i\right)\overline{cov}\left(\beta_2^i, B_2^i\right) = \left(1 + \frac{var(\varepsilon_{f_1})var(f_2)}{DET_1}\right)^2\left(\overline{var}\left(\beta_2^i\right)\overline{cov}\left(\beta_1^i, \beta_2^i\right) - \overline{cov}\left(\beta_1^i, \beta_2^i\right)\overline{var}\left(\beta_2^i\right)\right) = 0.$$ Hence, $w_2 = 0$.

***Proof of Proposition A2***

In the first pass, we regress the return only on factor 1 with regression $\boldsymbol{R^i} = \alpha^i + \beta_1^{i*}\boldsymbol{f_1^*} + \boldsymbol{\varepsilon}^i$, even if the true model follows regression $\boldsymbol{R_t}' = \alpha_t + \lambda_{1t}\widehat{\boldsymbol{\beta}}_1^{*\prime} + \lambda_{2t}\widehat{\boldsymbol{\beta}}_2^{*\prime} + \boldsymbol{\nu_t}$ . Hence, as $T$ converges to infinity, we obtain

$$\hat{\beta}_1^{i*} \rightarrow \frac{cov(\tilde{f}_1, R^i)}{var(\tilde{f}_1)} = \frac{cov(\tilde{f}_1, \alpha^i + \beta_1^{i*} f_1^* + \beta_2^{i*} f_2^* + \varepsilon^i)}{var(f_1) + var(\varepsilon_{f_1})}.$$

By definition, $f_1^* = f_1$, $cov(f_1^*, f_2^*) = 0$. Applying regularity assumptions about the measurement error and regression residuals results in

$$\frac{cov(\tilde{f}_1, \alpha^i + \beta_1^{i*} f_1^* + \beta_2^{i*} f_2^* + \varepsilon^i)}{var(f_1) + var(\varepsilon_{f_1})} \rightarrow \frac{var(f_1)}{var(f_1) + var(\varepsilon_{f_1})} \beta_1^{i*} = \frac{\beta_1^{i*}}{c}.$$

In the second pass, we run regression (A4), but the true model is

$$\boldsymbol{R_t}' = \alpha_t + \lambda_{1t}\widehat{\boldsymbol{\beta}}_1^{*\prime} + \lambda_{2t}\widehat{\boldsymbol{\beta}}_2^{*\prime} + \boldsymbol{\nu_t},$$

where $\lambda_{1t} = f_{1t}^* - E(f_1^*) + \gamma_1^* = f_{1t} - E(f_1) + \gamma_1$, and $\lambda_{2t} = f_{2t}^* - E(f_2^*) + \gamma_2^*$.

With the regularity assumptions, as both $N$ and $T$ go to infinity, the estimated coefficient for factor 1 in regression (A8) becomes

$$\hat{\lambda}_{1t} \rightarrow \frac{\overline{cov}(\frac{\beta_1^{i*}}{c}, R^i)}{\overline{var}(\frac{\beta_1^{i*}}{c})} = c\,\frac{\overline{cov}(\beta_1^{i*}, \alpha_t + \lambda_{1t}\widehat{\beta}_1^{i*\prime} + \lambda_{2t}\widehat{\beta}_2^{i*\prime} + \nu_t)}{\overline{var}(\beta_1^{i*})}.$$

Since $\overline{cov}(\beta_1^{i*}, \beta_2^{i*}) \rightarrow 0$ as $N$ goes to infinity (A7), and $\nu_t$ is uncorrelated with $\beta_1^{i*}$,

$$\frac{\overline{cov}(\beta_1^{i*}, \alpha_t + \lambda_{1t}\widehat{\beta}_1^{i*\prime} + \lambda_{2t}\widehat{\beta}_2^{i*\prime} + \nu_t)}{\overline{var}(\beta_1^{i*})} \rightarrow \lambda_{1t}$$