Testing Asset Pricing Model with Non-Traded Factors:

A New Method to Resolve (Measurement/Econometric) Issues in Factor-Mimicking Portfolio

Kuntara Pukthuanthong
University of Missouri
Email: pukthuanthongk@missouri.edu

Richard Roll
California Institute of Technology
Email: rroll@caltech.edu

Junbo Wang
Louisiana State University
Email: junbowang@lsu.edu

Tengfei Zhang
Louisiana State University
Email: tzhan23@lsu.edu

Abstract

We suggest the factor-mimicking portfolio (FMP) of a nontraded factor should jointly minimize the mispricing component of stock returns in a beta-pricing model. We also propose a novel method for constructing FMPs and measure the risk premiums of nontraded factors. Several of the macroeconomic factors of FMPs constructed by our method are related to the cross-sectional covariance of individual stock or bond returns. Consumption growth, inflation, and the unemployment rate command equity premiums; consumption growth and industrial production command bond premiums.

Keywords: factor-mimicking portfolios, nontraded factors, risk premium

JEL classification: G10, G12, G11

1. Introduction

Perhaps the most important question in asset pricing is whether different average returns across assets are rewarded for risk. Firm characteristics, such as size, book-to-market ratio, momentum, investment, and profitability, are related to the cross-sectional differences in asset returns (Fama and French, 1993, 2016, 2018; Hou et al., 2015). However, whether they proxy for risk exposures still needs to be determined.

Theoretically, macroeconomics factors (e.g., Chen et al., 1986) and other nontraded factors (e.g., Adrian et al., 2014) capture risks in the economy and thus should also explain cross-sectional expected returns. However, observed changes in these factors contain measurement errors and provide only weak predictions of asset returns. To reduce factor noise, the previous literature (Huberman et al., 1987; Breeden et al., 1989; Ferson et al., 2006; Balduzzi and Robotti, 2008; Giglio and Xiu, 2021) has recommended the use of factor-mimicking portfolios (FMPs), which are constructed from traded assets to represent the underlying nontraded factors. The extant literature often uses FMPs when testing asset pricing models (e.g., Cooper and Priestley, 2011; Barillas et al., 2019; Pukthuanthong et al., 2019, henceforth PRS).[1] In this paper, we propose a new interpretation of FMPs and a novel method to construct FMPs that mitigates measurement error.

Our new interpretation of FMPs completes the classical theory. Classical FMP construction follows Huberman et al. (1987) and Breeden et al. (1989), where an FMP is chosen to maximize the correlation with the underlying factor (i.e., a maximum correlation portfolio). Specifically, the maximum correlation portfolio projects nontraded factors onto the space of the traded assets. The projection can preserve the assets' risk exposure; that is, the covariance between the asset returns and a factor is the same as the covariance between the asset returns and the maximum correlation portfolio of the factors. Hence, the FMP reflects the risk of the underlying factor.

This paper connects the FMP and both the risk and pricing effects associated with the underlying factor. Specifically, in a beta-pricing model, by replacing underlying factors with

[1] Roll and Srivastava (2018, p. 21) point out the other applications: "mimicking portfolios have many potential uses, including (though not limited to): (1) Evaluating active manager performance, (2) Substituting for a desired investment in illiquid assets, (3) Determining the true potential for improved diversification, (4) Understanding the sources of past return volatility, (5) Predicting the likely level of future return volatility."

FMPs, we choose a portfolio that jointly minimizes the mispricing component of stock returns, with respect to the risk exposure of the underlying factors. The solution to the above minimization problem is called the least mispriced portfolio.

The least mispriced portfolio offers two benefits: First, conventional wisdom suggests that a factor representing the risk of assets is associated with a return premium. However, the recent empirical literature suggests that risk factors might not generate significant premiums (PRS, 2019; Lettau and Pelger, 2020; Kozak et al., 2020). To test whether a nontraded factor is related to asset risk and can price assets in the cross section, one should ensure that the FMP reflects both the risk and the pricing effects of the underlying factor. While the maximum correlation portfolio takes care of the risk effect, whether it also reflects the asset pricing effect is unclear. Our theory provides this connection.

Second, the least mispriced portfolio can be used to derive three FMP construction methods. The first method is the cross-sectional approach. We adopt the Fama-Macbeth (1973) cross-sectional regression each period and then use the time series of estimated coefficients as FMPs. The second method is a time-series approach, where nontraded factors are regressed on contemporaneous returns of preselected assets, and the fitted values are taken as returns on an FMP. The third method is the sorting-by-beta approach, where stocks are sorted into portfolios by their factor loadings (betas). Then a long-short portfolio between the top and bottom deciles is the FMP.[2]

When the underlying factor contains measurement errors, and when the estimated factor loading contains an estimation error, empirically constructed FMPs are subject to several methodological difficulties as will be described below. These difficulties make inferences for asset pricing tests unreliable. To cure the unreliability, we propose a new method that comprises four procedures.

The first procedure alleviates factor contamination. Specifically, when there is a measurement error in the underlying factor, and the underlying factor is correlated with another risk factor, we show that the constructed FMP is a linear combination of the underlying factor and

[2] Although top and bottom deciles are popular choices, empirical researchers sometimes use other cutoffs (see, e.g., Adrian et al., 2014).

the other risk factors in a multivariate cross-sectional method. Hence, the FMP represents not only the underlying factor but also the other factors. In other words, the risk premium associated with the FMP is contaminated by the other factor and defeats the purpose of creating the FMP. One potential solution to this problem is to extend the idea of Kan et al. (2013), who propose the use of the covariance between each factor and stock returns to replace beta loadings estimated from a multivariate time-series regression. Similarly, we suggest that FMPs for each factor should be created independently using the cross-sectional method, without controlling for other factors.

The second procedure addresses an errors-in-variables (EIV) issue in the estimated beta loading from the cross-sectional method. The issue is different from the well-known errors-in-variables bias. Specifically, we show that the error in the estimated beta loadings in the Fama-Macbeth regression can lead to a larger correlation between the FMPs and the underlying factor. To correct this problem, our approach relies on instrumental variables (IV) estimation with individual equities, following Jegadeesh et al. (2019). Under this approach, we divide the entire sample into even- and odd-numbered month subsamples and estimate betas in each subsample separately. Then betas from the even-numbered month subsample act as instrumental variables for the betas from odd-numbered months, or vice versa, in cross-sectional IV regressions with individual stock returns as a dependent variable.

The third procedure selects basis assets for FMP construction. Different papers have created FMPs from various candidate assets and the results depend on the basis assets that are chosen.[3] To avoid arbitrary basis asset selection, we propose the use of a large number of test assets: all individual stocks and/or bonds. However, a factor might only price a significant portion of assets, but not all assets. If we include all assets, the underlying factor can be uncorrelated with many assets and likely becomes a useless factor. This can induce spuriously large *R*-squares and

[3] Lamont (2001) proposes economic tracking portfolios using 13 basis assets that include eight industry-sorted stock portfolios, four bond portfolios, and a stock market return. Vassalou (2003) uses six equity portfolios sorted by size and book-to-market, the term spread, and the default spread. Kroencke et al. (2013) use equity portfolios sorted by size and book-to-market, as well as a momentum portfolio. Bianchi et al. (2017) use six size and book-to-market-sorted portfolios, plus the default and term spreads. Barillas and Shanken (2017) use 15 traded factors as basis assets. Maio (2018) uses the excess market return, the value spread, the term spread, and the S&P 500 price-to-earnings ratio. With the cross-sectional approach, Lehmann and Modest (1988) use size, the dividend yield, and variance-sorted portfolios, and Cooper and Priestley (2011) use 40 portfolios sorted by size, book-to-market, momentum, and asset growth. Pukthuanthong et al. (2019) use 50 portfolios sorted by size, book-to-market, momentum, investment, and operating profitability. Roll and Srivastava (2018) use eight exchange-traded fund (ETF) portfolios.

significant risk premiums in the asset pricing tests (Gospodinov et al., 2019). To mitigate this issue, we suggest an asset selection criteria that excludes uncorrelated assets. Specifically, we retain assets with the same signs between beta loadings estimated from the even-numbered month sample and the odd-numbered month sample. With this procedure, we are likely to select the factors that consistently affect asset returns over all the subsamples.

The last procedure attempts to reduce a measurement error. Given that an FMP has an excess return, Shanken (1992) shows that its average value is the risk premium estimate (we call it a one-stage risk premium estimation).[4] However, if the measurement error in the nontraded factor is large, the estimated risk premium can suffer from large estimation errors that affect the power of the hypothesis tests. To mitigate this problem, our last procedure applies the Fama-Macbeth regression using FMPs as factors. We show that using our estimated FMPs as factors can significantly alleviate the measurement error issue in the risk premium estimation.[5] Hence, we propose a two-stage method to estimate the risk premium of the underlying nontraded factors. In the first stage, we construct FMPs, and, in the second stage, we estimate the risk premiums associated with the FMPs. Given the EIV bias in the risk premium estimation, our second-stage cross-sectional regression also applies the IV method introduced by Jegadeesh et al. (2019).

In summary, our paper introduces a four-procedure (denoted by FP) method to address a series of issues in the FMP construction and asset pricing test. First, we apply a cross-sectional regression to a single-factor model to adjust for factor contamination. Second, we use the IV method to assuage the EIV problem. Third, we propose a basis asset screening approach. Finally, we reapply the cross-sectional-regression-IV-method to all factors and FMPs to estimate the risk premium.

We compare existing and newly introduced approaches to construct FMPs. To make a fair comparison, we address the error-in-variables bias to all existing FMP construction methods in both simulation and empirical tests. Specifically, we apply the IV method to estimate the risk premium for all FMPs (the second stage). For FMP construction (the first stage), we only apply

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4 We call this method the one-stage method, since the FMP creation and risk premium estimation occur in the same cross-sectional regression.

5 Creating FMPs using the cross-sectional method can mitigate the measurement errors in the underlying factor; thus, it is natural to believe that reapplying the cross-sectional method can further reduce the effect of measurement errors in risk premium estimation (Connor et al., 2015). We show that this is indeed the case.

IV for our own FMP because we want to compare our proposed FMPs with FMPs constructed by existing approaches. We test our FP method against several classical macro factors following the criteria proposed by Pukthuanthong et al. (2019). Specifically, we examine whether

1. the FMPs of macroeconomic variables are correlated with the systematic risk of returns, and

2. the FMPs command the risk premium in the cross section of individual assets.

For criterion 1, Pukthuanthong et al. (2019) propose that a genuine risk factor must be related to systematic risks (proxied for by the covariance matrix of returns).[6] We find that for cross-sectional approaches, most of the FMPs are related to the covariance of stocks. However, for time-series approaches, the FMPs fail to deliver systematic risk consistently. For criterion 2, our empirical results reveal that the FMPs of several macro variables are associated with significant risk premiums. Of all the approaches discussed above, our approach is the only one in which consumption growth, CPI, and unemployment rates are associated with significant risk premiums. Risk premiums estimated by existing cross-sectional, time-series, and sorting-by-beta approaches are, in general, insignificant.

We also construct FMPs from various approaches so that underlying nontraded factors can price individual corporate bonds. The only method of FMP construction that passes both criteria is the FP method. Using the FP FMP, we find that consumption growth, industrial production, bond market factors, and the default spread are associated with significant and positive risk premiums.

Although our method can improve the effectiveness of the asset pricing test for nontraded factors, some caveats should be mentioned. We list them below and propose practical solutions. The first issue is the severe measurement error in the underlying factor (Kleibergen, 2009). If the standard deviation of the measurement error is much larger than that of the underlying factor, the factor becomes uncorrelated with asset returns (useless factor). In this case, the asset pricing test statistics can have ill distributional properties.

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[6] Kozak et al. (2018) argue that the covariance of assets might not represent the risk. However, the risk factors should be associated with the covariance of the assets. Thus, examining whether the covariance of assets is related to FMPs can be viewed as a necessary condition to identify the risk factor.

To resolve this issue, we find (in the data) that the standard deviation of the measurement errors should not be not greater than five times the standard deviation of the underlying factor. Next, we examine whether the above measurement error is detrimental to FMP construction and asset pricing tests; to do so, we use our FP method. Additionally, construction errors in the FMPs can affect risk premium estimation and standard errors. For example, Jiang et al. (2015) show that construction errors in FMPs can lead to a large effect on standard errors for the asset pricing test. To examine whether these issues can be large in the finite sample, we resort to simulations. We find that, if the standard deviations of the measurement errors are not larger than five times the standard deviation of the true factors, the bias for the risk premium (we set the true risk premium to be zero as it is a null hypothesis of most of the asset pricing tests) is small, and the 5% critical value range of the *t*-ratios is consistent with that of standard normal distribution for the different underlying factors.

Finally, in the case of model misspecification,[7] the risk premium estimation can be biased (Lewellen et al., 2011; Giglio and Xiu, 2021), and the *t*-ratio can be spurious (Kan et al., 2013). Through simulations, we find that the statistical inferences are not significantly affected by the model misspecification for our asset pricing tests when the risk premiums of the underlying factors are zero. To control for model misspecification empirically, we adopt the two following methods. First, we control for Fama-French factors, the consumption-to-wealth ratio, and consumption volatility factors when estimating the risk premium of FMPs. Second, we apply Giglio and Xiu's (2021) three-pass method, which is designed for the asset pricing test under model misspecification. Applying both methods, we find that the model specification does not significantly affect the statistical inferences in our applications for the consumption growth factor.

Our contribution to the literature is threefold. First, we provide a new economic interpretation for FMPs, in addition to the maximal correlation portfolio proposed by Breeden et al. (1989) and Huberman et al. (1987). In line with our least mispricing portfolio theory, we also show that an effective FMP connects to both the risk and pricing effects of the underlying factors.

Second, we propose a new method to construct FMPs, called the FP approach. The FP approach resolves several econometric issues that hinder us from finding the underlying factor's

[7] This is a case when the factors are not the same as those supposedly included in the true model. See our Internet Appendix D (Table D2).

pricing effect. In contrast, existing cross-sectional approaches, as well as the sorting-by-beta and time-series approaches, are subject to substantial econometric issues.

Third, we perform a horse race among the FP method and existing FMP construction approaches in both the stock and bond markets. We find that the FP method is the winning horse for traded versions (FMPs) of macroeconomic factors, including consumption growth, CPI, and unemployment. The FMPs constructed by our method are associated with the covariance in asset returns. Unlike the alternative approaches, our FMPs have large and significant risk premiums in both equity and bond markets.

FMPs has been widely used. In our survey, several recent papers in asset pricing published in top-tier finance journals have applied factor-mimicking portfolios. For instance, Bessembinder et al. (2019) apply the time-series approach, following a long list of papers that apply this approach (see our footnote 4), to estimate benchmark returns and examine abnormal stock returns after corporate events. Engel et al. (2020) apply both time-series and cross-sectional approaches to the construction of factor-mimicking portfolios, which they call climate change hedge portfolios. Daniel et al. (2020) remove unpriced risk portfolios from characteristic-sorted portfolios. They find their characteristic-efficient portfolio generates a Sharpe ratio that is almost double in size of that from the characteristic-sorted portfolio. We contribute to the asset pricing literature and propose a new methodology to construct FMPs that address measurement errors and factor contamination.

Our paper builds on a large literature that tests the asset pricing model with FMPs. Balduzzi and Robotti (2008) conclude that using the time-series formulation of FMPs performs better in terms of estimating risk premiums than using the original factors with the one-stage cross-sectional approach. We provide a possible explanation for their findings, since the finite sample error is much larger for the one-stage cross-sectional approach. Rather than constructing mimicking portfolios for factors, Roll and Srivastava (2018) construct mimicking portfolios for individual stock returns. Fama and French (2020) construct factor-mimicking portfolios for characteristics using the cross-sectional approach and find that these characteristic-mimicking portfolios have better explanatory power over the average return than do characteristic-sorting-based factors (such as SMB, HML).

Our paper is related to nontraded factors, especially macroeconomic factors. Instead of using FMPs, Kleibergen and Zhan (2020) extend the Gibbons-Ross-Shanken statistic to identify the risk premiums of macro-risk factors and mostly find unbounded confidence sets for the consumption risk premium. The unbounded confidence sets include not only nonzero values (significance) but also zero (insignificance), so they conclude consumption growth appears uninformative for asset pricing. On other hand, we use FMP for the test. The pricing effect of the consumption growth factor can be identified using this method.

Our paper also joins a group of studies that address the problem of the FMP methodology. For the case of the maximum correlation FMP, Jiang et al. (2015) show that the estimation error in the weights of the mimicking portfolio affects the standard error of the risk premium estimates, and it is of first-order effect, especially when the model is misspecified. Our newly proposed FMP, the FP approach takes the above issues associated with the FMP construction into account.

2. The least mispriced portfolio

This section shows a novel theory of FMPs that connects the risk and pricing effects of the underlying factor and then discusses three implied econometric methods to construct FMPs.

2.1. The least mispriced portfolio theory

The FMP is constructed by minimizing the mispriced component of asset returns. *N* denotes the number of test assets. Let $\boldsymbol{R} = [R^1, \cdots, R^N]$,[8] an *N* by one vector, be the excess return of assets. A nontraded factor is represented by f.[9] We assume that the excess returns linearly depend on the projected factor f,

$$\boldsymbol{R} = \boldsymbol{\alpha} + \boldsymbol{\beta} f + \boldsymbol{\varepsilon}\,. \tag{1}$$

[8] Note that each entry of $\boldsymbol{R}$ is a random variable representing the returns of N assets. In a later section, we will use $\boldsymbol{\mathfrak{R}}$, a *T* by *N* matrix, to represent the matrix of the time-series realization of *N* assets.

[9] Note that f is a random variable. In later sections, we use the notation $\boldsymbol{f} = [f_1, \cdots, f_T]'$, a *T* by 1 vector, to represent the time-series realization of the nontraded factors. This convention is applied to all random variables defined in this paper.

Here, $\boldsymbol{\beta}$ (N by one) is the factor loading; $\boldsymbol{\alpha}$ (N by one) is the mispricing; $\boldsymbol{\varepsilon}$ (N by one) is the residual of the pricing model; and $\boldsymbol{\Omega}$ denotes its variance. Also, we assume that f contains only one factor; therefore, the residual, $\boldsymbol{\varepsilon}$, can be possibly correlated with other factors.

Our goal is to select a FMP that can minimize the mispricing component of the asset pricing model. Assume the risk factor can be approximated by an FMP, then $f = FMP + v = \boldsymbol{w}'\boldsymbol{R} + \boldsymbol{v},$ where $\boldsymbol{v}$ is the measurement error (with a mean of zero) of the risk factor, and $\boldsymbol{w}$ (N by one vector) represents the weight of the portfolio. The minimization problem can be written as follows:

$$\min_{\boldsymbol{w}} \widetilde{\boldsymbol{\alpha}}'\boldsymbol{\Sigma}\widetilde{\boldsymbol{\alpha}} \text{ where } \widetilde{\boldsymbol{\alpha}} = E(\boldsymbol{R}) - \boldsymbol{\beta}E(\boldsymbol{w}'\boldsymbol{R}). \quad (2)$$

Here, the weighting matrix, $\boldsymbol{\Sigma}$ (N by N), controls the relative importance of the mispricing components across assets.

Eq. (2) minimizes the mispricing component with respect to a single factor and solve for the portfolio weights of the FMP. It is possible to solve the same problem to obtain portfolio weights for multiple factors together. However, although these FMPs can jointly minimize the mispricing component of the multifactor model, each of them does not guarantee that the mispricing component of a single-factor model (with its underlying factor as the only factor) will be minimized. This is because, by controlling for other factors, the FMP obtained from the multifactor optimization problem might not extract the largest component of the return from the single-factor model. Evaluating the pricing effect of the portfolio computed from such weights can be complicated because the weights might not entirely reflect the pricing effect of each underlying factor.

The objective function can be written as follows:

$$\min_{\boldsymbol{w}} \widetilde{\boldsymbol{\alpha}}'\boldsymbol{\Sigma}\widetilde{\boldsymbol{\alpha}} = \left(E(\boldsymbol{R}) - \boldsymbol{\beta}E\left(\boldsymbol{w}'\boldsymbol{R}\right)\right)' \boldsymbol{\Sigma}\left(E(\boldsymbol{R}) - \boldsymbol{\beta}E\left(\boldsymbol{w}'\boldsymbol{R}\right)\right). \quad (3)$$

In the following proposition, we derive the solution to the above minimization problem.

Proposition 1: The optimum weight solution to the minimization problem (3) can be written as $\boldsymbol{w} = \frac{\boldsymbol{\beta}'\boldsymbol{\Sigma}E(\boldsymbol{R})}{\boldsymbol{\beta}'\boldsymbol{\Sigma}\boldsymbol{\beta}}E(\boldsymbol{R})(E(\boldsymbol{R})'E(\boldsymbol{R}))^{-1}$. Hence, the expected portfolio return is

$$E(\boldsymbol{R})'\boldsymbol{w} = \frac{\boldsymbol{\beta}'\boldsymbol{\Sigma}E(\boldsymbol{R})}{\boldsymbol{\beta}'\boldsymbol{\Sigma}\boldsymbol{\beta}}. \tag{4}$$

See Internet Appendix F for the proof.

Given that the portfolio can minimize the mispricing component of the test assets, we call it the least mispriced portfolio.

From Eq. (4), we know that if the cross-sectional expected returns are more highly correlated with their $\boldsymbol{\beta}$, then the numerator in Eq. (4) is larger. Thus, the optimum portfolio has higher expected returns. Intuitively, $\boldsymbol{\beta}$ reflects the sensitivity of the asset to the factor risk. If this factor can explain most of the cross-sectional variation of asset returns, the mispriced component is small. Hence, the expected return of the pricing component, which is the left-hand side of Eq. (4), should be larger.

Neoclassical theory (maximum correlation theory) suggests that FMPs can be constructed by projecting the factor onto the space of returns. Therefore, the factor can be decomposed into two parts: the first part is the projection (FMP) and the second part is uncorrelated with the space of returns. Hence, the covariance between the returns of any asset with the factor is the same as the covariance between the returns of the asset with the FMP, because the remaining component of the factor is uncorrelated with the asset returns. That is, the maximum correlation portfolio reflects the risk of the underlying factor. We present this theory in Internet Appendix A. In addition, Internet Appendix A provides the condition that the least mispriced portfolio theory is equivalent to the classical maximum correlation theory.

2.2. Implied methodology for FMP construction

In this section, we describe three methods derived from the least mispricing portfolio theory. These methods are based on the different choices of the weighting matrix, $\boldsymbol{\Sigma}$. In the asset pricing literature, all three approaches are widely applied. However, in Internet Appendix A, we show that not all of these methods are directly implied by the maximum correlation theory.

2.2.1. Time-series method

Let the weighting matrix be the inverse of the covariance matrix of testing the asset returns (i.e., $\boldsymbol{\Sigma} = \boldsymbol{V}^{-1}$). Thus, we have

$$var(f)\boldsymbol{\beta}'\boldsymbol{\Sigma}\boldsymbol{\beta}E(\boldsymbol{R})'\boldsymbol{w} = var(f)\boldsymbol{\beta}'\boldsymbol{\Sigma}E(\boldsymbol{R}) = cov(f,\boldsymbol{R})\,\boldsymbol{V}^{-1}E(\boldsymbol{R})\,. \quad (5)$$

The left-hand side is a scaled expected return of the least mispricing portfolio. The right-hand side represents the estimation method used to calculate the return. Equivalently, the return can be estimated by regressing the nontraded factor on returns of the test assets:

$$f = a + b\boldsymbol{R} + \boldsymbol{u}. \quad (6)$$

The fitted value of the above time-series regression is $cov(f,\boldsymbol{R})\,\boldsymbol{V}^{-1}\boldsymbol{R}$. Take the expected value; it is the same as the right-hand side of Eq. (5). Lamont (2001) is one of the leading papers using this approach; the paper constructs FMPs from 13 basis assets, which are portfolios formed by sorting individual firm characteristics.

2.2.2. Cross-sectional method

Let the weighting matrix be $\boldsymbol{\Sigma} = (\boldsymbol{I} - \boldsymbol{1}(\boldsymbol{1}'\boldsymbol{1})^{-1}\boldsymbol{1}')\boldsymbol{V}^{-1}(\boldsymbol{I} - \boldsymbol{1}(\boldsymbol{1}'\boldsymbol{1})^{-1}\boldsymbol{1}')$, where, $\boldsymbol{I}$ is the *N* by *N* identity matrix, and $\boldsymbol{1}$ is an *N* by one vector, with each entry being one. Replacing $\boldsymbol{\Sigma}$ in Eq. (4), we obtain

$$E(\boldsymbol{R})'\boldsymbol{w} = \frac{\boldsymbol{\beta}'\left(\boldsymbol{I}-\boldsymbol{1}(\boldsymbol{1}'\boldsymbol{1})^{-1}\boldsymbol{1}'\right)\boldsymbol{V}^{-1}\left(\boldsymbol{I}-\boldsymbol{1}(\boldsymbol{1}'\boldsymbol{1})^{-1}\boldsymbol{1}'\right)E(\boldsymbol{R})}{\boldsymbol{\beta}'(\boldsymbol{I}-\boldsymbol{1}(\boldsymbol{1}'\boldsymbol{1})^{-1}\boldsymbol{1}')\boldsymbol{V}^{-1}(\boldsymbol{I}-\boldsymbol{1}(\boldsymbol{1}'\boldsymbol{1})^{-1}\boldsymbol{1}')\boldsymbol{\beta}} = \frac{\overline{\boldsymbol{\beta}}'\boldsymbol{V}^{-1}\overline{E}(\boldsymbol{R})}{\overline{\boldsymbol{\beta}}'\boldsymbol{V}^{-1}\overline{\boldsymbol{\beta}}}. \quad (7)$$

Here, for any random variable $\boldsymbol{X}$, the notation $\overline{\boldsymbol{X}}$ is the demeaned $\boldsymbol{X}$, where the mean is taken across test assets. The left-hand side of Eq. (7) still represents the expected return of a scaled least mispricing portfolio. The right-hand side is the coefficient of regressing the expected return on its factor loadings across test assets. Specifically,

$$E(\boldsymbol{R}) = \alpha + \gamma\boldsymbol{\beta} + \boldsymbol{v}, \quad (8)$$

where the estimated coefficient γ is based on generalized least squares (GLS) with the weighting matrix $\boldsymbol{V}^{-1}$. Eq. (8) takes the same form as the right-hand side of Eq. (7).

In this case, the weighting matrix contains the covariance matrix of the asset returns. This matrix is more difficult to estimate when the number of test assets is large. A special scenario, in this case, is to set $\boldsymbol{V}^{-1} = \boldsymbol{I}$. The right-hand side of Eq. (7) becomes $\frac{\overline{\boldsymbol{\beta}}'\overline{E}(\boldsymbol{R})}{\overline{\boldsymbol{\beta}}'\overline{\boldsymbol{\beta}}}$. This corresponds to the coefficient of regression in Eq. (8) using ordinary least squares (OLS). Another special scenario is

Lehmann and Modest (1988, hereafter LM). They suggest using the diagonal matrix, which only consists of the variances of the residuals from the regression (1) to replace $\mathbf{V^{-1}}$.

2.2.3. Sorting-by-beta method

Suppose that we divide the assets into five groups by their factor loadings. Assume that asset 1 through asset $M = \frac{N}{5}$ are in the group with the smallest beta, and asset $4M + 1$ to asset $5M$ are in the group with the largest beta. Hence, the weighting matrix can be written as $\boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\Sigma_{11}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma_{22}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{\Sigma_{33}} \end{pmatrix}$, where $\boldsymbol{\Sigma_{11}} = \begin{pmatrix} -\frac{1}{\beta_1} & \cdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \cdots & -\frac{1}{\beta_M} \end{pmatrix}$ (an M by M matrix), $\boldsymbol{\Sigma_{22}} = \begin{pmatrix} 0 & \cdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \cdots & 0 \end{pmatrix}$ (an 3M by 3M matrix), and $\boldsymbol{\Sigma_{33}} = \begin{pmatrix} \frac{1}{\beta_{4M+1}} & \cdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \cdots & \frac{1}{\beta_{5M}} \end{pmatrix}$ (a M by M matrix). Replacing $\boldsymbol{\Sigma}$ in Eq. (4) results in

$$\frac{1}{M}\boldsymbol{\beta}'\boldsymbol{\Sigma}\boldsymbol{\beta}E(\boldsymbol{R})'\boldsymbol{w} = \frac{1}{M}\left(\sum_{i=4M+1}^{5M} E\left(R^i\right) - \sum_{i=1}^{M} E\left(R^i\right)\right). \tag{9}$$

The right-hand side of Eq. (9) is the difference between the average expected return of the high beta and low beta groups, which is the same as the scaled least mispriced portfolio on the left-hand side.

3. Measurement error in factors and econometric issues concerning FMP construction and asset pricing test

We derived methods for constructing the least mispriced portfolio in the previous section. Empirically, the measurement error in the nontraded factor and the estimation error raise issues with these methods. In this section, we discuss these issues and propose a novel approach to address them. We focus on the cross-sectional method, but we will also discuss the time-series and sorting-by-beta methods.

3.1. Measurement error and FMP construction

In this subsection, we examine the simple case in which the true expected asset returns depend on a single factor, which is nontraded. $\tilde{f}$ denotes the nontraded factor; f the underlying factor; and ε_f

the measurement error. The measurement errors in nontraded factors can lead to issues in the asset pricing test, and we examine these issue in this section. To simplify the analysis without loss of generality, assume that the measurement error has a mean of zero and is uncorrelated with the asset returns and the factors $cov(\varepsilon_f, \boldsymbol{R}) = 0$ and $cov(\varepsilon_f, f) = 0$. Moreover, assume that the true asset pricing model is $\boldsymbol{R} = \boldsymbol{\alpha} + \boldsymbol{\beta} f + \boldsymbol{\varepsilon}$; that is, the asset returns do not depend on the measurement error.

The FMP is constructed by the two-pass cross-sectional method. For a sample period of length *T* periods, let R_t^i be the return on asset *i* at time *t*, and $\tilde{f}_t$ be the factor at time *t*. Define $\boldsymbol{R}^i = [R_T^i, \cdots, R_T^i]'$, and $\tilde{\boldsymbol{f}} = [\tilde{f}_1, \cdots, \tilde{f}_T]'$. The first-pass estimates betas by running the following time-series regression for each asset:

$$\boldsymbol{R}^i = \alpha^i + \beta^i \tilde{\boldsymbol{f}} + \boldsymbol{\varepsilon}^i, \tag{10}$$

where α^i and β^i are regression coefficients, and $\boldsymbol{\varepsilon}^i = [\varepsilon_T^i, \cdots, \varepsilon_T^i]'$ is the regression residual. The second pass is a cross-sectional regression at each time point t (for $t \in \{1,2,\cdots,T\}$). Define $\boldsymbol{R_t} = [R_t^1, \cdots, R_t^N]$ and $\boldsymbol{\beta} = [\beta^{\boldsymbol{1}}, \cdots, \beta^{\boldsymbol{N}}]$. Then the regression can be written as

$$\boldsymbol{R_t}' = a_t + \lambda_t \widehat{\boldsymbol{\beta}}' + \boldsymbol{\eta_t}. \tag{11}$$

Here, we use $\widehat{\boldsymbol{\beta}}$ to represent the estimated value of the factor loading β^i, and the residual is $\boldsymbol{\eta_t} = [\eta_t^1, \cdots, \eta_t^N]$. The estimated coefficient, λ_t, is the return for the FMP at time *t*.

The key difference between Eqs. (10) and (11) and the cross-sectional regression on the traded factors is that $\tilde{f}$ contains measurement errors. With some mild regularity conditions on the measurement errors ε_f, as well as the factor and regression residuals, the next proposition shows that the FMP constructed by the two-pass method can adjust for the measurement errors when the sample size T is large.

Proposition 2: Assume that (1) measurement errors, ε_f, are uncorrelated with R^i for any asset *i*, uncorrelated with f, and uncorrelated with regression residual ε^i, (2) regression residuals are uncorrelated with factors f, and (3) beta ($\boldsymbol{\beta}$) is uncorrelated with the cross-sectional regression errors ($\boldsymbol{\eta_t}$). During the sample period, T converges to infinity; the estimated coefficient λ_t

converges to $c(f_t - E(f) + \gamma)$, where γ is the underlying factor risk premium; and $c = (var(f) + var(\varepsilon_f))/var(f)$ is a constant.

See Internet Appendix F for the proof.

This proposition is also shown in Section 6.2 of Balduzzi and Robotti (2008). We acknowledge that this proposition addresses mild, but not severe, measurement errors. If ε_f dominates and $var(f) \ll var(\varepsilon_f)$, we will observe that $\tilde{f}$ is close to being a useless factor because it is almost uncorrelated with all asset returns. Kleibergen (2009) and Kan and Zhang (2019) analyze the useless factor issue, where *c* goes to infinity. They find that, in such a case, the risk premium estimator will be spuriously significant, with non-standard-limiting behavior. In Proposition 2, *c* is finite and thus rules out the case in which the measurement error dominates. We further discuss these issues in Section 5 and show that the dominate measurement error issue might not be significant for our application.

From the proposition, we know that, when T is large, the estimated coefficient is a linear transformation of the factor without measurement error.[10] In particular, the FMP is scaled by a constant c. Intuitively, the measurement error in the nontraded factor leads to a scaling effect in the estimated beta coefficient in the first pass. Given that each stock's beta (which is the independent variable for the second-pass regression) is scaled up by the same constant, the estimated coefficient in the second-pass cross-sectional regression is scaled down by the same constant.

Moreover, from Proposition 2, the scaling effect on λ_t is homogeneous across time (by a constant number of c). Thus, the scaled FMP can still represent the same factor in the asset pricing test. To test the risk premium, we can calculate the average value of the coefficients over time as the risk premium estimates and calculate the Fama-Macbeth standard deviation of the regression coefficients [i.e., the average value of the coefficients is $\frac{1}{T}\sum_{t=1}^{T} c(f_t - E(f) + \gamma)$], and the Fama-Macbeth standard deviation of the coefficients is the sample standard deviation of $c[f_t - E(f) + \gamma)]$. Both the average and standard deviation are scaled by the same constant.

[10] When there is no measurement error ($\varepsilon_f = 0$), the factor at time *t* is f_t. Hence, the estimated coefficient in Proposition 2, $(c[f_t - E(f) + \gamma])$, is a linear transformation of the factor without measurement error.

Hence, the *t*-statistics formed by the estimate and the standard deviation given above are not affected by the constant number and thus converge to a normal distribution.

3.2. FMP contamination and a method for resolution

3.2.1 FMP contamination

Although the cross-sectional method for the one-factor model can remove the measurement error, we might need to create FMPs for various factors. The cross-sectional method that includes all factors together (Balduzzi and Robotti, 2008; Cooper and Priestley, 2011) can be ideally applied for FMP construction. However, similar to Kan et al. (2013), we find that if these factors are correlated, the FMP constructed by including correlated factors in the cross-sectional method is a linear combination of the underlying factor and correlated factors. Thus, that the constructed FMP is not a pure representation of the underlying factor leads to FMP-factor contamination.

As an illustration of the above point, let's assume that there are two factors (the two-factor assumptions apply throughout the analysis in this section), $\tilde{f}_1$ and $\tilde{f}_2$. Factors 1 and 2 have measurement errors denoted by ε_{f_1} and ε_{f_2}. Moreover, assume that the two factors and the measurement errors are correlated (i.e., $cov(f_1, f_2) \neq 0$ and $cov(\varepsilon_{f_1}, \varepsilon_{f_2}) \neq 0$). Employing the same vector notation used in Section 3.1, we can write the true regression model as

$$\boldsymbol{R^i} = \alpha^i + \beta_1^i \boldsymbol{f}_1 + \beta_2^i \boldsymbol{f}_2 + \boldsymbol{\varepsilon}^i. \qquad (12)$$

If factor 1 (2) has a measurement error, we must replace f_1 (f_2) by $\tilde{f}_1$ ($\tilde{f}_2$), its observed version, before being able to run regression (12). Then with the estimated beta coefficients, we compute the following cross-sectional regression:

$$\boldsymbol{R}_t{}' = \alpha_t + \lambda_{1t}\widehat{\boldsymbol{\beta}}_1{}' + \lambda_{2t}\widehat{\boldsymbol{\beta}}_2{}' + \boldsymbol{\eta}_t. \qquad (13)$$

If true factors and/or measurement errors are correlated, $\tilde{f}_1$ and $\tilde{f}_2$ are correlated. Thus, the proof for factor contamination when $\tilde{f}_1$ and $\tilde{f}_2$ are correlated is the same as the case in which f_1 and f_2 are correlated (from Kan et al., 2013). Below, we present one simplified case to illustrate the intuition supporting this idea. In this scenario, we assume that only factor 1 contains a measurement error, while factor 2 does not.

Proposition 3: In the first-pass regression (12), we assume that the measurement error of factor 1 is uncorrelated with the asset returns, regression residuals, and both factors; the measurement error of factor 2 is zero; the regression residuals are also uncorrelated with both factors, and the betas are uncorrelated with the cross-sectional regression errors.

(A) When sample size T converges to infinity, we have

$$\hat{\beta}_1^i \rightarrow \frac{var(f_1)var(f_2)-cov(f_1,f_2)^2}{DET_1}\beta_1^i \equiv B_1^i \ , \ \text{ and } \ \hat{\beta}_2^i \rightarrow \beta_2^i + \frac{var(\varepsilon_{f_1})(\beta_1^i cov(f_1,f_2)+\beta_2^i var(f_2))}{DET_1} \equiv B_2^i \ , \quad (14)$$

where $DET_1 = \left(var(f_1) + var\left(\varepsilon_{f_1}\right)\right) var(f_2) - cov(f_1, f_2)^2$.

In the second-pass regression (13),

(B) when both T and N (the number of test assets) converge to infinity, then

$$\hat{\lambda}_{1t} \rightarrow w_1\gamma_{1t} + w_2\gamma_{2t}, \quad (15)$$

where

$$w_1 = \frac{1}{DET_2}\frac{var(f_1)var(f_2)-cov(f_1,f_2)^2}{DET_1}\left(\overline{var}\left(B_2^i\right)\overline{var}\left(\beta_1^i\right) - \overline{cov}\left(\beta_1^i, B_2^i\right)\right)$$

$$w_2 = \frac{1}{DET_2}\frac{var(f_1)var(f_2)-cov(f_1,f_2)^2}{DET_1}\left(\overline{var}\left(B_2^i\right)\overline{cov}\left(\beta_1^i, \beta_2^i\right) - \overline{cov}\left(\beta_1^i, B_2^i\right)\overline{cov}\left(\beta_2^i, B_2^i\right)\right)$$

$$DET_2 = \overline{var}\left(B_1^i\right)\overline{var}\left(B_2^i\right) - \overline{cov}\left(B_1^i, B_2^i\right)^2$$

$$\gamma_{1t} = f_{1t} - E(f_1) + \gamma_1$$

$$\gamma_{2t} = f_{2t} - E(f_2) + \gamma_2.$$

Here, $\overline{var}$ and $\overline{cov}$ are the cross-sectional variance and covariance, respectively. The upper bar distinguishes them from their time-series companions. Moreover, γ_1 and γ_2 are the underlying risk premiums of factors 1 and 2.

(C) If $cov(f_1, f_2) = 0$, then $w_2 = 0$.

See Internet Appendix F for the proof. From Eq. (14) in Proposition 3(A), the estimated beta for factor 2 is a function of beta for factor 1. Moreover, when we employ the estimated betas in the cross-sectional regression, Eq. (15) in Proposition 3(B) shows that the constructed FMP for factor 1 is also affected by factor 2. In this case, the FMP contains an additional component from factor 2; thus, its risk premium reflects the excess returns associated with a combination of the two factors. This is not a desirable property for an FMP. In conclusion, we cannot construct a pristine FMP using the multivariate cross-sectional method.

When factors 1 and 2 are uncorrelated, then from Proposition 1 and Eq. (15), the FMP of factor 1 does not depend on factor 2. In the real world, it is very likely that factor 2 is correlated with factor 1. However, if one can create a modified factor 2 that is uncorrelated with factor 1 and use it in the FMP construction for factor 1 (i.e., run regressions (12) and (13) with factor 1 and the modified factor 2 to create $\hat{\lambda}_{1t}$), the resultant FMP will have the desirable property and represent the risk for factor 1. A classical approach to constructing two uncorrelated factors is to remove the effect of factor 1 from factor 2 and to use the remaining component of factor 2 (which is uncorrelated with factor 1) as a control. Specifically, in a regression of factor 2 on factor 1, the regression residual, being orthogonal to factor 1, can become the modified factor 2, which can then be used as a control in the cross-sectional method. When there is no measurement error in factor 1, this method works. However, given that factor 1 contains a measurement error, the following proposition shows that the residual of factor 2 is still correlated with factor 1. Thus, controlling for this residual leads to the same factor contamination problem present in Proposition 3.

Proposition 4: Assume that factors 1 and 2 are correlated. Let u_{12} be $f_2 - \frac{cov(\tilde{f}_1, f_2)}{var(\tilde{f}_1)} \tilde{f}_1$, which is the residual value of regressing factor 2 on nontraded factor 1 with measurement error. Then the correlation between u_{12} and f_1 is nonzero.

See Internet Appendix F for the proof.

3.2.2. A method for resolution

To resolve the issue of factor-FMP contamination, we extend the method of Kan et al. (2013). Specifically, we propose to construct an independent FMP for each factor, that is, using a one-factor cross-sectional regression approach, regardless of the other factors. We rely on the following assumption to show the validity of the two-pass regression using the one-factor model.

Assumption: When the number of assets is large enough, the factor loadings of two uncorrelated factors are also uncorrelated in the cross section. That is, if factors 1 and 2 (denoted by f_1^* and f_2^*) are uncorrelated, the factor loadings (denoted by β_1^{i*} and β_2^{i*} for asset *i*) satisfy

$$\overline{cov}(\beta_1^{i*}, \beta_2^{i*}) \rightarrow 0 \tag{16}$$

when the number of test assets, *N*, converges to infinity.

In Eq. (16), β_1^{i*} and β_2^{i*} represent the sensitivity of asset i on two uncorrelated factors, f_1^* and f_2^*, respectively. The assumption implies that the sensitivity of a typical asset to factor 1 is not correlated with the sensitivity of that asset to factor 2. Suppose consumption growth (f_1^*) and HML (f_2^*) are uncorrelated factors for stocks. The assumption implies that cyclical firms are not necessarily more likely to be value or growth firms. Note that this assumption is only about the uncorrelated factors. If consumption growth and HML are negatively correlated, cyclical firms are also likely to be growth firms. Empirically, we find that the average absolute correlation among factor loadings of orthogonalized (uncorrelated) factors across all individual stocks is only 8%, and the maximum absolute correlation is about 20%. This low correlation among factor loadings seems to be consistent with the assumption.[11]

Assume that the true model is

$$\boldsymbol{R^i} = \alpha^i + \beta_1^{i*}\boldsymbol{f_1^*} + \beta_2^{i*}\boldsymbol{f_2^*} + \boldsymbol{\varepsilon}^i,$$

where the two factors are uncorrelated. When we run the first-pass regression using one factor only, that is, we run the following regression,

$$\boldsymbol{R^i} = \alpha^i + \beta_1^{i*}\boldsymbol{f_1^*} + \boldsymbol{\varepsilon}^i,$$

the factor loading is the same as running the regression from the underlying factor model. In the second pass, the true model should be

[11] Assumption (16) is implicit to the sorting method widely applied in finance literature. For example, if we sort stocks on the consumption beta, the high-minus-low portfolio only captures the risk premium of consumption growth assuming the average loadings of other factors are the same among high or low groups. If a high consumption beta is associated with a lower beta on HML, the high consumption-beta group has a low beta in HML. Then the risk premium will reflect the HML factor; thus, the loadings of the other factors should not be correlated with the loading of consumption growth.

$$\boldsymbol{R_t}' = \alpha_t + \lambda_{1t}\widehat{\boldsymbol{\beta}}_1^{*\prime} + \lambda_{2t}\widehat{\boldsymbol{\beta}}_{\boldsymbol{2}}^{*\prime} + \boldsymbol{\nu_t},$$

where $\lambda_{1t} = f_{1t}^* - E(f_1^*) + \gamma_1^* = f_{1t} - E(f_1) + \gamma_1$ and $\lambda_{2t} = f_{2t}^* - E(f_2^*) + \gamma_2^*$.

When Eq. (16) is imposed on uncorrelated factors $\boldsymbol{f_1^*}$ and $\boldsymbol{f_2^*}$, and we run one-factor cross-sectional regression as follows:

$$\boldsymbol{R_t}' = \alpha_t + \lambda_{1t}\widehat{\boldsymbol{\beta}}_1^{*\prime} + \boldsymbol{\eta_t}, \tag{17}$$

then Proposition 5 shows that the estimated coefficient in this regression depends on f_1 only.

Proposition 5: Assume Eq. (16) holds. If the measurement error in factor 1 is uncorrelated with asset returns, regression residuals, and both factors, the regression residuals are also uncorrelated with both factors, the betas are uncorrelated with the cross-sectional regression errors, and the estimated coefficient from model (17) is

$$\hat{\lambda}_{1t} \to c[f_{1t} - E(f_1) + \gamma_1],$$

where $c = [var(f_1) + var(\varepsilon_{f_1})]/var(f_1)$, as sample period *T* and the number of test assets, *N*, converge to infinity.

See Internet Appendix F for the proof. Following the same example above, the estimated parameter in the second-pass regression is a good proxy for consumption growth risk as long as the factor loadings of the single-factor regression (with consumption growth) is uncorrelated with the factor loading of the value factor (i.e., the FMP is not contaminated by other factors).

3.3. EIV issues and the IV approach

It is well known that the cross-sectional regression is subject to an EIV bias when testing asset pricing models. In this section, we present another EIV issue. The measurement error in the estimated beta also affects the correlation between factors and their corresponding FMPs. To see this, suppose that $\boldsymbol{\beta}$ is estimated with an error *v*, so that $\widehat{\boldsymbol{\beta}} = \boldsymbol{\beta} + \boldsymbol{\nu}$. We first show that the true variance of the optimal FMP is larger than its estimated variance. To simplify this analysis without loss of generality, we assume that the weighting matrix is the covariance matrix of the stock returns ($\boldsymbol{\Sigma} = \boldsymbol{V^{-1}}$). Then the variance of the FMP with weight $\boldsymbol{w}$ from Proposition 1 can be written as $\boldsymbol{w}'\boldsymbol{V}\boldsymbol{w} = \left[\boldsymbol{\beta}'\boldsymbol{V}^{-1}\boldsymbol{\beta}\right]^{-1}$. The equality is established through plugging weight (from Proposition 1) into the variance. When we plug in the estimated beta, the variance becomes $\left[\widehat{\boldsymbol{\beta}}'\boldsymbol{V}^{-1}\widehat{\boldsymbol{\beta}}\right]^{-1}$. Thus, the estimated variance is smaller than the true variance because of the following inequality:

$$\left[\boldsymbol{\beta}'\boldsymbol{V}^{-1}\boldsymbol{\beta}\right]^{-1} > \left[\widehat{\boldsymbol{\beta}}'\boldsymbol{V}^{-1}\widehat{\boldsymbol{\beta}}\right]^{-1}. \qquad (18)$$

In Internet Appendix A, we show that the variance of the FMP is reciprocal to the correlation between the FMP and the factor (Eq. (A1)); thus, the correlation between the estimated FMP and the factor is empirically higher than its true value.

Classical theory proposes that the FMP should be a maximum correlation portfolio. A higher correlation between the FMP and the underlying factor is preferred. However, the inflated correlation can be misleading. When the measurement error of beta is large, the correlation can be significantly larger than the true correlations. Researchers might falsely conclude that the FMP represents a large component of the underlying factor, while the true representation can be much smaller.

The IV approach can adjust this issue. Imagine that we want to construct an FMP. We divide the total sample into odd- and even-numbered month subsamples. We run time-series regressions for the subsamples of odd- and even-numbered months separately, thereby estimating independent odd- and even-numbered month betas for each asset. With the odd-numbered month betas as IV betas and even-numbered month betas as EV (evaluation variable) betas, we construct the matrices for the betas of all assets: $\widehat{\boldsymbol{B}}_{\boldsymbol{IV}}$ and $\widehat{\boldsymbol{B}}_{\boldsymbol{EV}}$, where $\boldsymbol{B}$ is the $N \times (K+1)$ matrix containing all the betas augmented by a vector of one, that is, $\boldsymbol{B} = [\boldsymbol{1}, \boldsymbol{\beta}]$, where $\boldsymbol{1}$ is an N by one vector of ones.

Then we calculate a second-pass cross-sectional IV regression. For each even-numbered month, we run a cross-sectional regression, and the FMP at time t can be written as[12]

$$\widehat{\boldsymbol{\gamma}}_{\boldsymbol{t}} = (\widehat{\boldsymbol{B}}_{\boldsymbol{IV}}{}'\boldsymbol{V}^{-1}\widehat{\boldsymbol{B}}_{\boldsymbol{EV}})^{-1}\widehat{\boldsymbol{B}}_{\boldsymbol{IV}}{}'\boldsymbol{V}^{-1}\boldsymbol{r}_{\boldsymbol{t}}. \qquad (19)$$

[12] Empirically, we use the OLS-IV approach only, not the GLS-IV approach, because Roll and Ross (1994) find that only an OLS approach correctly and economically interprets the coefficient. In addition, the covariance matrix of individual assets is not invertible when the number of assets is larger than the number of time periods.

Here, $\boldsymbol{r_t}$ is the excess return for even-numbered months and is an *N* by one vector, and $\hat{\boldsymbol{\gamma}}_t$ is a *(K+1)* × *T* matrix containing all estimated FMPs augmented by the mispricing part in the first column. Correspondingly, for each odd-numbered month, we take the betas in the even-numbered months' subsample as the IVs and estimate the equation to obtain the FMP.

When the error contains no factor structure (Section 3.1), Jegadeesh et al. (2019) show that the IV approach can converge at the speed of $\sqrt{NT}$. When the error contains a factor structure (Section 3.2 and later), Jegadeesh and Noh (2014) show that the IV approach can converge at the speed of e^T, while the classical OLS method can only converge at the speed of $\sqrt{T}$. Hence, in both cases, the IV approach can adjust for the EIV issue. Our approach requires a large *T*. If *T* is small, the betas could be uninformative and noisy. Based on our unreported test, the IV method converges if T is greater than 200. Given that we have monthly macroeconomic factor data for more than 50 years, *T* is usually larger than 600; hence, the IV method should converge in this case.

In Internet Appendix B, we show that the correlation between the factors and FMPs will be deflated if we apply the IV method to construct the FMPs; that is, we provide a lower bound on the correlation between the FMP and its underlying noisy factor. The correlation for the lower bound can help us gauge the upper bound of the measurement error in the underlying factor.

Assume that we use the single-factor method to construct the FMP, and suppose that T converges to infinite, then the FMP should be perfectly correlated with f and uncorrelated with the measurement error term ε_f. To see this, we write the correlation between the observed underlying factor (that contains measurement error) and its FMP as

$$\rho = corr(\tilde{f}, FMP) = corr(f + \varepsilon_f, FMP) = \frac{cov(f+\varepsilon_f, FMP)}{\sqrt{var(f+\varepsilon_f)var(FMP)}} = \frac{\sqrt{var(f)}}{\sqrt{(var(f)+var(\varepsilon_f))}}. \qquad (20)$$

Thus, $\frac{\sqrt{var(\varepsilon_f)}}{\sqrt{var(f)}} = \frac{\sqrt{1-\rho^2}}{\rho}$. Given that $\frac{\sqrt{1-\rho^2}}{\rho}$ is a decreasing function of ρ, the lower bound of ρ implies the upper bound of $\frac{\sqrt{var(\varepsilon_f)}}{\sqrt{var(f)}}$. For example, if the lower bound of the correlation (ρ) is 20%, the standard deviation of the measurement error in the underlying factor is at most 4.9 times as big as the standard deviation of the underlying factor.

The upper bound on the measurement error can be important for us to evaluate whether a factor can be useless. For example, for a noisy factor, if the upper bound of the standard deviation of its measurement error over the standard deviation of the true factor is 4.9, and if the statistical property of the asset pricing test is not affected by the noise associated with the upper bound above, the factor might not be useless. In Section 5, we use simulations to examine the effect of the measurement error on our statistical inferences.

3.4. Two-stage method for risk premium estimation

A fundamental goal in constructing an FMP is to test whether the nontraded factor is associated with a risk premium. Since the FMP is an excess return, Shanken (1992) shows that its average value is the risk premium. Another method is to refit a cross-sectional regression to estimate the risk premium using the FMP as factors (following Connor et al., 2015). We call this method the two-stage method, in which the first stage is FMP construction for risk factors and the second stage is the risk premium estimation of FMP. The method to calculate the average value of the FMP returns is called the one-stage method. The following proposition shows that, although these two methods are asymptotically the same in a large sample, the finite sample error that stems from the measurement error (of the underlying factors) becomes smaller when we reapply a cross-sectional regression to test for a risk premium.

Proposition 6: (1) As T converges to infinity, the average value of the coefficients in regression (26) converges to the true risk premium times a constant: $\frac{1}{T}\sum_{t=1}^{T}\hat{\lambda}_{1t} \rightarrow c\gamma_1$. (2) When T is finite, in the average value of the coefficients ($\frac{1}{T}\sum_{t=1}^{T}\hat{\lambda}_{1t}$), the finite sample estimation error that comes from factor measurement error (ε_f) is of order $O(\frac{1}{\sqrt{T}})$. This is the result from the first stage of our two-stage method. (3) With the two-stage method, the finite sample estimation error that comes from the factor measurement error (ε_f) in the estimated risk premium is of order $O(\frac{1}{T})$, which is the result from the second stage of our two-stage method.

See Internet Appendix F for the proof.

If the measurement error for some nontraded factor is very large (such as macroeconomic factors), the estimation error in the one-stage method can still contain a large measurement error. A two-stage method can further mitigate the measurement error issue. Balduzi and Robotti (2009) compare the one-stage method with the two-stage method (although they use the time-series approach to construct the FMPs) and find that the two-stage method is superior. We provide a possible explanation for this finding from the effect of the finite sample estimation error.

The two-stage method also complements the Kan et al. (2013) method, specifically in adjusting the measurement error issue. Although their method avoids the factor contamination issue, they adopt a one-stage risk premium estimation method. Specifically, instead of estimating beta loadings, they use a single-factor model to estimate the covariance between a factor and individual stocks as risk exposure. Then they estimate the risk premium by running the Fama-MacBeth regression of the excess return on risk exposures. Since the measurement error in the one-stage method can be large when the measurement error of estimated beta is large, the statistical inference can be less powerful. Section 6.3.4 (Panel B, Table 5) shows that the estimated risk premiums based on Kan et al. (2013) are all insignificant, consistent with this proposition.

In the testing/second stage, we should incorporate all factors and FMPs to estimate risk premiums. Proposition 6 only proves the scenario with the single-factor model in the testing stage, but it is easy to extend it to the multifactor model. Also, to avoid the EIV bias of beta loadings in the risk premium estimation stage, we also apply the IV method following Jegadeesh et al. (2019) to estimate the risk premiums.

3.5. Asset selection

We propose to use a large number of test assets (e.g., individual stocks and/or bonds) following Jegadeesh et al. (2019). Ideally, all assets should be included. However, this can create issues if not all of these assets are correlated with the underlying factor. When the majority of assets are not correlated with the factor, the factor can be close to useless. In this case, Gospodinov et al. (2019) find that the estimated risk premium in a cross-sectional regression can be misleadingly large and significant. Therefore, it is necessary to select assets that are correlated with the factor (so the factor is not useless to the selected assets). The IV approach can select well-correlated assets because assets that are well correlated with the factor should have similarly estimated betas in the odd- and even-numbered samples. Thus, the sign of the IV and EV betas usually should be the same. Similarly, if assets are not driven by the factor, the IV and EV betas often will have different signs. Therefore, we should select only assets in which $\hat{\beta}_{IV}'\hat{\beta}_{EV} > 0$ or retain only those assets in which the IV and EV betas have the same signs.

The IV approach uses the beta loading of the factor in a subsample (from the even-numbered month subsample) as the IV for the beta of the factor in a different subsample (from odd-numbered months, or vice versa). Cautious readers might question whether the estimation error in the beta estimation can be severe, which amounts to the so-called "weak instrument problem" in econometrics. A weak IV problem arises when IV and EV are not correlated. By choosing the assets with $\hat{\beta}_{IV}'\hat{\beta}_{EV} > 0$ (which imposes a positive correlation), we can mitigate the weak instrumental variables problem.[13]

3.6. Summary of our method and a discussion of other methods

Based on the previous sections, we propose our four-procedure (FP) method as follows. First, we apply a cross-sectional regression on a single-factor model to avoid factor contamination. Second, we apply the IV method to mitigate the EIV issue. Third, we select basis assets to avoid the useless factor and weak IV issue. Finally, we reapply the cross-sectional regression IV method for all factors and FMPs to estimate the risk premium.

In the simulation in Section 5 and empirical work in Section 6, we will compare the proposed approach with various other existing methods. The classical methods in Section 2 contain the OLS cross-sectional method, the time-series approach, and the sorting-by-beta approach. In previous sections, we have described the issues present for the cross-sectional methods. The latter two methods suffer from the various issues described below.

We first discuss issues associated with the time-series method. The classical time-series method requires only a small number of assets. This can lead to a breakdown of the assumption in Eq. (16) since that equation is only reasonable for a large number of assets. Suppose that the number of test assets is large. In real data, the sample size (T) in Eq. (6) is finite. (For macroeconomic factors, the highest frequency is monthly, leading to roughly 600 months over 50

[13] A similar weak IV issue arises if the sample period (T) is limited. In this case, the estimated betas could be uninformative and/or noisy in a statistical sense. This is particularly important if the researcher has yearly data only for their proposed risk factor. The data set created by FRED is at a monthly frequency. So the weak IV issue can be mitigated. See https://research.stlouisfed.org/econ/mccracken/fred-databases/.

years. So $T = 600$ in this case.) Thus, there is an overfitting or even unidentifiable issue if N is close to or larger than T.

Even if Eq. (16) is satisfied, we show in the proposition below that an FMP created by the time-series method can still represent a combination of two factors.

Proposition 7: Let the true model be

$$\boldsymbol{R^i} = \alpha^i + \beta_1^{i*}\boldsymbol{f_1^*} + \beta_2^{i*}\boldsymbol{f_2^*} + \boldsymbol{\varepsilon}^i,$$

where the two factors are uncorrelated.

The regularity assumptions about measurement errors and regression residuals follow from Proposition 5. We estimate the coefficient in the regression below:

$$\boldsymbol{\tilde{f}_1} = a + \boldsymbol{b}\boldsymbol{\mathfrak{R}} + \boldsymbol{u}, \tag{21}$$

where $\boldsymbol{\mathfrak{R}} = [\boldsymbol{R}^1, \cdots, \boldsymbol{R}^N] = [\boldsymbol{R_1}; \cdots; \boldsymbol{R_T}]$, and ";" is the operator that stacks row vectors.[14] We construct the FMP as $\frac{1}{N}\boldsymbol{\hat{b}}\,'\boldsymbol{R}_t$. When the sample size and the number of test assets both converge to infinity, the FMP constructed above converges to $\frac{1}{N}\boldsymbol{\beta}_1^*\boldsymbol{V}^{-1}(a + \boldsymbol{\beta}_1^*\boldsymbol{f}_{1t} + \boldsymbol{\beta}_2^*\boldsymbol{f}_{2t})var(f_1)$.

See Internet Appendix F for the proof. As we can see from Proposition 7, the FMP still contains the other factor unless $\bar{E}(\frac{1}{N}\boldsymbol{\beta}_1^*\boldsymbol{V}^{-1}\boldsymbol{\beta}_2^*) \to 0$ as N goes to infinity. In a simplified scenario, $\boldsymbol{V}^{-1}$ is an identity matrix. The above equation becomes $\bar{E}(\beta_1^{i*}\beta_2^{i*}) \to 0$. Since $\bar{E}(\beta_1^{i*}\beta_2^{i*}) = \bar{E}(\beta_1^{i*})\bar{E}(\beta_2^{i*}) + \overline{cov}(\beta_1^{i*}, \beta_2^{i*})$, even if the factor loadings are uncorrelated (Eq. (16)), we still require the cross-sectional average of factor loadings to have a mean of zero. This is unlikely to be satisfied for most of the factors.

The sorting-by-beta method suffers from an EIV issue. To construct sorting-by-beta FMPs, we first estimate factor loadings (betas) for each asset, then sort assets by their betas, and group the assets into portfolios by the sorted betas. FMPs are the difference between the average returns of assets in the highest and the lowest beta groups. Because estimated betas contain estimation errors, the larger (smaller) betas are more likely to produce a positive (negative) estimation error.

[14] Recall that $\boldsymbol{R}^i$ is a T by one-column vector; $\boldsymbol{R_t}$ is a one by N-row vector; and $\boldsymbol{\mathfrak{R}}$ is a T by N matrix. For the given definition of ";", the two expressions of $\boldsymbol{\mathfrak{R}}$ are equivalent.

In an extreme case, where a major part of the estimated betas is an error, the sorting-by-beta approach is tantamount to sorting by error. Thus, even if the factor does price assets in the cross section, the difference in the average returns between the assets in the highest and the lowest estimated beta groups may not represent the difference of their exposures to the factor. The FMP created by this approach might not represent the pricing effect of the underlying factor. Hence, the sorting-by-beta method, like the cross-sectional approach, suffers from an EIV issue.

There are also other extensions for the cross-sectional methods. Lehmann and Modest (1988) introduce a weighted-least-squares method to construct the FMP. Instead of using the identity matrix as in the OLS approach, they suggest using the diagonal matrix consisting of the residual variances from the first-pass time-series regression as the weighting matrix.

Stein (1956) and James and Stein (1961) propose the shrinkage method to minimize root-mean-square errors (RMSEs) when at least three parameters are being estimated. Their shrinkage beta can be written as

$$\widehat{\boldsymbol{\beta}}_{\boldsymbol{Stein}} = \left(1 - \frac{(N-3)}{\|\widehat{\boldsymbol{\beta}}_{\boldsymbol{\sigma}}^{*}\|}\right)\widehat{\boldsymbol{\beta}}_{\boldsymbol{new}} + \bar{\hat{\beta}}, \tag{22}$$

where $\widehat{\boldsymbol{\beta}}_{\boldsymbol{\sigma}}^{*} = [\hat{\beta}_{\sigma}^{1}, \hat{\beta}_{\sigma}^{2}, \dots, \hat{\beta}_{\sigma}^{N}]$, and $\hat{\beta}_{\sigma}^{i} = \frac{\hat{\beta}^{i} - \bar{\hat{\beta}}}{\sigma^{i}}$, in which σ^{i} is the standard error of $\hat{\beta}^{i}$. Also, $\|\widehat{\boldsymbol{\beta}}_{\boldsymbol{\sigma}}^{*}\| = \sum_{i=1}^{N}(\hat{\beta}_{\sigma}^{i})^{2}$, $\bar{\hat{\beta}} = \frac{1}{N}\sum_{i=1}^{N}\hat{\beta}^{i}$ or the mean of $\hat{\beta}^{i}$, and $\widehat{\boldsymbol{\beta}}_{\boldsymbol{new}} = [\hat{\beta}^{1} - \bar{\hat{\beta}}, \hat{\beta}^{2} - \bar{\hat{\beta}}, \dots, \hat{\beta}^{N} - \bar{\hat{\beta}}]$.

Then the Stein-adjusted estimate is

$$\boldsymbol{\lambda_t} = (\boldsymbol{B}_{\boldsymbol{Stein}}^{'}\boldsymbol{B}_{\boldsymbol{Stein}})^{-\mathbf{1}}\boldsymbol{B}_{\boldsymbol{Stein}}^{'}\boldsymbol{R_t}, \tag{23}$$

where $\boldsymbol{B}_{\boldsymbol{Stein}}$ is the $N \times (K+1)$ matrix containing all the Stein shrinkage betas $\widehat{\boldsymbol{\beta}}_{\boldsymbol{stein}}$, augmented by a vector of one. Stein's method can reduce the mean-square error of the OLS estimator through the reduction of the standard error. Hence, the FMP constructed by this approach will be less volatile.

We will incorporate both the LM and Stein's approaches in the horse race. Note that for all these classical cross-sectional approaches, the convention is to run a multifactor regression to create the FMPs. However, doing so will lead to the factor contamination issue.

4. Data

In this section, we describe the data and variables used in this paper. Table 1 summarizes the descriptive statistics for these variables.

[Table 1 about here]

4.1. Stock return data

Monthly individual stock returns are from CRSP. The data start from January 1964 and run to March 2016 (627 months). Following the extant literature, we exclude stocks with prices less than \$1 or market capitalizations less than \$6 million. We also exclude stocks that have less than 60 continuous months of returns. After these exclusions, 10,833 stocks remain in our sample; there are 2,850 stocks in an average month; and total observations equal 1,784,351. The mean return of individual stocks over a risk-free rate (the one-month Treasury-bill rate) is 1.012% per month, but the median is 0.44% per month, indicating individual stocks enjoy very large positive returns.

4.2. Explanatory variables

Four macroeconomic variables obtained from the Federal Reserve Bank of St. Louis Research (FRED) website serve as our nontraded factors: (1) the growth in per capita consumption (DPCERAM1M225NBEA in FRED code), (2) the percentage change in the consumer price index (CPIAUCSL), (3) the percentage change in industrial production (INDPRO), and (4) the percentage change in the unemployment rate (UNRATE). Following Chen et al. (1986), we use innovations in these macroeconomic variables as factors. To measure innovations, we use the residuals from a first-order vector autoregression (VAR). Using first differences as innovations is also possible. However, as discussed by Boguth and Kuehn (2013), the residuals from VAR are a more conservative specification for risk exposures, although the results are robust to using first differences.

We also study factors for bonds, such as the default spread and term spread, downloaded from Robert Shiller's website. We download traded factors from Kenneth R. French's website (excess market return, small-minus-large market capitalization, and high-minus-low book/market portfolio returns). To construct the time-series factor-mimicking portfolios, we obtain 25 portfolios formed on size and book-to-market, 10 industry portfolios from Kenneth French's website, and 4 bond returns, which include 1-, 5-, and 10-year Treasury-bond yields and Moody's seasoned Baa corporate bond yield from FRED.

We also examine other consumption-related factors, including the CAY factor (the ratio of consumption to aggregate wealth proposed by Lettau and Ludvigson 2001) and the consumption volatility factor proposed by Boguth and Kuehn (2013). These factors are available from Martin Lettau's and Oliver Boguth's websites, respectively.

4.3. Corporate bond-return data

For corporate bonds, we use transaction records in the Trade Reporting and Compliance Engine (TRACE). TRACE provides corporate bond intraday trading prices, trading volumes, sell and buy indicators, and so forth. Our sample period is from August 2002 to June 2017. We follow the Bai et al. (2019) data-screening procedure and return-estimation approach. The monthly corporate bond returns are computed from the average quoted price at the end of the current month, accrued interest, and coupon payment for a month divided by the average quoted price at the end of the previous month or the beginning of the current month. A bond's excess return is the difference between its computed total return and the risk-free rate, where the latter is proxied for by the one-month Treasury-bill rate. Our final sample consists of 331,728 observations, and the average cross-sectional excess return is 0.389%, which is comparable to the Bai et al. (2019) sample. We include only bonds with at least 30 continuous monthly returns; 6,421 bonds remain in our final sample.

As for the explanatory variables, we first consider four nontraded macroeconomic factors. We subsequently add the default spread, the term spread, and the corporate bond market return. The default spread is the return difference between Moody's long-term corporate BAA-rated bonds and AAA-rated bonds. The term spread is the return difference between 10-year and 1-year Treasury bonds. The monthly corporate bond market return is the equally weighted average of the corporate bond returns in our sample.

5. Simulation

In this section, we compare, in the finite sample, the magnitude of the factor contamination of FMPs constructed using the methods described in Section 3.[15] Moreover, the measurement error

[15] In addition to factor contamination, the existing method also suffers from EIV bias when there is an estimation error in the factor loadings as well as in the basis asset selection. These issues are extensively evaluated in the literature on asset pricing tests. Since the issues are similar for FMP construction, we relegate the simulation of the measurement errors of beta loadings to Internet Appendix C. We find that the FMPs constructed by IV methods can yield risk premiums with a bias of less than 5%, but FMPs constructed by other methods suffer from severely biased estimations of the risk premium. See Table C1 in Internet Appendix C.

from the nontraded factor and the construction error of FMP affects the asset pricing test in the finite sample. Therefore, the statistical inferences of the asset pricing test also should be examined through the simulations.

Our motivation for the simulation stems from the issue of the dominating measurement error (Kleibergen, 2009). Specifically, suppose the standard deviation of the measurement error is much larger than the standard deviation of the underlying factor. In this case, the observed factor (including the measurement error) becomes uncorrelated with the asset returns, a scenario that can lead to large factor contamination and ill-behaved distributional properties for the asset pricing test.

To examine the impact of the measurement error, we first need to gauge its size. From Section 3, we know the correlation between the observed underlying factor and the FMP using the IV approach determines its lower bound. Ideally, when the sample period (T) is infinity, the FMP does not contain any measurement error. If T is large enough and if the correlation between the FMP and its underlying factor is large, the error in the underlying factor should be small. The correlations between the FMPs and the underlying factors range from 41% to 66%. Because of limited space, we report the results in Internet Appendix C. However, if the measurement error of the underlying factor is large and the sample size is finite, the FMPs still can be highly correlated with the measurement error (Proposition 6), leading to a misleadingly large correlation with the underlying factor. To resolve this issue, we create a second-stage FMP by reapplying the IV regression on each single-factor FMP constructed from the first three steps of our FP FMP. Proposition 6 shows that the second-stage FMP can significantly mitigate its correlation with the measurement error.[16] We find that the correlation between the second-stage FMP and the underlying factor is about 20% (see Internet Appendix C).

Since there is an estimation error in the FMP, and the FMP with the IV method understates the true correlation, the above correlation suggests that the true correlation between the underlying and observed underlying factors should be at least 20%. Applying Eq. (20) presented in Section

[16] For the FMP construction (first-stage), we use the single-factor IV approach, with stock selection. For the second-stage FMP, we use the same method, with the first-stage FMP replacing the observed factors and apply stock selection as well.

3.3, we find that the ratio of standard deviation between measurement error and true factor should be at most 4.9.

However, the above conclusion is only true based on the assumption that the measurement error is small in the second-stage FMP. It is possible that the measurement error is not small even if T is more than 600. Thus, by recursively applying the IV method to the second-stage FMP, we can create third- and fourth-stage FMPs, which can further reduce the impact of the measurement error. We find that the correlation between higher-stage FMPs with the underlying factors is similar to those for the second-stage FMPs (such as consumption growth, inflation, and unemployment rate), suggesting that the impact of measurement errors on the second-stage FMP is small enough. Therefore, one can reasonably assume that the maximum ratio of standard deviations between measurement error and underlying factor should be around 4.9.

Given that we have gauged the size of the measurement error, it is useful to examine its impact on factor contamination and statistical inference under the null (where the risk premium is zero).[17] We will describe our simulation procedures and results below.

5.1 Simulation to examine the factor contamination

5.1.1. Simulation procedure

Following the same notation used in Section 3, we write an observed factor ($\widetilde{f_t} = f_t + \varepsilon_{f,t}$) so that it contains two parts. The first part, f_t, is the true factor that resides in the space of excess returns, and the other part, ε_f , is the measurement error term that is uncorrelated with the excess return. The goal of the simulation is to examine the effectiveness of FMPs constructed by various methods to extract the true factor (f_t) from an observed factor ($\widetilde{f_t}$). If it is effective, the FMP should be highly corrected with the true factor (f_t) and not be correlated with another uncorrelated factor.

We use the four macro factors and three Fama-French factors (FF, henceforth) in the return-generating process. For the macro factors, we apply our single-factor IV approach with asset

[17] We do find that a very large measurement error can lead to a large bias in risk premium estimation when the true risk premium is nonzero. However, the risk premium estimation will not be affected when its true value is zero. In most asset pricing studies, we are mainly interested in the inferences on whether the factor significantly prices cross-sectional returns. The null hypothesis in this case is that the risk premium is zero. Hence, it is most important to examine the distributional properties of the test under this null hypothesis, which is the focus of the simulations in this paper.

selection twice to construct second-stage FMPs. These FMPs for macro factors are true factors in simulations. Since the three FF factors are traded factors, they contain no measurement errors. We assume that their original factors are the same as the true factors.

For each true macro factor, we orthogonalize the other true factors to make them uncorrelated. Orthogonalized factors are used for the data-generating process and for examining factor contamination.[18] $\boldsymbol{f}_t^{\perp}$ denotes the vector of all factors (the true factor and other orthogonalized factors). We calculate the mean, variance, and covariance (zero in this case) of the factors in $\boldsymbol{f}_t^{\perp}$. These values are used for all simulations.

We run the time-series regression for the returns of each asset (R_t^i) on the orthogonal risk factors ($\boldsymbol{f}_t^{\perp}$) to obtain the beta loadings ($\boldsymbol{B}^i$) and the residual, ε_{it}. Similar to the orthogonalized factors, we orthogonalize the beta loadings of the factors, which are expressed as $\boldsymbol{B}_i^{\perp}$. We orthogonalize the beta to satisfy the assumption in Section 3.2.2 (Eq. (16)), in which we assume that the cross-sectional correlation between the loadings of two orthogonalized factors converges to zero when *N* approaches infinity. In the real data, we find that, for each macro factor, the cross-sectional correlations between its factor loading and the loadings of other orthogonalized factors are small. Therefore, the assumption is close to being true in the real data. The mean, variance, and covariance (zero in this case) of the loadings, $\boldsymbol{B}_i^{\perp}$, are calculated for simulations.

With these orthogonalized factors and loadings, we proceed to the data-generating process of asset returns in the simulation. We first simulate orthogonalized factors using the Monte Carlo simulations and keep the mean, variance, and covariance of the factors the same as those from the data. For all simulated orthogonalized factors, we subtract the mean of these factors. The resultant factor is $\boldsymbol{f}_t^{true} = \boldsymbol{f}_t^{\perp} - \overline{\boldsymbol{f}^{\perp}}$. We also generate the factor loadings ($\boldsymbol{B}_i^{\perp}$) of all stocks for each factor from a multinomial normal distribution by keeping the same mean, variance, and covariance from the real data. The simulated return is computed as $R_{it}^s = \boldsymbol{f}_t^{true}\boldsymbol{B}_i^{\perp} + \varepsilon_{it}^s$, where ε_{it}^s is simulated

[18] We adopt Gram-Schmidt's orthogonalization process. Specifically, we regress the first control factor to consumption growth and use the residual as the proxy for the orthogonalized first control factor. Then we regress the second control factor on the orthogonalized first control factor and the consumption growth factor, and we use the residual term as the proxy for the orthogonalized second control factor. We follow the same procedure for the other control factors, so the resultant factor matrix has seven columns, and every column is orthogonal to the other columns.

from a normal distribution by keeping the same mean and variance of the regression residual for stock i, ε_{it}, from the real data.

Note that the orthogonalized factors are used for the data-generating process; researchers use the observed original factors for the asset pricing test. The observed original factors can be correlated, and there are measurement errors for the original macro factors. Given that only observed factors are used to construct the FMP, we need to simulate the observed factors. Since the three FF factors have no measurement errors, they are observed factors. The four macro factors contain measurement errors. Hence, the simulated observed factor is constructed by adding an error term—$\boldsymbol{f}_t^s = \boldsymbol{f}_t + \boldsymbol{v}_t$, with $\boldsymbol{f}_t$ and $\boldsymbol{v}_t$ independent—to the simulated return-related factor. The true factors, $\boldsymbol{f}_t$ (the factors not being orthogonalized), are simulated from a multinomial normal distribution with the same mean, variance, and covariance as those for the second-stage FMPs (assumed to be the true factors) in the real data. On the other hand, $\boldsymbol{v}_t$ is extracted from an independent normal distribution with a mean of zero. The standard deviation of $\boldsymbol{v}_t$ characterizes the size of the measurement error. We assume several scenarios. In the case that the measurement error is moderate, such as the correlation between the observed factors and when the true factor is 50%, the ratio of standard deviations between the measurement error and the true factor is 1.7 (by applying Eq. (20) and replacing FMP with the true factor since they are the same). However, the measurement error can be much larger for the macro factors. If the correlation between the observed factor and the underlying factor is only 20%, the standard deviation of v_t is 4.9 times that of the true factor, f_t. We also examine the cases in which the correlation is 10% or 30%, and the simulation results are in Internet Appendix D.

After simulating returns and observed factors with measurement errors, we apply various methods to construct FMPs for each macro factor. Since we perform a horse race among comparable approaches, we address the errors-in-variables issue. That is, we apply IVs to estimate the risk premium for all FMPs. For FMP construction, we only apply IVs for our proposed FMPs because we want to compare our proposed FMPs with the FMPs constructed from existing approaches.

Factor contamination occurs if the FMPs capture components that are not in its underlying factor but are found in other factors. We iterate the simulation 1,000 times and report the summary statistics for these correlations in Table 2.

5.1.2. Correlation between FMPs and their true factors

Table 2 reports our simulation results under the cases that the correlation between the true and observed underlying factors, ρ, is 50% and 20%, respectively. Table D1 in Internet Appendix D reports the results when the correlations are 10% and 30%. We first discuss the results when the correlation is 50%. Panel A in Table 2 reports the correlation between FMPs with their corresponding return-related factors (f_t). We do not examine the correlation between the FMP and the observed factor ($\widetilde{f_t}$), because the correlation depends on the variance of the measurement error, and the observed factors are not orthogonalized. Instead, the correlation between the FMP and the true factor can be as large as one. Taking consumption growth as an example, we find that the average correlation between the FMP from the FP method and the true consumption growth factor is 0.996. Hence, FMPs constructed by the FP method have a near-perfect correlation with the true factors.

The correlation between FMP_OLS and the true consumption growth factor is 0.729. FMP_Stein has the same correlation with the true factor as FMP_OLS because the Stein method only adds a scaling effect to the FMP_OLS. The averaged correlation between the FMP_LM and the true factor is 0.743, a value that is slightly higher than that for the FMP_OLS. These cross-sectional methods (such as OLS, LM, and Stein) could not yield FMPs with close-to-perfect correlations with true factors because of the factor contamination from using multiple regressions in the cross-sectional approach. Using a univariate regression with the asset selection method, the FP FMP alleviates these problems. The average correlation between FMPs constructed by the sorting-by-beta method and the true factors is small for 3 out of 4 factors. The time-series approach with the 25 FF portfolios as basis assets also has very low correlations with the true factors (0.099). The time-series method also suffers more from severe contamination issues (proposition 7); thus, low correlations are expected.

[Table 2 near here]

Panels B and C of Table 2 present the correlations between the FMPs for each macro factor and other orthogonalized factors.[19] In the first data column, we present the correlations for the

[19] The reason for not examining the correlation between the FMP of one macro factor and the other factor (which is not orthogonalized) is discussed below. Since the factors are correlated, for any macro factor, the correlation

consumption growth factors. The results for other macro factors are shown in consecutive columns, and they are qualitatively similar. The FMPs of consumption growth (CG) have six correlation coefficients with six other orthogonalized factors. We report the maximum and the average values across the absolute values of six correlation coefficients. Panel B lists the average values of the maximum value across 1,000 simulations. Panel C lists the mean values of the recorded average value across 1,000 simulations. Consistent with the analysis in Section 3, we find that the four-procedure FMP generates almost zero factor contamination. In Panel B of Table 2, on average, the maximal correlations between FMPs of any factor and the other true risk factors, on average, are 0.012 for the four-procedure method, 0.358 for FMP_OLS and FMP_Stein, 0.354 for FMP_LM, 0.627 for FMP_SB, and 0.041 for FMP_TS. In Panel C of Table 2, the average correlations between FMPs of any factor and the other true risk factors are 0.003 for FMP_FP, 0.164 for FMP_OLS and FMP_Stein, 0.168 for FMP_LM, 0.131 for FMP_SB, and 0.011 for FMP_TS. The magnitude is small for FMP_TS because of the large measurement error of the FMP as reflected by the low correlations between FMP_TS and their corresponding true risk factors in Panel A of Table 2.

Overall, these simulation results testify that the commonly used FMP construction methods (namely, sorting-by-beta, time-series, and multivariate cross-sectional methods) suffer from factor contamination. The method we propose (the FP FMP) in the univariate cross-sectional analysis with asset selection does not suffer from these issues.

We also examine the case in which the measurement error is large. Panels A, B, and C of Table 2 also show the results when the correlation is 20%, which corresponds to the case for the macro factors we examine in the real data.[20] Because of the large measurement error, the correlations between the FP FMPs and their corresponding factors drop (such as IP and UE). However, among all FMPs, FP FMPs perform best in terms of achieving the highest correlations

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between its FMP and other factors is not zero. But the nonzero correlation captures the component of the other factor that is also part of the underlying macro factors. For example, let consumption growth and market returns be correlated risk factors. If the FMP of the consumption growth factor is perfectly correlated with the true factor (implying that the FMP is not contaminated by other factors), it is still correlated with the market return. We examine the correlation with the orthogonalized factors. For example, suppose that the FMP of consumption growth is correlated with the orthogonalized market return, then the FMP captures the pricing component not belonging to market returns only, not other factors. In this case, the FMP is contaminated.

[20] We also report the results when the correlation is 0.3 and 0.1 in Table D1 in Internet Appendix D.

with the true factors. Moreover, only FP FMPs do not exhibit factor contamination even if the size of the measurement error is larger.

5.2. Simulation of the statistical inference of the risk premium under a large measurement error of factors

In the previous section, FMPs using our method are less correlated with the true factors when the measurement error is large. In this case, it cannot be eliminated by the one-stage Fama-Macbeth regression in the finite sample. Therefore, we propose the two-stage method to estimate the risk premiums of FMPs. However, the measurement error and the construction error can still affect the asset pricing test. Hence, in this subsection, we examine the inference of the risk premium estimated using our FP FMP.

We follow the same procedure from the previous subsection to simulate the stock returns and factors. We estimate the risk premium using our FP method over 1,000 trials. The mean of risk premium over these trials represents the bias of the estimation given that the true risk premium is zero. Then the distribution of 1,000 *t*-ratios is obtained. We report the mean risk premium and 2.5%, 5%, 50%, 95%, and 97.5% critical values of the *t*-ratio distribution in Table 3. Ideally, our proposed FP FMP should generate a risk premium that is close to the true risk premium, and the corresponding distribution of critical values should follow a normal distribution. Since the measurement error of the risk factor plays a crucial role in the statistical inference of the risk premiums of the FMPs, we report the results when correlations between the true and observed (noisy) risk factors are at the 30%, 20%, and 10% levels, respectively.

Panel A of Table 3 reports the mean of the estimated risk premiums of the FMPs across 1,000 simulations. When the correlation is equal to or larger than 20%, the bias of the risk premium estimation is close to zero, which is our preset true risk premium. The measurement error in the FMP matters in the finite sample only. We show in Table 3 that a moderate measurement error does not affect the risk premium estimation. However, when the correlation is equal to 10%, the bias of the risk premium estimation is much larger. For example, the estimated risk premium for CG is -0.05.

Panel B of Table 3 reports the critical values of *t*-statistics at the 2.5% to 97.5% level. When the correlation is at a 30% or 20% level, the distributions of critical values roughly follow the standard normal distribution. For example, when the correlation is at a 30% level, the critical value is -1.98 at the 2.5% level and is 1.97 at the 97.5% level, which is very close to the theoretical values of -1.96 and 1.96, respectively. However, when the correlation is only at the 10% level, the distribution of the critical values is far from the standard normal distribution. Specifically, the critical value to reject the hypothesis that the risk premium is different from zero at the 5% significance level is about 1.3, which is far below 1.96. This is also consistent with previous literature that shows a very large measurement error of risk factors can have spurious significant risk premiums.

In summary, when the measurement error of the risk factor is negligible (e.g., the correlation between noise and the true factor is larger than 20%, which corresponds to the real data), the bias in the risk premium and the standard deviation are small, and the *t*-ratio distribution is not very different from a standard normal distribution, in which the true risk premium is zero.

In Internet Appendix D (Table D2), we also examine the effect of the misspecification on the inference of the risk premium of our proposed FMPs. Specifically, we intentionally remove one FMP in the risk premium estimation stage (while the true data-generating process should include the factor associated with the FMP). The results are quite similar to the results without misspecifications, indicating the misspecification issue is not a severe concern for our proposed FP FMP, especially when the correlation between true and noisy observed factor is larger than 20%.

***Insert Table 3 around here ***

6. Applying FMPs in a test of the asset pricing model

This section applies FMPs constructed by various methods to test whether the underlying factors are risk related and can price assets. We focus on the FMPs of the four macro factors and augment this analysis with traded factors (the three FF factors) for comparison.[21] We examine the two criteria following Pukthuanthong et al. (2019) (PRS criteria):

[21] We also test the robustness by using alternative traded factors, such as the Carhart four factors, the FF five factors, and the FF six factors. We find these results are robust to these specifications. In some cases, we must drop the market factor to achieve robust results because of the strong correlation between the FMP of consumption growth and the market factors. Theoretically, consumption growth and the market factor should present a similar risk.

1. FMPs of factors are correlated with the systematic risk of returns, and

2. FMPs explain the cross-sectional of mean returns.

6.1. First criterion: Relation of FMPs with the covariance of asset returns

If the FMP of an underlying factor represents a risk factor, it should be correlated with the systematic risk of returns. Following Pukthuanthong et al. (2019), we test whether the FMP is related to the cross-sectional covariance of asset returns. We apply the asymptotic approach of Connor and Korajczyk (1988) (CK) to extract 10 principal components from the equities return series. The principal components of the covariance matrix of returns represent the systematic part of the asset returns. We then compute canonical correlations between the 10 CK principal components and the factor candidates and test the significance of these canonical correlations with the chi-squared statistic.

We examine this criterion for the FMPs constructed by various methods and their corresponding original factors as a comparison. The four original macroeconomic factors are those we have already considered above: CG, CPI, IP, and UE. The original traded factors are the three FF factors (MKT, SMB, and HML).

To pass this criteria, two conditions must be satisfied. We assume that an FMP strongly satisfies this criterion if it (1) is significantly related to any canonical variate in all decades or (2) has a mean *t*-statistic in the second row of each panel in Table 4 exceeding the one-tailed, 2.5% cutoff based on the chi-squared value, and (3) also has an average number of significant decade *t*-statistics exceeding 1.75 (bottom row of each panel).[22]

[Table 4 about here]

Notably, the four original macro factors do not pass, whereas the three FF factors pass this second criterion. FMPs constructed by all the cross-sectional methods and the sorted beta method satisfy this criterion. For FMP constructed by the time-series method, none of the macro factors passes the second criterion.

[22] Pukthuanthong et al. (2019) require an average number of significant decade *t*-statistics exceeding 2.5 from 10 factor candidates (one-fourth of the total number of factors). We have seven factor candidates; thus, 1.75 is from using the same proportion as theirs. Pukthuanthong et al (2019) say, "This is a conservative threshold to ensure we do not miss a true factor at our necessary condition stage. We focus on the significant canonical correlations, rather than all canonical correlations, because insignificant CCs imply that none of the factors matter, so using them would be over-fitting."

6.3. Second criterion: Risk premium estimation using FMPs

The FMP of an underlying factor that can price assets implies a significant risk premium. This section compares the extent to which various construction methods for FMPs produce different risk premium estimates. To avoid the EIV bias of beta loadings in the risk premium estimation stage, we apply the IV method following Jegadeesh et al. (2019) to FMPs constructed by various methods. Following Jegadeesh et al. (2019), we also winsorize extreme estimated coefficients from the Fama-MacBeth cross-sectional regressions at the 1.5% and 98.5% levels.[23]

The recent literature suggests that nontraded factors can have large measurement errors. When the measurement error is large, Kleibergen (2009) shows that the *t*-ratio distribution can be different from a standard normal distribution even if the null hypothesis is true. In this case, the standard Fama-Macbeth approach to test the third criterion cannot be applied. In Section 5 and Table 3, we show through simulations that this issue only starts to emerge in our applications when the correlation between the FMP and observed factor is smaller than 10%. When the correlation is at least 20%, these issues are not severe. In Panel A of Table 2, we gauge the correlation and find that the lower bound is 20%. Thus, the standard statistical inferences can be applied in this case.

6.3.1. Cross-sectional approaches

Panel A of Table 5 reports the risk premium estimates of FP FMPs and the FMPs constructed by other approaches. We select stocks whose betas in odd- and even-numbered months have the same signs. Only 463 (about 4.6%) of more than 10,000 stocks are never used to construct FMPs, with each factor using about 6,000 stocks. The asset selection also mitigates the weak IV issue. As we will show in Table 6, the correlation between IV and EV betas is much higher with asset selection.

[Table 5 about here]

The results show that risk premiums estimated by different methods often have dramatic disagreements in both magnitude and significance. For example, the risk premium for consumption growth is 0.203 (*t*-value = 2.843) using an OLS regression, while it is 0.164 (*t*-value = 3.238) using the FP method. The risk premiums for consumption growth for LM and Stein methods are smaller

[23] For the robust check, we also apply the classical Fama-MacBeth OLS regression. We find similar results, even though the estimated risk premiums are smaller.

than those for the FP. With OLS, LM, and Stein, the risk premium estimates for industrial production are negative, which is opposite to the theoretical prediction[24] and contrast markedly with the FP estimates. We find a negative risk premium for the unemployment rate. The risk premium for the unemployment rate should be negative because stocks with a positive unemployment beta can be viewed as hedging during economic downturns. Stocks with a negative unemployment beta are riskier because the returns for these stocks decrease during periods of high unemployment. For comparison, we estimate risk premiums for the three FF-traded factors. The signs and significance levels for the risk premium estimates do not vary much across methods.

When the model is misspecified, the test for macro factors can be misleading. Therefore, the first method to address this issue is to control for other factors. The first set of control variables is Fama-French factors. As revealed in Table 5, the FP risk premiums of most of the macro factors are still significant, even after including the three FF factors. But this is not true for the other FMP construction methods. Moreover, the risk premiums for the FF factors are virtually the same with any of the FMPs added into the estimation. We also present two other robustness tests to address the misspecifications issue in Section 6.5.

6.3.2. Time-series approach

Under the time-series approach, we perform the risk premium estimation with FMPs constructed by the Lamont (2001) method. Compared with the IV and OLS approaches, the estimated risk premium for consumption growth is negative only when we include FF factors, while the risk premium for industrial production is significantly negative but insignificant. This is opposed to the intuition that industry production and consumption growth should be associated with a positive premium.

6.3.3. Sorting-by-beta approaches

We estimate betas from time-series multivariate regressions and then sort the betas for stocks into 10 equally weighted deciles. We construct FMPs as the average return in the highest decile minus the average return in the lowest percentile (high-minus-low). The FMPs are used as factors to

[24] Giglio and Xiu (2021) obtain a similar result. The risk premium for IP is positive (although insignificant) with the four-procedure method. As a robustness check, we drop the FMP for the industrial production factor since it is not significant, and we find that the results for the other FMPs are virtually unaltered.

estimate risk premiums, which are reported in Panel A of Table 5. The estimated risk premium is significant only for CPI with or without including the three FF factors. CG and IP are significant only when we do not include FF factors.

6.3.4. Kan et al. (2013) approach

Kan et al. (2013) propose a method to control for the measurement error in the underlying factors. Instead of estimating beta loadings, the authors use the covariance between a factor and individual stocks as risk exposure. Then they estimate the risk premium by running a Fama-MacBeth regression of excess returns on risk exposures. Panel B of Table 5 reports the corresponding results. None of the four macro factors has significant explanatory power over the excess stock returns. This result is consistent with Proposition 6 and the simulation analysis in Section 5. The estimation error in the one-step approach can be enlarged by the measurement error, leading to a lower power for the asset pricing test.

6.4. Stock selection in the IV approach

As discussed in Section 3.5, returns for some stocks have little correlation with macro factors, thereby inducing a large dispersion between their betas in odd- and even-numbered months. Thus, the correlation between endogenous variables (EV betas) and instrumental variables (IV betas) could be very low in some cases, resulting in large variations in both the sign and the magnitude of the time-series FMP returns if the stocks with the low correlation with macro factors are selected. Therefore, we select only those stocks whose EV betas agree in sign with their corresponding IV betas to overcome the weak IV issue. Empirically, this uses 52% of the stocks, on average, for each factor, and only 6.5% of the stocks have never been used for all factors. For instance, 6,319 of 10,833 stocks are used to construct FMPs for consumption growth. The number of stocks used for CPI, industrial production, and the unemployment rate is 6,240, 4,288, and 6,501, respectively. Table 6 reports the correlation between EV and IV betas, measured as the correlation between EV and IV betas for the full sample and our adjusted sample.

[Table 6 about here]

We first compare the correlations between EV and IV betas when constructing FMPs (Panel A of Table 6) and compare the correlations between EV and IV betas when estimating the risk premiums using the two alternative FMPs (Panel B of Table 6). In Panel A, we can see that the correlations between EV and IV betas are very low for macro factors when using every stock. None of them is larger in absolute value than 0.1, and, even worse, 3 of the 4 are negative. The IVs seem to be very weak in this case. Instead, using the truncated sample that includes only stocks that have EV and IV betas with the same sign significantly improves the correlation. (For instance, the correlation between the EV and the IV betas for the FMP1 of consumption growth is 0.455.)

Panel B of Table 6 illustrates the benefits of mitigating the weak instrument problem on the risk premium test. Risk premiums are estimated using separate FMPs: FMP1 with IV and EV sign agreement and FMP2 with no restriction. The signs of the correlations between the EV and IV betas for industrial production (IP) and the unemployment rate (UE) are negative for FMP2, which is opposite of our assumption that betas are consistent across even- and odd-numbered months subsamples. The negative correlation is the result of weak IV problems. Consistent with our conjecture, the sample selection procedure strengthens the IV method.

6.5. Robustness: Giglio and Xiu's (2021) three-pass method and other controls

Although we find that the risk premium for consumption growth is significant when using its associated FMP, the estimated risk premium can be biased (Lewellen et al., 2011; Giglio and Xiu, 2021), and the *t*-ratio can be spurious (Kan et al., 2013; Jiang et al., 2015) when the model is misspecified. To address this issue, we first control for the Fama-French three factors in Table 5 and find that the results are robust to these controls. We also control for various combinations of the Fama-French six factors, and the results are robust in most of the specifications. Moreover, we can apply the three-pass method of Giglio and Xiu (2021) to estimate the risk premiums of our FP FMP. Internet Appendix G reports the results. Consistent with our main findings in Table 5, we find that consumption growth has significant risk premiums.

Lettau and Ludvigson (2001) derive a conditional consumption capital asset pricing model (CAPM) that can explain the average stock returns in the cross section; the authors use the consumption-to-wealth ratio (CAY) as a control variable. Table 7 reports the estimated risk premiums when adding the CAY factor. In all but two specifications, the CAY factor is associated with a negative risk premium and is always insignificant. The consumption growth factor has a significantly positive risk premium for the FP FMP. This result is consistent with our earlier results that use four macroeconomic factors. Following Lettau and Ludvigson (2001), we include an interaction term between CAY and consumption growth; its risk premium is insignificant. CG remains significant at the 5% level for FP and at the 10% level for the OLS and sorting-by-beta methods. When we include the three FF factors, CG is significant at the 1% level for FP but is either insignificant or has the opposite sign for the other approaches.

[Table 7 about here]

Boguth and Kuehn (2013) find that consumption volatility, supposedly a proxy for macroeconomic uncertainty, is also a source of risk and has a negative risk premium. In unreported results, we also add consumption volatility as a control variable. We find that the risk premium of the consumption volatility factor is negative but insignificant in all specifications across all FMP construction methods. In contrast, the risk premium for consumption growth is still significant. Therefore, using FMPs of consumption growth, we obtain results confirming that consumption growth is a robust risk factor that can explain the cross-sectional stock returns conditional on other consumption-related factors.

Our overall conclusion for U.S. equities is that FMPs constructed by our approach pass the two criteria of PRS (2019). Moreover, the FP FMPs dominate FMPs constructed by other methods in producing larger and more significant risk premium estimates. With the FP method, consumption growth, inflation, unemployment rate, and the three FF factors can explain cross-sectional stock returns.

7. Using FMPs to test risk premiums in the corporate bond market

Bond returns are related to firm fundamentals and are affected by the business cycle (Ludvigson and Ng, 2009). Fama and French (1993) propose two traded bond factors, namely, the default and term spread. Gebhardt et al. (2005) find that the default spread significantly explains cross-sectional bond returns even after controlling for bond characteristics, such as duration and rating. In contrast, Bai et al. (2019) find that attributes such as value-at-risk and rating dominate the default spread and term spread. Bessembinder et al. (2008) suggest that a broad bond market return, unexpected GDP growth, and unexpected inflation explain excess abnormal bond returns. Following these papers, we evaluate the FMPs for four macroeconomic factors, a broad bond market return (MKT_B), the default spread (DS), and a term spread (TS).

We use various FMP construction methods: FP, OLS, LM, Stein, time series, and sorting by beta. Following the PRS criteria, we first analyze the relation between FMPs and the principal components of the covariance matrix of individual corporate bond returns. We focus on one criterion (the *t*-statistic of significant canonical correlation) because our sample period for bonds is about 15 years. The results are similar to those for equities. Panel A of Table 8 presents the results. FMPs constructed by all approaches, except the time-series approach pass this criterion. For the time-series approach, only consumption growth and shock in CPI pass.

Panel B of Table 8 presents the results of estimating the risk premium for corporate bond returns in a similar vein as for equity returns. For FP FMPs, consumption growth and industrial production have significant risk premiums. The risk premiums of the macro factors associated with time-series Stein, LM, OLS, time-series, and sorting-by-beta FMPs are in general insignificant.

[Table 8 about here]

8. Conclusion

A voluminous literature constructs FMPs as the traded versions of non-traded factors and then applies them to estimate risk premiums. Our paper proposes a new economic explanation of FMPs, delves into the issues for existing methods, and offers a new method for FMP construction. Our four-procedure method encompasses a single-factor model, IV estimation, asset selection, and two-stage risk premium estimation. We also examine the macro factors using FMPs based on the criteria of PRS (2019) in terms of risk effect and pricing effect.

Simulation results show that many existing methods suffer from several econometric issues, whereas our method can adjust for these issues. Empirically, we apply our four-procedure method to estimate risk premiums for a series of nontraded factors. By applying the FMP method to the construction of FMPs for four classical macroeconomic factors, we find that consumption growth, CPI, and the unemployment rate have significant risk premiums for the stock market, and CG and industrial production have a significant risk premium for the corporate bond market. These conclusions cannot be obtained from the same factors constructed by other existing approaches in the literature.

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Table 1. Descriptive statistics.

This table reports the summary of statistics for the main variables, including the number of observations, mean, median, standard deviation, and percentiles (1st, 5th, 25th, 75th, 95th, and 99th). Panel A reports the statistics for excess stock returns and its explanatory variables. For the stock return, we have 10,833 stocks in total and 626 months of data. The stock returns are over a risk-free rate (1-month Treasury-bill rate). The explanatory variables include the consumption growth rate (CG), the consumer price index (CPI), industrial production (IP), the unemployment rate (UE), the excess stock market return (MKT), a small-minus-big size portfolio (SMB), a high-minus-low book-to-market portfolio (HML), and the consumption-to-wealth ratio (CAY). Panel B lists statistics for corporate bond returns and related explanatory variables. For corporate bond returns, we have 6,421 bonds and 179 months of data. The bond returns are over a risk-free rate (one-month Treasury-bill rate). In addition to the four macro variables (CG, CPI, IP, UE), bond market excess return (MKT_Bond), default spread (DS), and term spread (TS) are taken as explanatory variables. MKT_B is the equally weighted return of all corporate bond returns in our sample in excess of the risk-free rate. DS is a default spread, measured by the return difference between Moody's long-term corporate BAA-rated bonds and AAA-rated bonds. TS is a term spread, measured by the return difference between 10-year Treasury bonds and 1-year Treasury bonds. The sources of these data are described in detail in Section 4.

Panel A: Statistics for stock returns and its explanatory variables

	N	Mean	Median	SD	1st	25th	75th	99th
Stock return	1,784,351	1.012	0.440	13.341	–31.764	–5.194	6.209	42.179
CG	626	0.018	0.006	0.528	–1.557	–0.303	0.305	1.344
CPI	626	0.007	–0.006	0.248	–0.695	–0.124	0.145	0.594
IP	626	0.004	0.018	0.699	–1.985	–0.374	0.373	1.932
UE	626	0.003	0.001	0.161	–0.408	–0.097	0.107	0.403
MKT	626	0.490	0.785	4.466	–11.804	–2.100	3.450	11.178
SMB	626	0.229	0.130	3.108	–6.695	–1.520	2.050	8.435
HML	626	0.349	0.310	2.819	–8.097	–1.160	1.710	7.930
CAY	626	–0.002	–0.002	0.021	–0.046	–0.015	0.015	0.034

Panel B: Statistics for bond returns and its explanatory variables

	N	Mean	Median	SD	1st	25th	75th	99th
Bond return	331,728	0.389	0.236	2.544	–5.764	–0.349	1.089	7.248
CG	179	0.021	0.021	0.362	–1.248	–0.164	0.238	0.880
CPI	179	0.004	0.020	0.278	–0.875	–0.127	0.132	0.678
IP	179	–0.027	–0.003	0.648	–1.909	–0.359	0.330	1.355
UE	179	0.002	–0.003	0.154	–0.378	–0.098	0.105	0.376
MKT_bond	179	0.380	0.390	1.875	–6.031	–0.367	1.094	7.276
DS	179	1.093	0.960	0.469	0.579	0.853	1.218	3.090
TS	179	1.805	1.870	1.003	–0.374	1.235	2.590	3.368

Table 2. Simulation: The impact of measurement error on the relation between FMPs and their true factors.

This table shows the simulation results for the effectiveness of FMPs constructed by various approaches as a proxy of true risk factors. We generate simulated factors using a return-related factor added by a normally distributed measurement error. We generate simulated returns using orthogonalized true risk factors multiplied by orthogonalized true beta loadings. With the simulated return and simulated risk factors, we construct FMPs using the six methods described in Section 3.6. The details of the simulations are discussed in Section 5. Panel A shows the correlations between FMPs and their true factors, ρ. Panel B presents the maximum correlations between FMPs for a macro factor with other factors. Panel C presents the averaged correlations between FMPs for a macro factor with other factors. The last column of each table is the average value of the values of the four factors. In each table, we report two cases that the correlation between true and observed factors, $corr(\tilde{f}, f)$, is 50% and 20%, respectively. The values in each table are mean values across 1,000 simulations. The sample period is January 1964 to March 2016. To be included in the sample, individual stocks must have at least 60 continuous months of returns on CRSP. The macro factors include unexpected consumption growth (CG), unexpected changes in the CPI (CPI), unexpected changes in industrial production (IP), and unexpected changes in the unemployment rate (UE). Internet Appendix D reports that the correlation between true and observed factors, ρ, is 30% and 10%.

Panel A: Correlation between FMPs and their true factors

	$corr(\tilde{f}, f)$ = 50%				
	CG	CPI	IP	UE	Average
FMP_FP	0.996	0.995	0.927	0.917	0.959
FMP_OLS	0.729	0.806	0.813	0.944	0.823
FMP_Stein	0.729	0.806	0.813	0.944	0.823
FMP_LM	0.743	0.821	0.813	0.925	0.826
FMP_SB	0.998	0.740	0.172	0.151	0.515
FMP_TS	0.099	0.092	0.067	0.060	0.079
	$corr(\tilde{f}, f)$ = 20%				
	CG	CPI	IP	UE	Average
FMP_FP	0.974	0.968	0.770	0.746	0.864
FMP_OLS	0.798	0.849	0.576	0.773	0.749
FMP_Stein	0.798	0.849	0.576	0.773	0.749
FMP_LM	0.820	0.858	0.518	0.696	0.723
FMP_SB	0.993	0.702	0.126	0.102	0.481
FMP_TS	0.038	0.037	0.026	0.022	0.031

Panel B: Maximal correlation between FMPs of a target factor and other true risk factors

	$corr(\tilde{f}, f)$ = 50%				
	CG	CPI	IP	UE	Average
FMP_FP	0.003	0.005	0.030	0.009	0.012
FMP_OLS	0.360	0.531	0.401	0.141	0.358
FMP_Stein	0.360	0.531	0.401	0.141	0.358
FMP_LM	0.369	0.503	0.365	0.177	0.354

FMP_SB	0.019	0.658	0.928	0.904	0.627
FMP_TS	0.002	0.037	0.073	0.050	0.041
	$corr(\tilde{f}, f)$ = 20%				
	CG	CPI	IP	UE	Average
FMP_FP	0.005	0.008	0.040	0.006	0.015
FMP_OLS	0.220	0.297	0.553	0.258	0.332
FMP_Stein	0.220	0.297	0.553	0.258	0.332
FMP_LM	0.189	0.256	0.612	0.318	0.343
FMP_SB	0.023	0.643	0.813	0.868	0.587
FMP_TS	0.002	0.015	0.030	0.019	0.017

Panel C: Average correlation between FMPs for a target factor and other true risk factors					
	$corr(\tilde{f}, f)$ = 50%				
	CG	CPI	IP	UE	Average
FMP_FP	0.002	0.001	0.005	0.003	0.003
FMP_OLS	0.245	0.149	0.169	0.094	0.164
FMP_Stein	0.245	0.149	0.169	0.094	0.164
FMP_LM	0.240	0.148	0.163	0.122	0.168
FMP_SB	0.011	0.122	0.160	0.233	0.131
FMP_TS	0.001	0.007	0.015	0.022	0.011
	$corr(\tilde{f}, f)$ = 20%				
	CG	CPI	IP	UE	Average
FMP_FP	0.002	0.002	0.008	0.003	0.004
FMP_OLS	0.133	0.110	0.159	0.139	0.135
FMP_Stein	0.133	0.110	0.159	0.139	0.135
FMP_LM	0.117	0.110	0.157	0.170	0.138
FMP_SB	0.011	0.117	0.160	0.224	0.128
FMP_TS	0.001	0.003	0.007	0.009	0.005

Table 3. Simulation: The impact of measurement error on the inference of the risk premium.

This table shows the statistical inference of the risk premiums of FMPs estimated by our proposed method. We set the true risk premium to zero in the simulation. Panel A reports the mean value of the estimated risk premiums of FMPs. Panel B reports the critical values of the *t*-statistic at different percentiles. We report three cases in which the correlation between the true and observed factor, $corr(\tilde{f}, f)$, is 30%, 20%, and 10%, respectively. The values in each table are mean values across 1,000 simulations. The last column of Panel B is the simple average of the values of the four factors. The number of sample periods is 630. To be included in the simulations, individual stocks must have at least 60 continuous months of returns on CRSP. The macro factors include unexpected consumption growth (CG), unexpected changes in the CPI (CPI), unexpected changes in industrial production (IP), and unexpected changes in the unemployment rate (UE).

Panel A: The bias of the risk premium estimation

$corr(\tilde{f}, f)$	CG	CPI	IP	UE
30%	-0.001	0.002	0.002	0.000
20%	-0.002	-0.002	0.013	0.001
10%	-0.052	-0.011	0.058	-0.038

Panel B: The critical value of the risk premium estimation

	$corr(\tilde{f}, f) = 30\%$				
Percentile	CG	CPI	IP	UE	Average
2.5%	-2.046	-1.971	-1.890	-2.004	-1.978
5%	-1.673	-1.663	-1.688	-1.564	-1.647
10%	-1.277	-1.295	-1.281	-1.232	-1.271
50%	-0.021	0.003	0.003	-0.002	-0.004
90%	1.234	1.206	1.307	1.216	1.241
95%	1.546	1.620	1.610	1.661	1.609
97.5%	1.934	1.978	1.869	2.115	1.974
	$corr(\tilde{f}, f) = 20\%$				
Percentile	CG	CPI	IP	UE	Average
2.5%	-2.005	-2.155	-1.881	-2.014	-2.014
5%	-1.600	-1.646	-1.547	-1.598	-1.598
10%	-1.276	-1.222	-1.176	-1.225	-1.225
50%	-0.010	-0.087	0.099	0.001	0.001
90%	1.220	1.212	1.216	1.216	1.216
95%	1.588	1.572	1.597	1.586	1.586
97.5%	1.860	1.898	2.047	1.935	1.935
	$corr(\tilde{f}, f) = 10\%$				
Percentile	CG	CPI	IP	UE	Average
2.5%	-0.514	-1.042	-1.846	-1.872	-1.319
5%	-0.336	-0.706	-1.153	-1.260	-0.864
10%	-0.186	-0.461	-0.768	-0.875	-0.572
50%	0.000	0.002	0.011	-0.003	0.002

90%	0.237	0.439	0.736	0.799	0.553
95%	0.424	0.738	1.246	1.261	0.917
97.5%	0.683	0.962	1.860	1.795	1.325

Table 4. Relation with the covariance of asset returns.

This table reports the canonical correlations between FMPs and the principal components (PCs) of the covariance matrix of individual stocks. The factor candidates include FMPs constructed using the methods of FP, OLS, Lehmann and Modest (1988), Stein (1956), and Lamont (2001). As a comparison, we also include the canonical correlation results for the original risk factors. The PCs are extracted, as explained in Pukthuanthong et al. (2019), using the Connor and Korajczyk (henceforth CK, 1988) cross-sectional method. We summarize the significance levels of factor candidates. The following procedure is implemented to derive the significance levels of each factor candidate: First, for each canonical pair, the eigenvector weights for the 10 PCs are taken and the weighted average PC (which is the canonical variate for the 10 PCs that produce the canonical correlation for this particular pair) is constructed. Then a regression using each CK PC canonical variate as the dependent variable and the candidate factor realizations as the seven independent variables is run over the sample months. The *t*-statistics from the regression then give the significance level of each candidate factor. There are 10 pairs of canonical variates in each decade and a canonical correlation for each one; thus, there is a total of 50 regressions (10 regression per decade). The first row (Avg *t*) presents the mean *t*-statistic over all canonical correlations. The second row (Avg *t* sig. CC) reports the mean *t*-statistic when the canonical correlation itself is statistically significant. The third row (# decades) reports the average number of significant canonical correlations over the five decades. The critical rejection levels for the *t*-statistics are 1.65 (10%), 1.96 (5%), and 2.59 (1%). We assume that an FMP satisfies this criterion if it (1) is significantly related to any canonical variate in all decades or has a mean *t*-statistic in the second row that exceeds the one-tailed, 2.5% cutoff based on the chi-squared value, or (2) has an average number of significant *t*-statistics exceeding 1.75 (the third row of each panel). *t*-statistics breaching the 5% (1%) critical level appear in boldface. The factors that pass the necessary conditions are highlighted in gray. See Section 2 for a description of each method.

	FMP				Equity factors		
	CG	CPI	IP	UE	Rm.Rf	SMB	HML
				Original			
Avg *t*	1.14	1.15	1.10	1.01	10.66	6.70	3.31
Avg *t* (sig. CC)	1.25	1.37	1.04	1.14	**22.80**	**14.19**	**6.87**
# decades	1.40	1.40	1.40	0.60	2.80	2.80	2.60
				FMP_FP			
Avg *t*	2.43	2.57	1.89	2.22	9.71	5.00	3.30
Avg *t* (sig. CC)	**3.00**	**3.27**	**2.10**	**2.57**	**13.41**	**6.49**	**4.25**
# decades	3.00	3.20	2.20	2.80	3.40	3.80	3.20
				FMP_OLS			
Avg *t*	2.50	2.53	2.05	2.66	10.07	5.45	3.42
Avg *t* (sig. CC)	**3.01**	**3.12**	**2.41**	**3.39**	**13.80**	**7.18**	**4.46**
# decades	2.80	3.00	2.80	3.20	3.20	4.00	4.20
				FMP_LM			
Avg *t*	2.28	2.52	2.07	1.93	3.35	3.77	3.03
Avg *t* (sig. CC)	**2.72**	**3.13**	**2.28**	**2.30**	**4.21**	**4.79**	**3.97**
# decades	3.20	3.20	3.20	1.80	3.00	4.40	3.20
				FMP_Stein			
Avg *t*	2.50	2.53	2.05	2.66	10.07	5.45	3.42
Avg *t* (sig. CC)	**3.01**	**3.12**	**2.41**	**3.39**	**13.80**	**7.18**	**4.46**
# decades	3.40	3.40	3.60	3.00	3.00	4.00	3.00
				FMP_time series			
Avg *t*	1.39	1.57	1.37	1.40	9.79	6.74	3.25
Avg *t* (sig. CC)	**2.01**	**2.09**	1.63	1.70	**19.45**	**13.17**	**6.09**
# decades	2.00	1.80	1.80	1.80	3.40	3.20	3.60
				FMP_SB			
Avg *t*	2.01	2.80	2.16	2.15	6.54	5.42	5.23
Avg *t* (sig. CC)	**2.21**	**3.20**	**2.38**	**2.36**	**8.07**	**6.68**	**6.46**
# decades	2.40	4.00	3.20	3.20	3.40	3.60	3.40

Table 5. Risk premiums in equity market using factor-mimicking portfolios.

Panel A reports the risk premium estimates using the Fama-MacBeth regression. For cross-sectional methods, we assume betas are constant. For the FP method, we use betas in even-numbered months as instruments for betas in odd-numbered months and apply the IV method with sample adjustment to obtain FMPs (assets that have $\hat{\beta}_{IV}{}' \hat{\beta}_{EV} > 0$). Then we apply the IV method for a second time to test the risk premiums of these FMPs (FP). For other methods, such as OLS, Lehmann, Modest (LM), and Stein, we apply the corresponding approach (OLS, Lehmann, LM, Stein) to obtain factor-mimicking portfolios and use these methods to run Fama-MacBeth regressions again to test risk premiums. For the time-series (TS) approach, we use time series to create FMPs and then estimate the risk premiums for these FMPs using the OLS method in the second-pass regression. For the sorting-by-beta (SB) method, we estimate betas from time-series multivariate regressions and then sort the betas for each factor into 10 deciles. We then construct FMPs as the arithmetic average returns in the highest decile minus the average in the lowest percentile (HML). The FMPs are used as factors to estimate the risk premium, which is reported in the table. In Panel B, the sample period is January 1964 to March 2016. We use individual stocks that have at least 60 continuous months of returns on CRSP. The risk factors include four macroeconomics variables, the consumption growth rate (CG), unexpected CPI changes (CPI), unexpected changes in industrial production (IP), and unexpected changes in the unemployment rate (UE). MKT is the excess market return (proxied for by the value-weighted return of all CRSP firms in the United States); SMB is the FF small-minus-big size factor; and HML is the FF high-minus-low book-to-market factor. The values in parentheses are *t*-statistics. **p* < 0.1; ***p* < 0.05; ****p* < 0.01.

Panel A: Risk premium of various approaches

	Intercept	CG	CPI	IP	UE	MKT	SMB	HML
				Cross-sectional approach				
FP	0.464***	0.164***	–0.017	0.009	–0.022**			
	(3.980)	(3.238)	(–0.920)	(0.192)	(–2.000)			
OLS	0.617***	0.203***	-0.068**	-0.089	-0.078			
	(4.907)	(2.843)	(-2.371)	(-0.660)	(-1.352)			
LM	0.553***	0.103***	-0.034**	-0.02	-0.027**			
	(4.944)	(2.586)	(-2.341)	(-0.508)	(-2.291)			
Stein	0.617***	0.122***	-0.053**	-0.037	-0.044			
	(4.907)	(2.843)	(-2.371)	(-0.660)	(-1.352)			
FP	0.515***					0.553**	0.281*	–0.474***
	(4.805)					(2.281)	(1.816)	(–3.411)
OLS	0.426***					0.599***	0.06	-0.409***
	(4.306)					(3.079)	(0.450)	(-2.806)
LM	0.342***					0.723***	0.019	-0.390***
	(3.430)					(3.651)	(0.157)	(-2.958)
Stein	0.426***					0.598***	0.054	-0.405***
	(4.306)					(3.079)	(0.450)	(-2.806)
FP	0.653***	0.136**	–0.048**	0.094	–0.028**	0.545**	0.262*	–0.487***
	(5.985)	(2.070)	(–2.132)	(1.606)	(–2.134)	(2.269)	(1.835)	(–3.766)
OLS	0.424***	-0.057	-0.039**	-0.058	0.006	0.732***	0.062	-0.387***
	(3.833)	(-1.110)	(-2.276)	(-0.999)	(0.440)	(3.709)	(0.506)	(-3.084)
LM	0.213	-0.232**	-0.046**	0.072	-0.015	0.923***	0.009	-0.486***

	Intercept	CG	CPI	IP	UE	MKT	SMB	HML
	(1.335)	(-1.985)	(-2.140)	(0.973)	(-0.670)	(4.068)	(0.061)	(-3.042)
Stein	0.424***	-0.281	-0.092**	-0.204	0.015	0.736***	0.063	-0.446***
	(3.833)	(-1.110)	(-2.276)	(-0.999)	(0.440)	(3.709)	(0.506)	(-3.084)
Time-series approach								
Model 1	2.154**	-0.152	0.057	-0.274	0.036			
	(2.318)	(-1.004)	(0.540)	(-1.059)	(0.790)			
Model 2	-2.129*					3.176**	1.316	-1.112
	(-1.882)					(2.466)	(1.361)	(-1.383)
Model 3	-0.166	-0.211**	0.1	-0.097	0.037	1.798**	-0.371	-1.322***
	(-0.223)	(-2.017)	(1.114)	(-0.525)	(1.466)	(2.076)	(-0.783)	(-2.577)
Sorting-by-beta approach								
Model 1	0.705***	0.879***	-0.516**	-0.449*	-0.304			
	(5.270)	(3.011)	(-2.251)	(-1.692)	(-1.065)			
Model 2	0.549***					0.717**	0.253	-0.605**
	(5.382)					(2.315)	(0.832)	(-2.116)
Model 3	0.637***	0.253	-0.843***	0.107	-0.348	0.683**	0.244	-0.664**
	(5.886)	(0.682)	(-3.002)	(0.320)	(-1.111)	(2.181)	(0.779)	(-2.350)

Panel B: Risk premium estimation using Kan et al.'s (2013) approach

Intercept	CG	CPI	IP	UE	MKT	SMB	HML
0.814***	0.170	-0.125	-0.093	0.206			
(4.715)	(1.486)	(-0.674)	(-1.291)	(0.625)			
0.355***	-0.066	0.079	-0.052	0.114	0.023***	0.012	-0.010
(4.246)	(-0.736)	(0.439)	(-0.749)	(0.377)	(2.698)	(1.057)	(-0.890)

Table 6. The strength of instrumental variables.

This table presents the strength of instrumental variables measured as the correlations between EV betas and IV betas. Panel A presents the correlations in constructing the FMPs, and panel B presents the correlations between IV and EV betas for the second stage, where we estimate the risk premiums of FMPs. The sample with "EV*IV>0" includes only stocks whose EV beta and IV beta have positive signs denoted by "FMP1." "All" includes all individual stocks, denoted by "FMP2." In the second stage, we use all stock returns as our sample and either FMP1 or FMP2 from the first stage to estimate risk premiums. The sample period is January 1964 to March 2016. To be included, an individual stock must have at least 60 contiguous monthly returns in CRSP. See Table 1 for the notation of the factor candidates we use. We consider three specifications, four macro factors, three Fama-French factors, and combined four macro factors along with FF three factors.

Panel A: Correlations between IV and EV betas in the first stage (constructing FMPs)

	Sample	CG	CPI	IP	UE	MKT	SMB	HML
FMP1	EV*IV>0	0.477	0.616	0.577	0.606	0.638	0.572	0.756
FMP2	All	-0.006	0.045	-0.025	-0.019	0.474	0.353	0.311

Panel B: Correlations between IV and EV betas in the second stage (estimating the risk premiums of FMPs)

	FMP	CG	CPI	IP	UE	MKT	SMB	HML
Macros	FMP1	0.455	0.266	0.252	0.229			
	FMP2	0.112	0.440	-0.384	-0.295			
FF3	FMP1					0.343	0.314	0.358
	FMP2					0.571	0.514	0.435
Combined	FMP1	0.136	0.172	0.207	0.160	0.276	0.279	0.294
	FMP2	0.007	0.081	-0.147	-0.018	0.473	0.386	0.305

Table 7. Estimated risk premiums with FMPs for consumption growth and CAY

This table shows the estimated risk premiums for consumption growth and the CAY factor of Lettau and Ludvigson (2001) and with and without the three FF factors. CAY is the log ratio of consumption to aggregate wealth. FMPs are constructed for each factor using four methods [FP, OLS, time series, and sorting by beta (OLS-SB)]. CG is the unexpected consumption growth rate. MKT is the excess market return; SMB is the FF small-minus-big size factor; and HML is the FF high-minus-low book-to-market factor. We obtain the monthly CAY from Martin Lettau's websites. The monthly sample is from January 1964 to March 2016. The *t*-values in parentheses are based on Newey-West standard errors. *$p < 0.1$; **$p < 0.05$; ***$p < 0.01$.

	Intercept	CAY	CG	CG*CAY	MKT	SMB	HML
FP	0.751***	–0.171*					
	(4.002)	(–1.757)					
OLS	0.802***	-0.103*					
	(5.152)	(-1.680)					
Time series	0.534*	-0.355					
	(1.861)	(-0.982)					
SB	0.871***	-0.236					
	(5.102)	(-1.052)					
FP	0.566***		0.125**	–0.003			
	(3.761)		(2.017)	(–0.022)			
OLS	0.764***		0.052	0.017			
	(3.382)		(1.179)	(0.189)			
Time series	0.145		0.073	0.14			
	(0.369)		(1.037)	(1.519)			
SB	0.800***		0.447*	0.353			
	(5.444)		(1.654)	(1.147)			
FP	0.498***	–0.08	0.150**	–0.03			
	(3.384)	(–0.920)	(2.391)	(–0.229)			
OLS	0.520***	-0.136**	0.058*	0.001			
	(4.151)	(-2.139)	(1.811)	(0.013)			
Time series	0.214	-1.625***	0.004	-0.008			
	(0.620)	(-3.977)	(0.084)	(-0.124)			
SB	0.770***	-0.575***	0.502**	-0.114			
	(5.358)	(-2.798)	(2.069)	(-0.369)			
FP	0.584***	0.135	0.238***	–0.197	0.635**	0.361**	–0.504***
	(5.452)	(1.145)	(2.692)	(–1.165)	(2.494)	(2.166)	(–3.169)
OLS	0.388***	-0.161	-0.086**	-0.055	0.668***	0.172	-0.389***
	(3.703)	(-1.570)	(-2.054)	(-0.625)	(3.387)	(1.381)	(-3.157)
Times series	-1.347	0.049	-0.07	0.168	2.804*	-0.741	0.393
	(-1.089)	(0.044)	(-0.480)	(1.096)	(1.818)	(-0.690)	(0.800)
SB	0.812***	-1.919	-0.813	1.092	0.897	-0.11	-0.339
	(4.344)	(-1.350)	(-0.661)	(0.879)	(0.904)	(-0.061)	(-0.416)

Table 8. Testing FMPs to explain individual corporate bond returns.

This table shows the four criterion tests for FMPs constructed from various approaches and with individual corporate bond returns as the basis assets. Panel A presents canonical correlations of FMPs with asymptotic PCs using the same approach described in Table 4 for equities. Panel B shows risk premiums for corporate bond returns estimated with FMPs as factors. These factors are described in Table 1. The sample period is August 2002 to June 2017. The numbers in parentheses are *t*-statistics. $^*p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$.

Panel A: Canonical correlations with asymptotic PCs and the significance levels of factor candidates

	CG	CPI	IP	UE	MKT	DS	TS
			Original				
Avg *t*	1.18	1.04	1.96	1.03	4.98	1.55	1.09
Avg *t* (sig. CC)	1.51	0.98	4.90***	0.72	16.09***	2.34**	0.98
			Cross-sectional approach				
			FMP_FP				
Avg *t*	5.35	5.02	4.75	2.12	4.54	4.51	3.52
Avg *t* (sig. CC)	5.35***	5.02***	4.75***	2.12**	4.54***	4.51***	3.52***
			FMP_OLS				
Avg *t*	4.48	4.43	6.90	3.54	10.12	3.10	3.14
Avg *t* (sig. CC)	5.07	5.01	8.03	3.98	11.67	3.23	3.63
			FMP_Stein				
Avg *t*	4.48	4.43	6.90	3.54	10.12	3.10	3.14
Avg *t* (sig. CC)	5.07	5.01	8.03	3.98	11.67	3.23	3.63
			FMP_LM				
Avg *t*	3.41	3.55	5.44	2.02	7.00	3.27	2.26
Avg *t* (sig. CC)	4.42	4.63	7.38	2.30	9.54	3.96	2.54
			Time-series approach				
Avg. *t*	1.99	1.88	1.33	0.98	2.60	2.26	1.55
Avg *t* (sig. CC)	2.39	2.53	1.75	0.69	3.53	2.77	1.79
# of sig	2	2	2	1	4	5	3
			Sorting-by-beta approach				
Avg. *t*	5.25	4.64	4.56	4.88	6.99	5.10	3.84
Avg *t* (sig. CC)	6.00	5.04	5.07	5.68	8.05	5.94	4.33
# of sig	4	6	4	6	4	6	3

Panel B: Estimated risk premiums using FMPs

	Intercept	CG	CPI	IP	UE	MKT_B	DS	TS
				Cross-sectional approach				
FP	0.892***	0.291***	–0.049	0.586***	–0.090	0.376*	0.176**	–0.355
	(3.838)	(3.082)	(–0.589)	(2.838)	(–1.174)	(1.870)	(2.275)	(–1.389)
OLS	0.081	-0.035	0.003	0.082	-0.013	0.178	0.042	-0.012
	(1.045)	(-0.546)	(0.076)	(0.776)	(-0.551)	(1.131)	(0.599)	(-0.100)
LM	-1.400	-0.291	-0.060	0.023	0.311	1.161	-0.315	-0.286
	(-1.145)	(-0.514)	(-0.132)	(0.035)	(0.734)	(0.977)	(-0.461)	(-0.271)
Stein	0.081	-0.125	0.005	0.156	-0.052	0.182	0.107	-0.066
	(1.045)	(-0.546)	(0.076)	(0.776)	(-0.551)	(1.131)	(0.599)	(-0.100)

				Time-series approach				
Time series	-0.594**	0.033	0.027	-0.465***	0.057	1.409***	0.288***	1.284***
	(-2.339)	(0.509)	(0.405)	(-3.073)	(1.488)	(3.071)	(3.243)	(3.783)
				Sorting by beta approach				
Sorting by beta	0.083	-0.837**	-0.069	0.292	-0.012	0.325	1.041***	0.006
	(0.720)	(-1.986)	(-0.208)	(1.033)	(-0.059)	(1.164)	(3.333)	(0.020)