Changing Expected Returns Can Induce Spurious Components in Autocorrelations*

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Abstract

Shifts in expected returns (*ER*) can cause a bias in measured autocorrelations, and the resulting spurious component is usually positive for infrequent regime shifts in *ER*. We demonstrate this point with examples and investigate its prevalence in equity and corporate bond data. In a key contribution, we use shifts in *ex ante* expected return estimates from options prices, factor models, and analysts' forecasts to investigate our premise. As predicted, absolute shifts in expected returns are indeed strongly and positively related to autocorrelations. We also show that shifting *ER* implies spurious *cross*-autocorrelation and find supporting evidence for this phenomenon as well.

Keywords: Efficient Markets, Serial Correlation, Spurious Results
JEL classifications: G10, G14

Introduction

For many reasons, expected returns (henceforth, *ER*) on assets change over time. For instance, if bonds have risk premia related positively to term, their *ER* decline as they mature. Equities are also subject to changing *ER* because lines of business evolve, firms issue more debt or retire some, they become larger, more mature and have longer and more familiar records of performance; there are probably many other driving reasons for *ER* shifts.

The aim of our paper is to investigate the idea that return autocorrelations (henceforth, *AR*) can have spurious components when returns are lumped together across regimes with different *ER*. Within each regime, when the *ER* is relatively constant, return *AR* can be close to zero, yet combining different regimes and computing an unconditional *AR* will frequently find it to be positive. As we show in simulations, this bias can be substantial, particularly when return volatility is low. The effect, however, is illusory in that no profit can be earned on the seemingly non-zero serial correlation.[1]

To give a simple example, consider a stock that has no debt for a given decade and then borrows large amounts during the next decade.[2] Its debt to total asset ratio is, say, 50%, in the second decade, so it is quite a bit riskier, and its market beta is larger. Just for illustration, assume its beta is 1.0 in the first decade and then, using a simple leverage adjustment with no taxes, is 2.0 in the second decade. Suppose its *ER* in the first decade is the broad market's average, say six percent per annum or 0.5% per month. Assuming that the riskless rate is zero for ease of illustration, its *ER* in the second decade is twice the market's, or 1% per month. Each of the three panels of Figure 1 shows simulated monthly returns for two decades that involve

[1] We present how our paper differs from the studies in this branch in the detailed literature review to follow.

[2] This is not unusual among US industrial firms; see DeAngelo and Roll (2015).

doubling of the *ER* in the second decade, and with different levels of return volatility in the three panels.

Panel A illustrates the scatter diagram of returns in adjacent months when the volatility of returns is 0.1% per month. The problem is readily apparent. There are two distinct (and obvious) clusters centered on means of .5% and 1%. The return *AR* within the first (second) decade is -0.028 and 0.008; neither is statistically significant. In contrast, the unconditional *AR* with all observations lumped into a single calculation is 0.864 and is highly significant. (The solid plotted line depicts the unconditional OLS regression of returns on first-order lagged returns while the two dashed lines depict the conditional regressions.) Although this simple illustration deals with first-order *AR*, it should be obvious that second- and higher-order *AR* will also be spuriously overstated. This will continue until the *AR* lag relates a return in the second decade to a return in the first, i.e., it relates t to $t-s$ where t has one *ER* and $t-s$ has the other. For *AR* orders greater than this point, the bias will be negative, not positive.

The observed monthly return on a stock is its expected plus its unexpected return; if the volatility of the latter is relatively large, the bias in *AR* might not be so obvious. To illustrate, Panel B of Figure 1 considers a monthly standard deviation of 0.2%, or double that in Panel A. The scatter is now considerably less obvious. The first-order return *AR* remain the same within each constant mean regime (indeed, *AR* is unaffected by *ER* whatever its value) but the unconditional *AR* falls from 0.864 to 0.621. This again might be misleading to an observer who does not notice the clustering and might wish to exploit a seeming profit.

Going further into a more realistic situation with abundant volatility, an individual stock's return standard deviation is about 30% per annum, or, with no serial correlation, about 9% per month, which is much higher than the volatilities in Panels A and B of Figure 1. Indeed, as Panel C below shows, it is really impossible to discern any clustering with such a level of volatility; the unconditional *AR* has fallen to −0.007.[3] Yet the clustering is still there. The underlying data are identical except for the volatility. The *ER* are still 0.5% and 1%, respectively, in the two decades, but the noise is so overwhelming that the clustering is completely hidden.

The simple examples in Figure 1 are undoubtedly too simple. There is little reason why a firm should have a constant *ER* in one decade and another constant in a second decade. Indeed, changes in *ER* must evolve for some firms at a rather steady pace while those of other firms are sudden (such as when the firm issues substantially more debt), which could occur anywhere in the record of observations. In addition to the endemic volatility of unexpected returns, such diverse experiences of various firms seem likely to render detection all that more difficult.

We should also mention that the phenomenon under study in this paper affords a method for detecting changes in *ER*. If markets are efficient, serial correlation should be small and insignificant. Consequently, under the assumption of market efficiency, return *AR* is intrinsic evidence that *ER* has changed during the sample. Note that the argument that variations in *ER* can influence *AR* is not new, as we see in the next section. A novel observation in this paper is the observation that it can create spurious *AR*, with a bias that is *generally positive (assuming, as we show, that regime shifts in ER are infrequent).* We provide analytical verification of this observation, and investigate whether the data provide evidence in favor of the existence of spurious components to *AR*.

Our second contribution in this paper is to link *AR* to shifts in *ex ante* measures of *ER*, obtained not only from realized returns, but also from options markets, analysts' price targets, and a standard factor model. We are not aware of earlier attempts to measure *AR* bias using actual ex ante metrics for *ER*.

[3] The solid and dashed regression lines in Panel C cannot be distinguished from the horizontal axis.

Literature review

We now review the main studies on *AR*. First, examining autocorrelation in portfolios, Fama and French (1988), Ball and Kothari (1989), and Poterba and Summers (1988) document positive *AR* over short horizons and negative *AR* over long horizon. The last study finds this evidence in both the US and 17 international stock indexes. Richardson and Stock (1989) stipulate that the joint testing of the hypothesis that portfolio autocorrelations at various lags jointly equal zero is hampered when using long-horizon data. This is due to the paucity of truly independent observations.

The evidence and explanation of the *AR* of individual stock returns, which is the focus of this study, is different from those of portfolio returns. While daily returns of market indices exhibit pronounced positive *AR*, individual stocks exhibit on average only slightly positive first-order *AR*. Fama (1965, 1976), French and Roll (1986), and Lo and MacKinlay (1988,1990) find that short-horizon individual security returns tend to be positively autocorrelated with no empirical evidence of significant *AR* for higher lags. They also show the returns of many securities are negatively autocorrelated, but larger firms' stocks tend to exhibit weak positive *AR* (see Chan, 1993; Sias and Starks, 1997; Chordia, Roll, and Subrahmanyam 2005). French and Roll (1986) show that the *AR*(1) of the returns of the largest three equities of NYSE and Amex stocks are positive. Atchison et al (1987) report average *AR* close to zero.

Outside the US, Safvenblad (2000) examines the *AR* of individual stocks traded on the Stockholm Stock Exchange (SSE) and finds a positive *AR* on average with larger firms exhibiting a higher *AR*. In less liquid markets, positive return *AR* has been observed in several markets such as Austria (Huber, 1997), Finland (Berglund and Liljeblom, 1988), Israel (Ronen, 1998), and Malaysia and Singapore (Laurence, 1986).

Autocorrelation can vary with the length of time horizon. The literature has shown the time-series of short-horizon returns of individual stocks and portfolios reveal intriguing properties. Campbell (1987), French and Roll (1986), Keim and Stambaugh (1986), Conrad and Kaul (1988,1989), Lo and MacKinlay (1988) and Mech (1993) show that weekly and monthly portfolios returns are significantly and positively autocorrelated and that a positive *AR* is inversely related to firm size.

The underlying causes of a short-term *AR* could be different from those inducing a long-term *AR*. Ball and Kothari (1989) argue that trading frictions explain short-term *AR*. They argue that short intervals are characterized by noise that masks the impact of risk shifts and nonstationary *ER*. Studying daily returns of stocks, Brown, Harlow, and Tinic (1988) argue that short-term negative serial correlation is caused by bad information whose effects are reversed in prices. Campbell et al. (1993) postulate that, if the main motive for trading is informationless hedging, then extreme short-term stock returns, positive or negative, will tend to be reversed when they are associated with large trading volume.

Over longer intervals, Fama and French (1988), French, Schwert, and Stambaugh (1988), and Poterba and Summers (1988) present simple frameworks in which aggregate returns show negative serial correlation. The settings assume that the aggregated *ER* is autocorrelated but mean-reverting and that revisions in these returns are independent of revisions in aggregate expected future dividends. Fama and French (1988) and Poterba and Summers (1988) argue *AR* is weak for the daily and weekly holding periods but stronger for long-horizon return. A large negative *AR* for return horizons beyond a year suggests that predictable price variation due to mean reversion accounts for large fractions of 3-5-year return variances. Fama and French (1988) posit a mean reverting process for *ER*. They argue that expected returns are positively correlated in the short term but mean revert in the long run, thus generating negative serial correlation over long horizons. Their paper is different from ours in two ways. First, we do not posit any particular process for *ER*;

i.e., any change in *ER* will suffice. Second, they do not remark that changes in *ER* produce positive *AR*s over short horizons. In Section I, we show algebraically why this happens. Third, a predictable process for *ER* allows for profit enhancement by astute longer-term trading, while we argue that no profit is possible relative to the conditional *ER* in each period.[4] Ball and Kothari (1989) provide supporting evidence of negative serial correlation over five years and argue the changing risks should be more apparent at longer horizons.

Using weekly returns of size-based portfolios over the 1962-85 period, Conrad and Kaul (1988) characterize the time pattern of *ER* by a stationary first-order autoregressive process. They document strong and positive first-order *AR* for all portfolios and even second-order *AR* for the smaller stock portfolios. They dismiss microstructure as an underlying cause. However, they do not mention (1) that non-stationary *ER* alone is sufficient to produce the positive *AR*, or (2) that the observed positive *AR* is spurious in the sense that it does not provide a profit opportunity. In short, they present empirical evidence that *ER* is non-constant and that *AR* is positive but they do not show that the former is the cause and the latter is the effect.

Conrad and Kaul (1989) extend Conrad and Kaul (1988) by modeling the monthly *ER* as a decaying function of the weekly *ER*. They show the mean reversion in the shorter-horizon (weekly) *ER* explains a significant proportion of the variation in the monthly *ER* and that the observed return variance is explained by the variation of the *ER*. The impact of the latter is more pronounced for small firm portfolios. Our paper is different from these papers. Specifically, we make the point that a change in *ER* over time, for any reason, will produce spurious short-horizon *AR*s if the computation of the *AR* uses a constant mean. We say the computed *AR* is spurious

[4] Roll (1970) applies a similar argument to treasury bonds and finds that much of the serial dependence in raw T-Bill returns is accounted for by changes in the level of the one-period yield. See also Anderson (2011) and Anderson et al. (2013).

because it does not imply a profit opportunity which would be zero in an efficient market if the conditional mean were used rather than a constant mean.

Campbell (2018, p. 138) lists three distinct sources of autocovariance in *ex post* returns: (a) the positive covariance between dividend news and revisions in expected returns, (b) the negative capital loss that occurs when expected returns unexpectedly increase, and (c) the direct autocovariance of expected returns.[5] We do not consider (a) or (b) above, because our main point is that structural (permanent) changes in *ER* can induce unconditional return autocovariance, as distinct from the conditional autocovariances induced by these rationales. Also, regarding (c), there are numerous reasons for intertemporal movements in *ER*, such as permanent shifts in a firm's risk profile or alterations in the business climate or in the firm's policies, *inter alia*, going beyond a well-specified stochastic process for *ER* In other words, our focus is generally on unconditional autocorrelation over lengthy samples associated with large and permanent changes in *ER*.

Brennan and Wang (2010) make a different point on autocorrelations: When prices depart from fundamentals, but the departure is zero on average, expected returns are biased due to Jensen's inequality. They show that this bias in expected returns is related to autocorrelations in factor model residuals. We instead make the point that changing expected returns affect return autocorrelations. Beyond the differences from earlier papers delineated above, of course, we also advance the literature by considering the relation between *AR* shifts and changes in actual estimates of *ER* from options prices and analysts' price forecasts.[6]

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[5] See also Campbell (1991), and Pástor and Stambaugh (2009, 2012).

[6] Alti, Johnson, and Titman (2022) explore how persistence in ER can affect conclusions on whether anomaly-sorted portfolios truly exhibit long-lasting premia. They do not, however, use specific ER proxies. Bogousslavsky (2016) argues that investors' rebalancing horizons in response to liquidity shocks influence true autocorrelations. DeMiguel, Nogales, and Uppal (2014) argue that a vector autoregressive model across multiple stocks comprehensively captures true levels of auto- and cross-dependence in returns. In contrast to these papers, we show how variations in *ER* can cause *spurious AR*.

Cross-autocorrelations

Although our studies focus on time-series *AR*, a large group of studies from 1990s focuses on cross-autocorrelations (cross-*AR*). Lo and MacKinlay (1990) and Mech (1993) present cross-*AR* patterns in an attempt to investigate the sources of contrarian profits. They argue that the returns on a portfolio of small stocks is correlated with lagged returns of a large stock portfolio, but not vice versa.

The most common explanation of cross-*AR* is that the time series of stock prices are not sampled synchronously. Atchison et al (1987), Lo and MacKinlay (1988, 1990) and others argue that some of the cross-autocorrelations might be attributed to nonsynchronous trading problems, but to claim all of them suffer that requires markets to be unrealistically thin. Boudoukh, Richardson and Whitelaw (1994) show the cross-*AR* between large and small stock portfolio can be explained by the *AR* of small stock portfolios.

Controlling for firm size, Chordia and Swaminathan (2002) show daily and weekly returns on high volume portfolios lead returns on low volume portfolios. Nonsynchronous trading or low volume portfolio *AR* cannot explain this finding, but it appears that the traded returns of low volume portfolios react more slowly to information in market returns. Avramov, Chordia and Goyal (2006) show that controlling for trading volume, illiquidity causes negative cross-*AR* for individual stocks as price pressures caused by non-informational demands for immediacy are accommodated. Lewellen (2002) relates negative cross-autocorrelations across portfolios to cross-sectional momentum.

In Section VI we show that shifting *ER* across assets create spurious components to cross-AR that depend on the loadings of the assets on risk-factors. We test for the existence of such components, and find empirical support.

I. The algebra for *AR* across regimes with different expected returns

In this section, we present the basic details for the impact of shifting *ER* on *AR*. We first present a simple case with two regimes, and then move on to the multiple regime case.

I.1 Two regimes

In the simplest case we consider, there are two disjoint regimes, A and B, which have different *ER* and (possibly) different variances. The first, Regime A, lasts for a fraction α of all observations and the subsequent Regime B follows and lasts for the complementary fraction. If *ER* in the two regimes are denoted, respectively, μ_A and μ_B, the unconditional *ER* is $\bar{\mu} = \alpha\mu_A + (1-\alpha)\mu_B$.

An expected first-order unconditional autocovariance for regime A, computed with the unconditional *ER*, is $E[(R_{A,t} - \bar{\mu})(R_{A,t-1} - \bar{\mu})]$,[7] where $R_{A,t}$ is the return during Regime A at time t and, of course, μ_A =E$[R_{A,t}]$ with analogous expressions for Regime B.[8] This autocovariance can also be expressed as

$$E[\big(R_{A,t} - \mu_A + \mu_A - \bar{\mu}\big)\big(R_{A,t-1} - \mu_A + \mu_A - \bar{\mu}\big) =$$
$$E\big[\big(R_{A,t} - \mu_A\big)\big(R_{A,t-1} - \mu_A\big)\big] + (\mu_A - \bar{\mu})\big[E\big(R_{A,t} - \mu_A\big) + \big(R_{A,t-1} - \mu_A\big)\big] + (\mu_A - \bar{\mu})^2. \quad (1)$$

If markets are fully efficient, the first term in equation (1) is zero because it is a <u>conditional</u> autocovariance. In the second term, both expectations within the bracket are also zero. Hence the entire autocovariance is simply $(\mu_A - \bar{\mu})^2$. The analogous autocovariance in Regime B is $(\mu_B - \bar{\mu})^2$. Both of these autocovariances are zero if

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[7] Note that this is a population as opposed to a sample covariance; the latter can deviate because of sampling error. We work with population values throughout this section.

[8] For now, we ignore the transition observation between the two regimes. It will be covered in detail subsequently.

there is no difference in *ER* between Regimes A and B, otherwise, they are (spuriously) positive.

The unconditional variance of returns in Regime A is

$$E(R_{A,t} - \bar{\mu})^2$$
$$= E(R_{A,t} - \mu_A + \mu_A - \bar{\mu})^2 = \sigma_A^2 + 2(\mu_A - \bar{\mu})E\big(R_{A,t} - \mu_A\big) + (\mu_A - \bar{\mu})^2\,, \qquad (2)$$

where σ_A^2 denotes the conditional variance of returns during Regime A. As with the autocovariance, the second term in equation (2) is zero. Hence the unconditional variance of returns in Regime A consists of the sum of the conditional variance plus the squared difference between the conditional *ER* in A and the unconditional *ER*. There is an analogous argument for Regime B.

To obtain the first-order *AR* coefficient, ρ, we simply weight-average the terms from Regimes A and B in the numerator and denominator, while noting that the population variances of returns and lagged returns are identical. The resulting expression is

$$\rho = \frac{\alpha(\mu_A - \bar{\mu})^2 + (1-\alpha)(\mu_B - \bar{\mu})^2}{\alpha[\sigma_A^2 + (\mu_A - \bar{\mu})^2] + (1-\alpha)[\sigma_B^2 + (\mu_B - \bar{\mu})^2]}. \qquad (3)$$

Provided that the two regimes share the observations (i.e., $0 < \alpha < 1$), the *AR* is always positive. The expression can be simplified because $\mu_A - \bar{\mu} = (1-\alpha)(\mu_A - \mu_B)$ while $\mu_B - \bar{\mu} = \alpha(\mu_B - \mu_A)$. Substituting and collecting terms, we obtain

$$\rho = \frac{\alpha(1-\alpha)(\mu_A - \mu_B)^2}{\alpha\sigma_A^2 + (1-\alpha)\sigma_B^2 + \alpha(1-\alpha)(\mu_A - \mu_B)^2}\,. \qquad (4)$$

There are some interesting special cases. For instance, as one regime becomes much longer than the other, either α or $(1-\alpha)$ approaches zero and the *AR* approaches zero. Also, if the regimes are equal in length, as we assumed them to be in some of our empirical work, the *AR* coefficient simplifies further to

$$\rho = \frac{\left[\frac{\mu_A - \mu_B}{2}\right]^2}{\left[\frac{\sigma_A^2 + \sigma_B^2}{2}\right] + \left[\frac{\mu_A - \mu_B}{2}\right]^2}. \tag{5}$$

In any case, the coefficient is non-negative and will be biased upward if the two regimes have different *ER*. Note that since the denominator contains return variances, greater volatility in return innovations implies less *AR* bias.

Table 1 verifies the patterns we have already discussed by computing equation (5) with a variety of *ER* and volatilities (assuming equal volatilities in the two regimes). For small levels of unexpected return volatility (the first few rows of the table) and large changes in *ER* (the last few columns), there is substantial bias in the *AR* coefficient, rising to a level of more than 0.5 in an extreme case. Conversely, when volatility is substantial, there is little bias. It is still positive but would be hard to detect in a finite sample.

In the above analysis, we have not discussed *AR* for an observation that occurs just as regimes shift. This cannot matter much when the sample is long and there is only one regime change. However, if regimes shift frequently, the autocovariance is affected. To see this, a first-order unconditional autocovariance for an observation that occurs when a shift occurs from A to B is

$$E[R_{B,t} - \bar{\mu})(R_{A,t-1} - \bar{\mu})] = E[\left(R_{B,t} - \mu_B + \mu_B - \bar{\mu}\right)\left(R_{A,t-1} - \mu_A + \mu_A - \bar{\mu}\right). \tag{6}$$

Reasoning along the same lines following equation (1) above, and collecting terms, the resulting autocovariance is $(\mu_B - \bar{\mu})(\mu_A - \bar{\mu}) = -\alpha(1-\alpha)(\mu_A - \mu_B)^2$, which is negative for $0 < \alpha < 1$. Hence changes of regime offset the positive *AR* bias discussed above. If such changes are more frequent than continuations over any given time sample, the *AR* could even be biased negatively.

In general, if γ denotes the fraction of observation pairs (i.e., pairs of t and $t-1$) that are in the same regime, the overall autocovariance is $(2\gamma - 1)\alpha(1-\alpha)(\mu_A - \mu_B)^2$. The fraction $(2\gamma - 1)$ is a measure of regime persistence. It is close to +1 when regime shifts are infrequent while it is closer to -1 when regimes shift back and forth repeatedly and often.

Unlike the autocovariance, the unconditional variance is not affected by regime shifts. Equation (2) above still holds for the unconditional variance of returns in Regime A, regardless of how they are interspersed over time, and there is an identical valid expression for the unconditional variance of observations in Regime B. Hence, the general expression for the *AR* coefficient when a fraction $1-\gamma$ of the observations coincide with a regime shift from $t-1$ to t is, analogously to equation (4),

$$\rho = \frac{(2\gamma - 1)\alpha(1-\alpha)(\mu_A - \mu_B)^2}{\alpha\sigma_A^2 + (1-\alpha)\sigma_B^2 + \alpha(1-\alpha)(\mu_A - \mu_B)^2}. \tag{7}$$

According to equation (7), the *AR* coefficient in an efficient market is biased unless there are just as many reversals of regimes as continuations; in that case $\gamma = 1/2$ and $\rho = 0$ but this is probably a rather serendipitous situation.

It is interesting to speculate on whether actual regimes are more likely to persist or exhibit frequent reversals. In the case of bonds, persistence seems more likely because *ER* decline with shorter maturities and with lower inflation, which has historically been rather stable. For equities, driving influences such as leverage, product lines, or industry structure suggest persistence. In contrast, episodes of rapid change, such as periods around earnings announcements, could generate reversals, i.e., short-term risk and higher *ER* falling back to a normal level after each episode (see Savor and Wilson, 2016). General empirical investigations of persistence and/or reversal would seem to be a useful and important topic for future research.

I.2. General treatment of multiple regimes (more than two)

A natural extension of our theory is to examine the influence of multiple regimes on the return *AR* coefficient. We require an expression corresponding to equation (7) with more than just regimes A and B. Assume that we know there are exactly *K* different possible sequential *ER* regimes but we do not know when any particular one of the *K* becomes the prevailing regime. If each can occur only once during a time series with *T* observations, the persistence parameter discussed previously is $\gamma = 1-(K-1)/T$. However, persistence could be larger if regimes re-occur and arrive randomly because a "shift" could simply leave the old regime in place.

Let α_t denote the fraction of observations in the t^{th} regime, t=1,…,K, and $\sum_{t=1}^{K}\alpha_t = 1$. Then the return *AR* coefficient in an efficient market is a generalization of equation (3), viz.,

$$\rho = \frac{\gamma\sum_{t=1}^{K}\alpha_t(\mu_t-\bar{\mu})^2 + (1-\gamma)\,\frac{\sum_{t=1}^{K-1}[(\mu_{t+1}-\bar{\mu})(\mu_t-\bar{\mu})]}{(K-1)}}{\sum_{t=1}^{K}\alpha_t[\sigma_t^2+(\mu_t-\bar{\mu})^2]}\,, \tag{8}$$

where μ_t is now the conditional *ER* in Regime t while the unconditional *ER* is $\bar{\mu} = \sum_{t=1}^{K}\alpha_t\,\mu_t$. Unlike our earlier development with just two regimes, the arguments of the second summation in the numerator of equation (8) are not invariably negative; indeed, if regime shifts tend to sequentially move in the same direction, most of them or perhaps all of them except one, (when μ_t moves from less than $\bar{\mu}$ to greater than $\bar{\mu}$), could be positive. Alternatively, if most regime shifts tend to be reversals, these terms could be mostly or all negative. In other words, the sign of each element in this second summation is determined by whether both μ_t and μ_{t+1} are on the same side of $\bar{\mu}$ or on opposite sides. On the other hand, the first summation in the numerator of equation (8) is strictly non-negative, so the *AR* coefficient's sign is decreased by the prevalence of reversals, with frequency $1-\gamma$, and by whether terms in the second summation in the numerator of equation (8) tend to be negative.

Fitting equation (8) to actual data would appear to be a daunting task, though perhaps not impossible. Since the number of regimes K is an unknown, one would have to search over values of K from 1 to T while for each such value, find the best fit for the other $3K$+1 parameters (γ, and K values for a, σ, and μ, with the time subscript omitted for convenience). We leave a detailed estimation of equation (8) for future research, but note here that large K, the second summation in the numerator of equation (8) is dominated by the first. Moreover, the entire fraction of equation (8) is increasing in the first summation, which is a transformation of the volatility of *ER*. This suggests a relation between *AR* bias and the volatility of various *ER*. In the next subsection, we offer a detailed treatment of the link between *ER* volatility and *AR*.

I.3.The volatility of ER

We next show the algebra for *ER* volatility. At the outset, we note that the presence of bias in serial correlation is not an artifact of the specific stochastic process followed by the mean. Even if the mean follows a standard drift with diffusion, our assertion still holds. More generally, our arguments hold as long as there is *some* variation in *ER*; the specific process does not matter.

However, algebraic details are more transparent under assumptions about the process that drives the temporal evolution of the *ER*. For example, it could change monthly but be a constant over the days within each month. When expected returns follow a general *AR*(1) process, they obey

$$\mu_t = c + \phi\mu_{t-1} + \sigma\varepsilon_t \, , \tag{9}$$

where the symbols are as follows:

μ_t: the expected return in period t,

ϕ: the autoregressive parameter; $-1 < \phi < 1$ for stationarity,

σ: the volatility of the $AR(1)$ process,

ε_t: a mean zero standardized IID perturbation.

Let $\bar{\mu}$ be the long-run expected return, which, asymptotically, equals $\frac{c}{(1-\phi)}$[9]. It will be convenient below to rewrite equation (9) slightly as

$$\mu_t - \bar{\mu} = c + \phi(\mu_t - \bar{\mu}) + (\phi - 1)\bar{\mu} + \sigma\varepsilon_t$$
$$= c + \phi(\mu_{t-1} - \bar{\mu}) + \frac{(\phi-1)c}{1-\phi} + \sigma\varepsilon_t = \phi(\mu_{t-1} - \bar{\mu}) + \sigma\varepsilon_t. \quad (10)$$

Write the observed return as

$$R_t = \mu_t + \xi_t \ , \quad (11)$$

where the symbols denote:

R_t: the return in period t,

ξ_t: the unexpected return in t, a mean zero IID perturbation that includes risk volatility.

By assumption, $E[\xi_t\varepsilon_{t-j}] = 0\forall j$ and $[\xi_t\xi_{t-j}] = 0\forall j \neq 0$. Now, the first order autocovariance is given by

$$Cov = E[(R_t - \bar{\mu})(R_{t-1} - \bar{\mu})] = E[(R_t - \mu_t + \mu_t - \bar{\mu})(R_{t-1} - \mu_{t-1} + \mu_{t-1} - \bar{\mu})] \ . \quad (12)$$

[9] This result can be confirmed by recursive substitution; i.e., $\mu_t = c + [c + \varphi\mu_{t-2} + \varepsilon_{t-1}] + \varepsilon_t$ and so on, which delivers $\mathrm{E}[\mu_t] = c(1 + \varphi + \varphi^2 + \cdots) \cong c/(1-\varphi)$ because $E(\varepsilon) = 0$ and $\varphi^t\mu_0 \cong 0$ for large t and $|\varphi| < 1$.

Expanding equation (12), we have

$$Cov = E[(R_t - \mu_t)(R_{t-1} - \mu_{t-1})] + E[(R_t - \mu_t)(\mu_{t-1} - \bar{\mu})] + E[(\mu_t - \bar{\mu})(R_{t-1} - \mu_{t-1})] + E[(\mu_t - \bar{\mu})(\mu_{t-1} - \bar{\mu})].$$

Substituting, the terms on the right-hand side become

$$E[\xi_t \xi_{t-1}] + E[\xi_t(\mu_{t-1} - \bar{\mu})] + E[(\mu_t - \bar{\mu})\xi_{t-1}] + E\{[\phi(\mu_{t-1} - \bar{\mu}) + \sigma\varepsilon_t](\mu_{t-1} - \bar{\mu})\}.$$

By assumption, the first three terms are zero. The fourth term, denoted by ν, is

$$\nu \equiv \phi E[\mu_{t-1} - \bar{\mu}]^2, \tag{13}$$

which is strictly positive if there is some variation in *ER* and ϕ is positive. The expectation in equation (13) is the conditional variance of *ER* as of period $t-1$. Thus, equation (13) models the (bias in) autocovariance as a function of the volatility of the expected return, σ_μ^2. For most realistic applications we expect the autoregressive parameter ϕ to be positive. A constantly sign-flipping expected return is not for the most part realistic. Under $\phi > 0$ the spurious component of *AR* is positively related to the volatility of *ER*. This is an implication we test below.

II. The basic empirical evidence

In this section, under the assumption that markets are efficient (so that true serial correlations are zero), we ask the following question: Is there evidence that serial correlations do have an *ER* induced bias? As the analysis indicates, such biased components manifest themselves in their links with shifts in *ER*. Therefore, we consider the empirical connection between measured serial correlations and shifts in *ER*.

II.1. Data

We collect monthly returns data from CRSP for common stocks listed on NYSE, AMEX, and NASDAQ, for the period 1966-2000. We use monthly data to mitigate the potential biases associated with nontrading and the bid-ask effect in daily data. Data with missing values are discarded. We also often use the midpoint of closing bid and ask prices to avoid these biases. CRSP provides bid and ask prices starting in 1982, so our sample with midpoint-based returns starts in that year and ends in 2020. We use standard methods as in Fama and French (1992) to match with Compustat in tests which use financial statements.

II.2. A first look

We take an admittedly preliminary look at the data by computing the unconditional return *AR* over the entire sample of data available for each stock described in Section II.1. We then use average realized returns as proxies for *ER* and consider the link between *AR* and shifts in the mean returns.

More specifically, we split each stock's sample in half and compute the mean return from the observations in each half. Next, we take the absolute difference between each half's mean return and rank those absolute differences across stocks, sorting them into ten deciles, where decile 1 (10) has stocks with the lowest (highest) absolute return difference. Within each decile, we compute an equally weighted average return *AR*. The results are shown in Figure 2.

There is clearly an upward trend in *AR*, albeit non-monotonic, from the lowest to the highest decile of absolute mean return differences. This clear pattern is intriguing because the absolute return difference between two half-samples of each stock is a rather crude measure of changes in *ER*. The means are sample means, not

ER, and there is no good reason why true changes in *ER* should be manifest in half-samples. Yet, Figure 2 seems to portray exactly the pattern we would have anticipated *a priori*.

The other feature of Figure 2 is that *AR* are negative on average for every decile, which can be caused by bid/ask bounce. This phenomenon can be mitigated by simply eliminating the last trading day of each month. We adopt this simple cure to find the results shown in Figure 3. We find that the average *AR* in every absolute return difference decile increases in Figure 3. This suggests that bid/ask bounce does affect monthly *AR*, though the patterns in AR across *ER*-shift deciles remain unchanged.

II.3. Transformed autocorrelation, bid/ask quote midpoints and industry clustering

To transform *AR* into a variable that is not bounded by negative and positive unity, we perform the following standard procedure:

$$Transformed\ AR = TAR = \frac{AR*\sqrt{T-2}}{\sqrt{1-AR^2}}.$$

The transformed *AR* is not bounded, and we employ this throughout the regression analyses to follow. For ease of interpretation, we standardize an absolute change in *ER* (thereafter, $|\Delta ER|$) or the standard deviation of *ER* to have a zero mean and unit variance and then relate the transformed *AR* to either one of them.

For all the equity sample cross-sectional results reported within the main paper from here on, we use mid-points of closing bid/ask quotes instead of transaction prices, in order to mitigate the impact of bid/ask bounce on *AR*[10]. Further, we cluster

[10] We use closing transaction prices for corporate bonds, as bid-ask quotes are more likely to be stale for such securities.

all standard errors by industry (first two digits of the SIC code for each stock. This is to mitigate the impact of cross-correlation in error term. Appendix A provides results using a Feasible Generalized Least Squares approach which uses a general variance/covariance matrix of the error terms. [11]

II.4 Implementation

Our basic requirement is that *AR* is computed with returns that are over shorter horizons than the *ER*. Since we compute *AR* from monthly returns, we split our sample into two equal halves. In this section, we use the realized mean return in each half as the *ER* proxy. To compare different groups of stocks, we follow common practice (e.g., Fama and French, 1992) and compute separate results that exclude financial firms and firms with negative book values.[12]

Table 2 Panel A presents the slopes and corresponding *t*-statistics from cross-sectional regressions of unconditional *AR* on absolute changes in mean returns computed from the first and second half of available observations for each stock.[13] As we can see, for every subsample, the slope is positive and highly significant with a *p*-value less than 0.001. Note, however, that the intercept is negative, suggesting a baseline negative autocorrelation at the monthly level, even using mid-points of closing quotes. Nonetheless, the positive slopes on $|\Delta ER|$ support our premise that *ER* shifts cause a positive bias in *AR*. For robustness, Appendix A, Table A1 reports the results of the Table 2 regressions using Feasible Generalized Least Square regressions (FGLS) of *AR* on $|\Delta ER|$. We find that the qualitative conclusions remain virtually unchanged under this alternative procedure.

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[11] We apply the command "proc mixed data=PRS method=ml" in SAS to perform FGLS. This approach allows for heteroskedasticity as well as cross-correlation.

[12] Financial firms have 4-digit SIC codes of 6000 to 6999.

[13] To reduce the impact of bid/ask bounce, we use the midpoint of closing bid and ask quotes.

In Panel A, the R^2 is greater than 1.19% in all specifications. This is the pattern to be anticipated if markets are efficient and there have been changes in *ER* for at least some firms. A reasonable question that arises is whether we can further test whether the *AR* we observe actually has a spurious component. To address this issue, we note that the right side of equation (5) is the bias in *AR* (ρ), which should be zero in efficient markets. We therefore estimate the empirical relation between *AR* and ρ. To calculate the latter, we simply use the sample counterparts of the moments on the right-hand side of equation (5), defining regimes A and B as the first and second halves of the sample, respectively, for each stock. In Panel B, we cross-sectional regress the *AR* of individual stocks on their ρ and find the slope to be highly significant. Further, the *t*-statistics and R^2's rise relative to those in Panel A, which is what we would expect, since equation (5) measures the bias more precisely than a simple absolute shift in *ER*.[14]

II.5 Market index AR

To supplement these findings with a broad index, we compute the *AR* of the S&P 500 and its ρ, i.e., the right-hand side of equation (5), using daily data over the 1927-2020 period. Specifically, we first divide the sample into non-overlapping five-year and two-year subsamples. To obtain ρ, we first estimate a Quant-Andrews breakpoint; i.e., the maximally probable breakpoint within each subperiod (for details, see Quandt, 1960; Andrews, 1993; Andrews and Ploberger, 1994). The approach tests a specified equation for one or more structural breakpoints during a time series sample by performing the single breakpoint Chow (1960) test across every two adjacent observations. Those Chow tests are then aggregated into one statistic for a test against the null hypothesis of no breakpoints anywhere in the sample. We divide

[14] Ex post, stocks that have risen or declined by large amounts could conceivably exhibit both positive autocorrelation and a shift in mean return in the second half relative to the first half of their time sample. To address this, we trim the sample at the 10th and 90th percentile of full sample return, and find that the results are unchanged.

each subsample of S&P 500 returns into two parts around the identified Quant-Andrews breakpoint. This allows the calculation of ρ from the two parts.

We regress subsample *AR*s on subsample $\rho's$ in time series, consisting of 47 subsamples in the two-year case and 13 in the five-year case.[15] In results not tabulated for brevity, we find that *t*-statistics using Newey-West standard errors with four lags are 1.91 and 3.18, respectively, which suggests less movement in the *ER* of the S&P than in the average individual stock.[16]

Although all of the above results are consistent with the phenomenon under study, realized mean returns are imperfect *ER* proxies. We therefore complement our analysis by next computing *ex ante* expected returns using various approaches. These approaches use options prices, factor models, and analysts' price targets to estimate *ER*, and use only currently available data to do so. This avoids the look-ahead issue inherent in realized means as *ER* proxies.

III. Analyses based on ex ante expected returns

We now describe and apply three methods to estimate *ex ante* expected returns.

III.1 Using options prices

The first approach to ex ante *ER* is based on Martin and Wagner (2019) (MW), who derive a formula for the *ER* on a stock in terms of the risk-neutral variance of the market and the stock's excess risk-neutral variance relative to that of the average

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15 We recognize that the index is not a traded asset and may exhibit positive autocorrelation due to non-synchronous trading. Although this could create noise in our regression estimation; it is not obvious why this component might be related to ρ.

16 Newey and West (1987) suggest using the lag-length to equal the integer portion of $4(T/100)^{(2/9)}$, where T is the number of observations. Our choice of four lags is slightly more conservative. The results remain intact with longer lags; over five-year (two- year) subsamples, the *t*-statistics are 2.04 (3.24) and 2.07 (3.37) for five and six lags, respectively.

stocks. Their parameters are computed from index and stock option prices. We estimate *ER* of stock i at time t ($E_t R_{i,t+1}$) from their equation (17) as follows:

$$E_t R_{i,t+1} = \left(R_{f,t+1} * \left[SVIX_t^2 + 0.5\left(SVIX_{i,t}^2 - \overline{SVIX}_t^2\right)\right]\right) + R_{f,t+1},$$

where $SVIX_{i,t}^2$ is the risk-neutral variance of firm i, $SVIX_t^2$ is the risk-neutral variance of the index, $\overline{SVIX}_t^2$ is the average risk-neutral variance, and $R_{f,t}$ is the gross riskless return. MW provide the data on $SVIX_t$, $SVIX_{i,t}$, $\overline{SVIX}_t^2$, and $R_{f,t}$.[17]

We apply the MW approach for several reasons. First, their *ER* is based on market prices rather than historical financial characteristics. It is more parsimonious than relying on multiple characteristics, whose association with expected returns is still inconclusive. Second, their approach does not rely on regressions, but on just the three measures of risk-neutral variance. Thus, it is not subject to bias arising from the estimation of factor loadings. Third, it makes specific predictions about the relationship between *ER* and the three measures of risk-neutral variance whereas the relationship between characteristics and *ER* is less deterministic and varies with sample period. The caveat is that the data do not start until 1996 when Option Metrics became available and the number of covered stocks is just above 800. Relying on the authors' data, our sample period ends in 2013 and is limited to firms that are constituents of the S&P500.

III.2 Factor models

Our second approach is to use factor models. The multi-factor arbitrage pricing theory (APT) of Ross (1976) or Merton's (1973) ICAPM can be expressed in two equivalent forms. The basic form expresses the total return on asset i at time t as

$$R_{i,t} = E_{i,t} + \beta_{i,1}\tilde{f}_{1,t} + \ldots \beta_{i,K}\tilde{f}_{K,t} + \tilde{\varepsilon}_{i,t}\,, \tag{14}$$

[17] https://onlinelibrary.wiley.com/doi/full/10.1111/jofi.12778

where the K systematic risk factors each have a zero mean; $E_{j,t}$ is the ER on asset i at time t (it can be time-varying); and the last term is the idiosyncratic risk component. If factor risk premiums (denoted by λ's) are stochastic, while the betas are constant, the equation for expected returns at time t is

$$E_{i,t} = \lambda_{0,t} + \beta_{j,1}\, \lambda_{1,t} + \ldots \beta_{i,k}\, \lambda_{k,t}\ . \tag{15}$$

It seems completely plausible that risk premiums are time-varying; why not? For demographic and many other reasons, aggregate risk tolerances can change over time. Further, the volatilities of risk factors can change for macroeconomic reasons. All this implies that AR bias can be induced in securities due to risk premia that fluctuate over time.

There are various factor models. For illustration purposes, we apply five- and six-factor Fama-French models (FF5 and FF6). The FF5 model is described in Fama and French (2015), whereas FF6 simply supplements FF5 with a momentum factor (*UMD*). In our application, we perform the following steps (using the FF5 factors as an example):

1. For any month j, we estimate factor loadings over the months $j-1$ to $j-60$ and save these loadings for each month j. That is, for each stock, we estimate factor loadings for FF5 ($\beta_{k,i,j}$) in every month using rolling data for 60 months where i, k and j stand for factor k, firm i, and month j:

$$R_{i,j} - R_{f,j} = \alpha + \beta_{R_m - R_f}(Rm - Rf_j) + \beta_{HML} HML_j + \beta_{SMB} SMB_j + \beta_{RMW} RMW_j + \beta_{CMA} CMA_j + \varepsilon_{i,j}.$$

2. For each month j, we then run the Fama-MacBeth regression of monthly

returns on factor loadings measured at time $j-1$ over the past 48 months and save the coefficients $\lambda_{k,i,j}$ of stock i for factor k in each month j.[18]

$$R_{i,j} = \hat{\alpha}_j + \hat{\lambda}_{R_m-R_f,i,j}\, \hat{\beta}_{R_m-R_f,i,j-1} + \hat{\lambda}_{HML,i,j}\hat{\beta}_{HML,i,j-1} + \hat{\lambda}_{SMB,i,j}\hat{\beta}_{SMB,i,j-1} + \hat{\lambda}_{RMW,i,j}\hat{\beta}_{RMW,i,j-1} + \hat{\lambda}_{CMA,i,j}\hat{\beta}_{CMA,i,j-1} + \eta_{i,j}.$$

3. For the *ER* prediction, we use the average Fama-MacBeth coefficients and intercepts over the past 48 months ($j-48$ to $j-1$) from Step 2 and the current estimate of factor loadings from Step 1 to predict month *j*'s *ER*:

$$ER_{i,j} = \frac{\sum_{j=t-48}^{t-1} \hat{\alpha}_{i,j}}{48} + \sum_k \left[\frac{\sum_{j=t-48}^{t-1} \hat{\lambda}_{k,i,j}}{48} \times \hat{\beta}_{k,i,j-1}\right].$$

III.3 Analysts' price targets

The third method is *ER* estimated from price targets forecasted by analysts. For this approach, we follow Engelberg, McLean, and Pontiff (2020). Specifically, we collect analysts' 12-month price targets from IBES and use these to impute an expected return. These data are available monthly from the IBES starting in 1999.

For each stock-month in our sample, we use the most recent price target issued by each analyst over the last 12 months and compute the median across all such targets. The *ER* estimate (when available) is then the expected price appreciation implied by the median target price relative to the actual transaction price as of the end of the month.[19] The *ER* is set to be missing if there is no target price available.

III.4 Implementation

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[18] We follow Brennan, Chordia, and Subrahmanyam (1998) and Haugen and Baker (1996) in using the past 48 rolling months. The factor are from Ken French's website (tinyurl.com/337sy3md).

[19] We ignore dividend yields for convenience.

We now present results using the three methods of calculating ER described above. In each case, we split the sample into two halves, compute the monthly average ER for each half, and regress the full sample monthly AR for each stock on the absolute change in monthly average ER across the two samples ($|\Delta ER|$). Based on the analysis in Section I.3, we also include the full sample standard deviation (volatility) of the monthly ER for each stock as an additional explanatory variable.

Table 3 shows the results of ER estimated by the MW approach. The $|\Delta ER|$ and volatility of ER induce AR at 10% and 1% significance levels, respectively. The economic magnitude of the change in AR is more material: A one standard deviation increase in $|\Delta ER|$ and volatility of ER increase AR by 10.8% and 22.6%, respectively. The impact of ER volatility on AR is almost three times stronger than that of $|\Delta ER|$. Compared to the mean AR of -2.88%, $|\Delta ER|$ and volatility of ER are economically and strongly impactful.

Table 4 presents the results for FF5 and FF6. This estimation allows us to expand the sample to the 1982 to 2020 period and the results become stronger relative to those in Table 3. In particular, both $|\Delta ER|$ and the volatility of ER estimated by FF5 and FF6 are significant at 1% and their impact is economically meaningful. A one standard deviation change in either quantity increases AR by 8% and 10.6%, respectively for FF5 (7.5% and 9.7%, respectively for FF6). The economic impact of volatility is about 1.3 times larger than that of $|\Delta ER|$.

Table 5 presents the results of ER derived from analysts' price targets. These results continue to support our conjecture. Specifically, both $|\Delta ER|$ and volatility of ER are significant at the 1% level. Economically, the impact of $|\Delta ER|$ and volatility of ER are higher relative to FF5 and FF6; a one standard deviation change increases

AR by 9.8% and 15.1%, respectively.[20] This is consistent with the view that the forward looking analysts' predictions have have less embedded noise than the factor model counterparts.

IV. Tests with regime shifts

In the previous sections, we examine how *AR* increases with $|\Delta ER|$ in two equally split sample periods. In this section, we formally identify regime shifts for individual stock returns and factors using the Quandt-Andrews test (henceforth, QA, and mentioned in Section II.5), as well as the Bai-Perron test (henceforth, BP; see Bai, 1997; Bai and Perron, 1998, 2003a and 2003b). We apply the QA approach using all breakpoints as well as the most statistically significant breakpoints. For the latter, we set the QA parameters so that we infer only a single breakpoint for each stock over the entire available sample, the most significant one in terms of the largest absolute change in means. This is tantamount to applying a single Chow test.

After detecting the largest regime shift (breakpoint) using the QA test, we compute the absolute difference in *ER* (using our previous proxies) before and after the breakpoint. We cross-sectionally regress the unconditional *AR* against this absolute difference. For all breakpoints from the QA test, Table 6 reports a slope coefficient *t*-statistic of over 3.3 for *ER* estimated from various approaches except the Martin and Wagner (2019)'s approach, thereby supporting the notion that changes in *ER* increase sample unconditional *AR*. In an alternative test, we retain only the regime shifts that are significant according to the QA test and perform similar regressions with them. The *t*-statistics between 3.4 to 5.9 for *ER* computed from

[20] To address issues arising from cross-sectional correlation of errors, we re-estimate all regressions of Table 5 while clustering errors at the industry level, using the 48 industry-classification at Kenneth French's website (tinyurl.com/337sy3md). We also provide the Feasible Generalized Least Squares (FGLS) versions of the results in Tables 3 to 5 within Appendix A (Tables A2 to A4). The results are virtually unchanged.

realized means, FF5-FF6, and analysts' price targets, again support the same inference.

Lastly, we detect all and significant breakpoints with the BP approach. The ideas behind the BP implementation are described in Hothorn and Zeileis (2008). We find again that the absolute change in returns is significantly related to unconditional *AR*; regression slope *t*-statistic of 2.78 and 2.97 for all and significant breakpoints of observed returns, respectively. The results of the BP test for FF5 and FF6 and price targets continue to yield significance.[21] Overall, our premise of bias in *AR* due to shifting *ER* receives reasonable support, considering the modest number of observations for the *ER* backed out from options markets.[22]

V. Corporate bonds

Our premise should hold for bonds as well, as these have natural variations in term premia, and, in turn, *ER*. We collect corporate bond data from TRACE and follow the approach of Bai, Bali, and Wen (2019) (BBW) for data cleaning and estimation of bond returns. Our data span June 2002 to December 2019. We lose about 10 years of observations from the three-step procedure below to estimate expected bond returns. To maximize the number of observations, we do not aggregate bonds across different grades within a firm but perform an analysis at the bond level.

We use two proxies for *ER*, realized mean returns and expected returns from a standard bond factor model. For the latter approach, we use the BBW factors and follow the steps below to compute expected returns for bonds.[23] The steps are similar

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[21] The sample sizes differ across the different *ER* methods not only because of the availability of the *ER* proxy, but also because the relevant procedure fails to detect a structural break in the return time-series for several stocks. Such stocks are omitted from the regressions.

[22] Feasible Generalized Least Squares (FGLS) estimation of the Table 6 regressions leaves the results virtually unchanged; details are available from the authors.

[23] In an alternative approach, we use the bond market factors used in Fama and French (1993) and find similar results.

to what we apply to estimate expected returns for stocks in Section III. However, since we have shorter series of bond data, we slightly modify the requirements in steps 1 and 2 of Section III.2 to be 36 months and in step 3 of that section to be 12 months. Thus, our steps to estimate bond expected returns are the following:

1. For any month j, we estimate factor loadings over the months j–1 to j–36 and save these loadings for each month j. That is, for each bond, we estimate loadings ($\beta_{k,i,j}$) on the BBW factors in every month using rolling data for 36 months where k, i, and j stand for factor k, firm i, and month j. In the estimation equation below, *MKTbond*, *DRF*, *CRF*, and *LRF* pertain to excess bond market return, default risk, credit risk, and liquidity risk factors:[24]

$$R_{i,j} - R_{f,j} = \alpha + \beta_{MKTbond}(MKTbond_j) + \beta_{DRF}DRF_j + \beta_{CRF}CRF_j + \beta_{LRF}LRF_j + \varepsilon_{i,j}.$$

2. For each month j, we then run the Fama-MacBeth regression of monthly returns on factor loadings measured at time $j-1$ over the past 36 months and save the coefficients $\lambda_{k,i,j}$ of bond i for factor k in each month j.

$$R_{i,j} = \hat{\alpha}_j + \hat{\lambda}_{MKTbond,i,j}\,\hat{\beta}_{MKTbond,i,j-1} + \hat{\lambda}_{DRF,i,j}\hat{\beta}_{DRF,i,j-1} + \hat{\lambda}_{CRF,i,j}\hat{\beta}_{CRF,i,j-1} + \hat{\lambda}_{LRF,i,j}\hat{\beta}_{LRF,i,j-1} + \eta_{i,j}.$$

3. For the *ER* prediction, we use the average Fama-MacBeth coefficients and intercepts over the past 12 months ($j-12$ to $j-1$) from Step 2 and the current estimate of factor loadings from Step 1 to predict month j's bond *ER*.

$$ER_{i,j} = \frac{\sum_{j=t-12}^{t-1}\hat{\alpha}_{i,j}}{12} + \sum_k\left[\frac{\sum_{j=t-12}^{t-1}\hat{\lambda}_{k,i,j}}{12} \times \hat{\beta}_{k,i,j-1}\right].$$

[24] The data are available on Jennie Bai's website (http://www.jenniebai.com/).

We begin by using realized mean returns as the *ER* proxy in a similar vein as in Table 2. We split the sample into two equal periods. The results are reported in Panel A of Table 7. In this panel, we limit each bond to have data for no less than 4 months, which leaves us with 21,358 bonds. We find that higher $|\Delta ER|$ and volatility of *ER* are associated with higher *AR* at the 1% level.

Next, we estimate expected returns using the BBW factor model. The results appear in Table 7 Panel B. In this case, we need each bond to have monthly returns data for 84 months to reliably estimate *ER*. We are left with 1,137 bonds. Our conclusion remains intact, although the degree of significance is lower. $|\Delta ER|$ and volatility of *ER* continue to be positively associated with *AR*.[25]

VI. Cross-autocorrelations

Our basic idea also applies to cross-autocorrelation. Stocks could display positively biased cross-autocorrelation when their expected returns are correlated and change over time. For example, two stocks with similar betas from the market model should have larger measured cross-autocorrelations because their expected returns (driven by the market's expected return and transmitted by their betas from the market model) should have stronger co-movement. Higher beta stocks have more volatile returns, both total and expected; consequently, higher beta quintiles should display more positive cross-autocorrelations unless there is no movement in the market's expected return.

[25] Table A5 of Appendix A shows that similar results obtain using Feasible Generalized Least Squares (FGLS). In that table, we also show similar results with simple averages of monthly yields to maturity and durations as *ER* proxies. For the latter measure, the assumption is that bond *ER* should be positively related to its interest rate exposure (duration). Also, for both panels of Table 7, clustering standard errors by company instead of industry makes no material difference to the results.

The algebra of cross-autocorrelation is very similar to that presented in Section I, which derives the first-order autocorrelation for a single asset whose expected return (*ER*) changes once, from Regime A to Regime B. The only addendum is that we now have two assets, both of which experience a regime change at the same time. Once this is done for a single regime change, the further generalization to multiple changes, to shifts in regimes (both presented also in Section I), and to non-simultaneous regime changes, are straightforward and so we do not present these analyses here.

Suppose that assets i and j both have a concurrent regime change from A to B. The conditional expected returns for i are denoted $\mu_{i,A}$ and $\mu_{i,B}$ and similarly for asset j. As before, Regime A lasts for a fraction α of all observations and Regime B lasts for the complementary fraction. Hence, the unconditional *ER* for i is $\bar{\mu}_i = \alpha\mu_A + (1-\alpha)\mu_B$ and similarly for j. A first-order unconditional cross-autocovariance in Regime A, computed with the respective unconditional *ER* is $E[(R_{i,A,t} - \bar{\mu}_i)(R_{j,A,t-1} - \bar{\mu}_j)]$ with an analogous expression for Regime B. Note also that a cross-autocovariance can be computed in two ways, with j either lagging or leading i. Taking the former approach, the cross-autocovariance can be written as

$$\begin{aligned} &E[(R_{i,A,t} - \mu_{i,A} + \mu_{i,A} - \bar{\mu}_i)(R_{j,A,t-1} - \mu_{j,A} + \mu_{j,A} - \bar{\mu}_j) \\ &= E[(R_{i,A,t} - \mu_{i,A})(R_{j,A,t-1} - \mu_{j,A})] + (\mu_{i,A} - \bar{\mu}_i)E(R_{j,A,t-1} - \mu_{j,A}) \\ &\quad +(\mu_{j,A} - \bar{\mu}_j)(R_{i,A,t} - \mu_{i,A}) + (\mu_{i,A} - \bar{\mu}_i)(\mu_{j,A} - \bar{\mu}_j). \end{aligned} \tag{16}$$

If markets are efficient, the first three terms in equation (16) are zero because they involve, respectively, a conditional cross-autocovariance and two conditional expectations about conditional means. Hence, the entire cross-autocovariance is the fourth term, $(\mu_{i,A} - \bar{\mu}_i)(\mu_{j,A} - \bar{\mu}_j)$. There is an analogous expression for the cross-autocovariance in Regime B.

Unlike the situation for a single asset in Section I, where the analogous remaining term was a square, the sign of this remaining term in equation (16) depends on whether the two assets change expected returns in the same direction from Regime A to Regime B. This would be the situation, for example, if the returns on both assets are driven by the market model (with positive betas) and the only change between regimes is in the market's expected return. However, there could be other cases where a negative cross-autocovariance might occur for two competing firms that had opposite responses to a concurrent alteration in economic circumstances. In either case, the resulting computed cross-*AR* does not indicate a profit opportunity.

The unconditional variance of returns for asset i in Regime A is

$$E(R_{i,A,t} - \bar{\mu}_i)^2 = \sigma_{i,A}^2 + 2(\mu_{i,A} - \bar{\mu}_i)E(R_{i,A,t} - \mu_{i,A}) + (\mu_{i,A} - \bar{\mu}_i)^2 \, , \tag{17}$$

where $\sigma_{i,A}^2$ denotes the conditional variance of returns for asset i during Regime A. The second term in equation (17) is zero. There are analogous expressions for asset j and Regime B. The first-order cross-autocorrelation coefficient (cross-*AR*) across the two regimes, denoted by ρ_C, weight averages across regimes the remaining expression from equation (16) in the numerator and the square roots of equation (17) in the denominator, to obtain, after collecting terms and simplifying,

$$\rho_C = \frac{\alpha(1-\alpha)(\mu_{i,A}-\mu_{i,B})(\mu_{j,A}-\mu_{j,B})}{\left\{\left[\bar{\sigma}_i^2+\alpha(1-\alpha)\left(\mu_{i,A}-\mu_{i,B}\right)^2\right]\left[\bar{\sigma}_i^2+\alpha(1-\alpha)\left(\mu_{j,A}-\mu_{j,B}\right)^2\right]\right\}^{1/2}} \, , \tag{18}$$

where the weight-averaged conditional variance is denoted $\bar{\sigma}_i^2 = \alpha\bar{\sigma}_{i,A}^2 + (1-\alpha)\bar{\sigma}_{i,B}^2$ for asset i and similarly for asset j. Appendix B shows that when (i) asset returns conform to the market model, (ii) all betas are positive, and (iii) only market expected returns shift across the regimes, the right-hand side of equation (18) increases in the betas of the two stocks.[26] Thus, the implication is that the measured cross-*AR* should increase across portfolios sorted by ascending beta.

To assess the empirical extent of the above effect, we select stocks that have at least 30 monthly returns and compute the market model beta of each stock using all available months. We then sort stocks by their betas into quintiles and within a quintile, sort stocks by their permanent company number (*PERMNO*). We compute the cross-autocorrelations for $N/2$ sorted pairs in each quintile, where N is the number of stocks in a quintile. To avoid unnecessary sampling dependence, we use each stock only once, with its adjacent partner by the next lower *PERMNO*, instead of twice (with the partners both above and below it). Then the average cross-autocorrelation is computed for the $N/2$ pairs in each quintile and transformed into an *AR* between $R_{i,t}$ and $R_{j,t-1}$ and another between $R_{i,t-1}$ and $R_{j,t}$. As shown in Table 8, consistent with equation (18), the average *AR* generally increases with beta.[27] Further, we have verified that the differences in cross-*AR* are statistically different across the extreme beta groups, with p-values of less than 0.001.

PERMNO might be related to firm characteristics. To guard against this possibility, we randomly draw, without replacement, pairs of stocks within each beta quintile and compute their cross-*AR*. The results are given in Table 8, which generally shows a monotonic pattern. Specifically, the average cross-*AR* generally increases with beta except for the last case where the average is highest in Q4 (0.352) and is slightly lower in Q5 (0.330). Again, the value in Q4 is statistically higher than that in Q1 with a p-value less than 0.001.

VII. Conclusion

[26] Intuitively, the numerator is proportional to the product of the betas times the squared change in the expected market return. Dividing the numerator and denominator by the product of the betas and substituting for the total variance from a market model, it can be seen that the denominator decreases in each individual beta.

[27] The *AR* transformation is given in the equation of Section II.3.

The goal of our paper is to demonstrate the following: An absolute change in expected return (*ER*) or volatility in *ER* can induce bias in measured return autocorrelation, *AR*, and that the direction of this bias is generally positive regardless of the direction of shifts in *ER*, provided regime shifts in *ER* are infrequent. The bias is generally an increasing function of either the absolute change in *ER* or the volatility of *ER*. Thus, under the null of efficient markets, measured *AR* should be positively related to variations in expected returns. We demonstrate this with simple examples and derive analytics that verify the phenomenon. We find evidence of bias in *AR* using shifts in *ex ante* metrics for *ER* obtained from options markets, analysts' price targets, and standard factor models.

Although *AR* bias is mitigated by volatility in unexpected returns, we find significant evidence supporting the existence of such bias for individual US equities and bonds. We also show that potential for bias also applies to cross-autocorrelation. We observe that stocks in higher beta quintiles, whose expected returns change more with movements in the market-wide expected return, indeed exhibit more positive cross-autocorrelation, as suggested by our theory.

In future work, one could broaden our enquiry by looking into other possible measures of shifts in *ER* (i.e., $|\Delta ER|$). These include but are not necessarily limited to: A. Slow drifts in mean returns over time; B. Events that precipitate sudden ΔER (such as mergers, debt issuance or retirement, etc.); C. EGARCH models. In each case, we would be able to estimate the statistical significance of ΔER and thereby sort stocks into groups that are more or less likely to have biased return *AR*. Finally, the phenomenon applies to any asset class and it would be interesting to ascertain which classes are subject to it.

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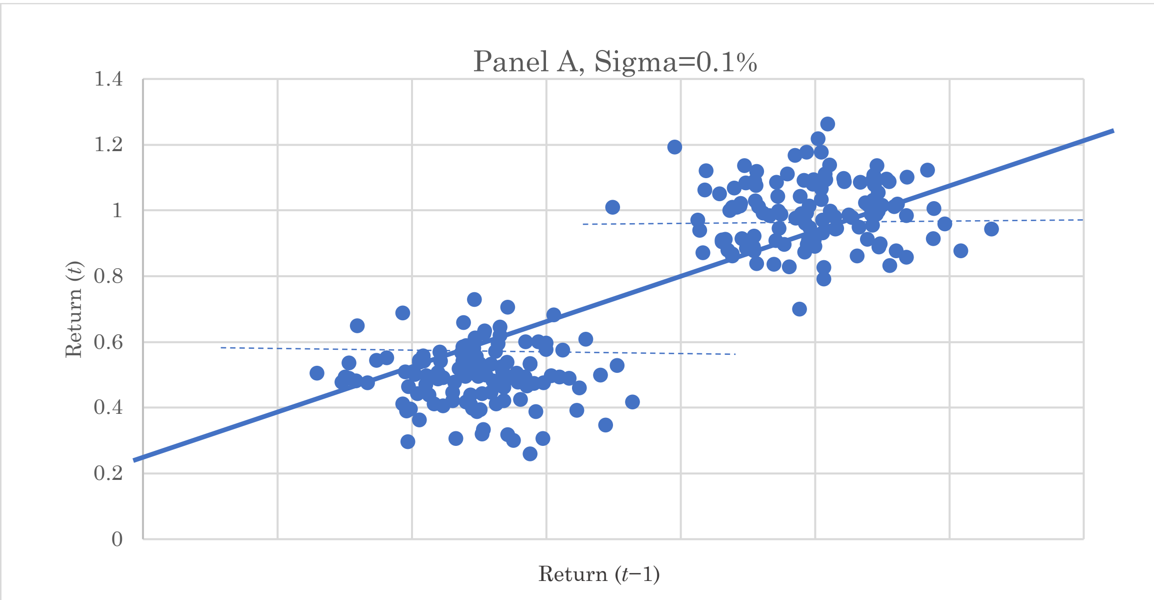
Figure 1. Simulated scatter diagram for successive returns with two regimes that differ <u>only</u> in *ER*. In one regime, the *ER* is 0.5% per period and in the other regime, it is 1% per period. The three panels illustrate the impact of volatility, in standard deviation per period, which is 0.1%, 0.2% and 8.66%, respectively, in Panels A, B and C. The dashed lines show the conditional autoregression within each regime while the solid line shows the unconditional autoregression using all observations regardless of regime.



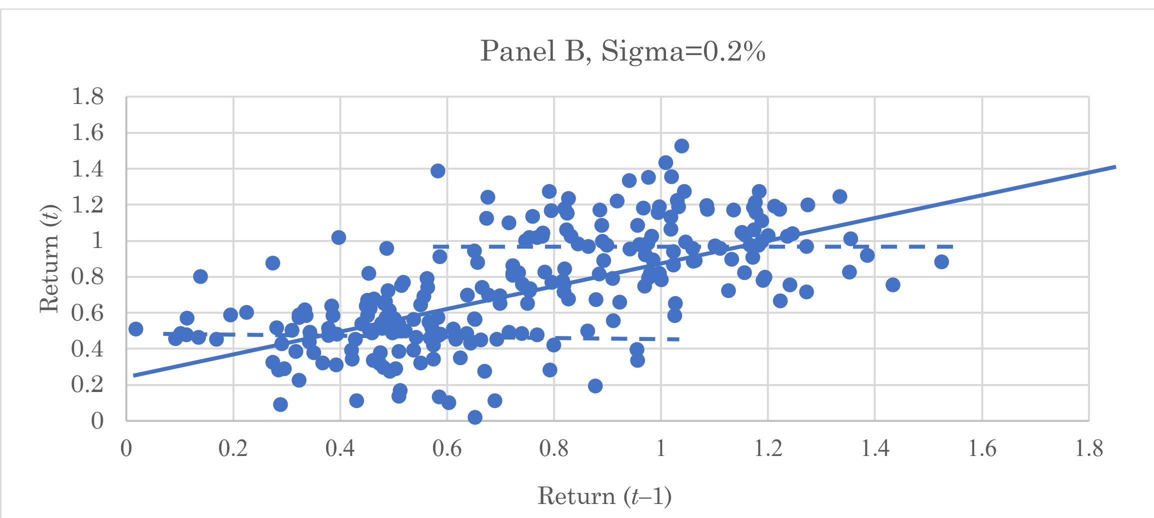


Figure 1, contd.

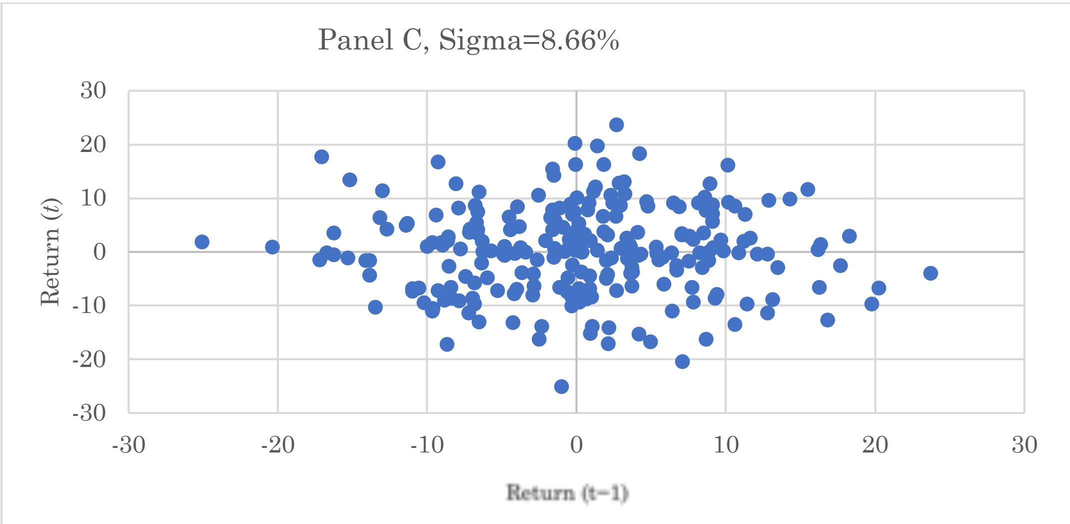


Figure 2. Autocorrelations by differences in mean returns. For each non-financial firm common stock listed on the NYSE, AMEX, and NASDAQ, with at least 2 years of presence in Compustat, the unconditional monthly return first-order *AR* is computed over the entire available sample. Then the stock's sample is divided in half and mean returns are computed from each half. The absolute difference between the two halves' mean returns is computed and ranked across stocks, then sorted into deciles, where decile 1 (10) has stocks with the lowest (highest) absolute mean return difference. An equally-weighted average return *AR* is computed for each decile and is plotted here.

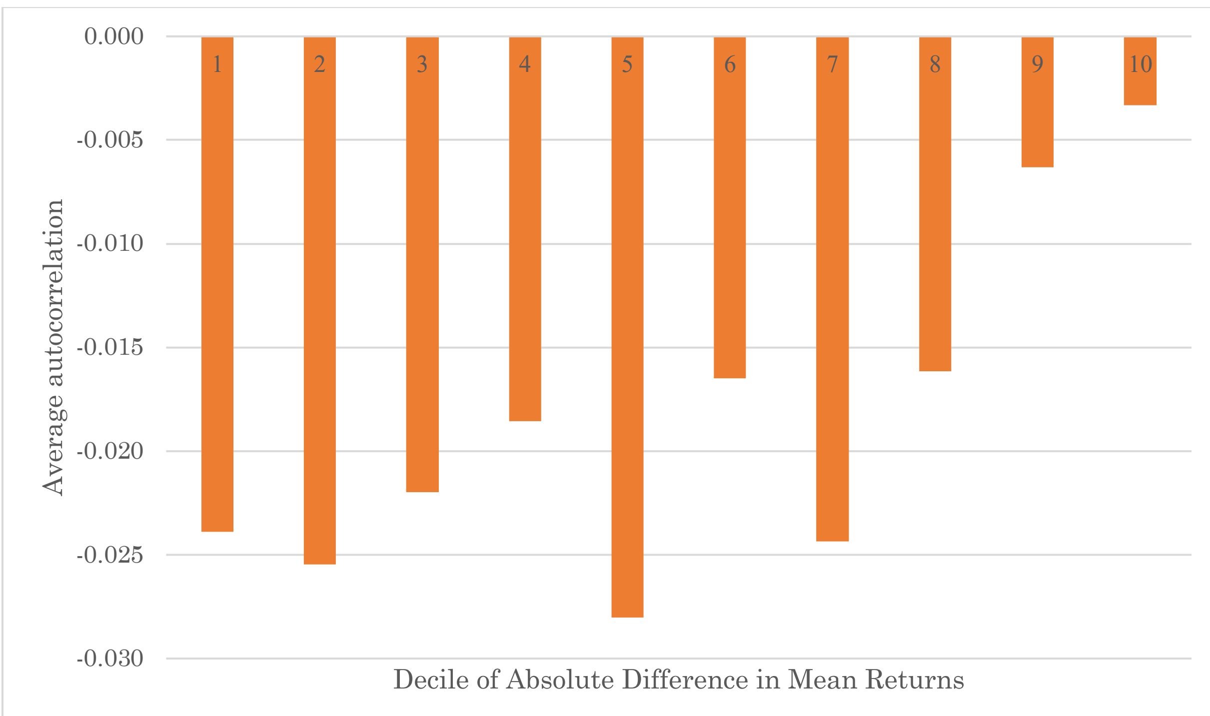


Figure 3. Autocorrelation by mean returns, excluding the last trading day of the month. For each non-financial firm common stock listed on the NYSE, AMEX, and NASDAQ, with at least 2 years of presence in Compustat, the unconditional monthly return first-order *AR* is computed over the entire available sample while excluding the last trading day of each month. Then the stock's sample is divided in half and mean returns are computed from each half. The absolute difference between the two halves' mean returns is computed and ranked across stocks, then sorted into deciles, where decile 1 (10) has stocks with the lowest (highest) absolute mean return difference. An equally-weighted average return *AR* is computed for each decile and is plotted here.

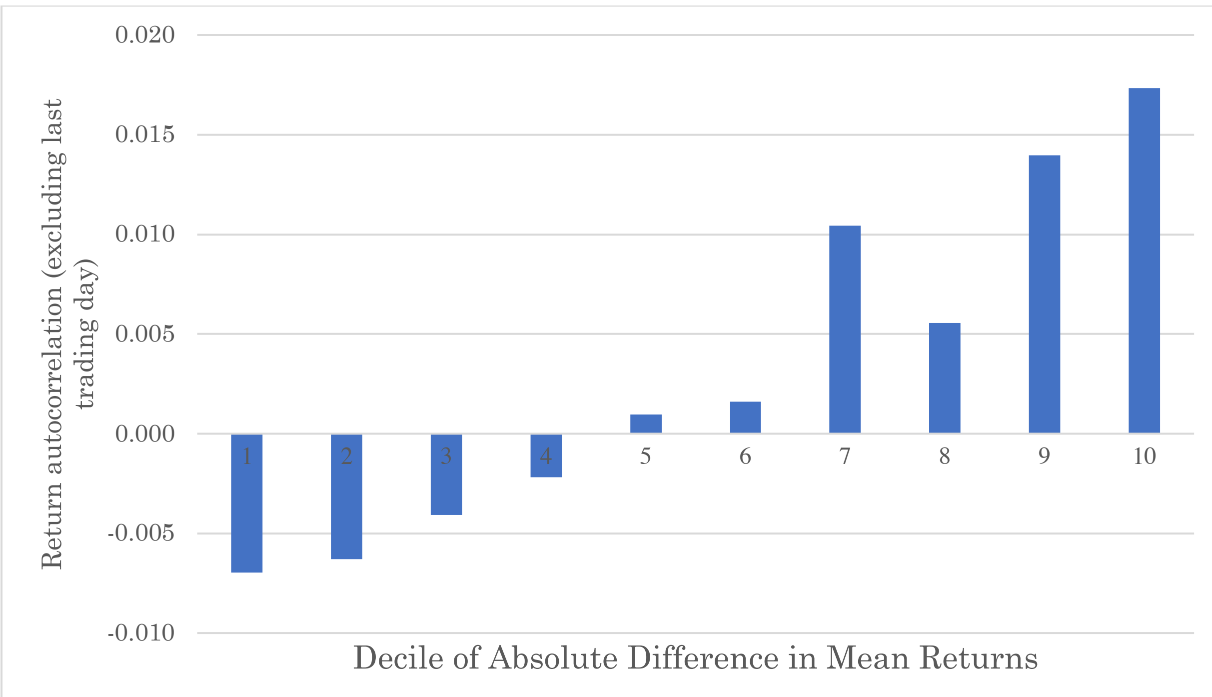


Table 1. Theoretical Spurious Return Autocorrelation Coefficients for Different Combinations of Return Volatility and Expected Return. This table is generated from simulated returns with two regimes that differ only in *ER*. Sigma in the left-most column is the standard deviation of returns per period in percent per annum.

Sigma	Expected Return Difference, (% per annum)						
	0	5	10	15	20	25	30
0	N/A	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	0	0.0383	0.1324	0.2478	0.3600	0.4581	0.5397
10	0	0.0099	0.0368	0.0761	0.1233	0.1745	0.2267
15	0	0.0044	0.0167	0.0353	0.0588	0.0859	0.1153
20	0	0.0025	0.0094	0.0202	0.0340	0.0502	0.0683
25	0	0.0016	0.0061	0.0130	0.0220	0.0327	0.0448
30	0	0.0011	0.0042	0.0091	0.0154	0.0229	0.0315
35	0	0.0008	0.0031	0.0067	0.0113	0.0170	0.0234
40	0	0.0006	0.0024	0.0051	0.0087	0.0130	0.0180

Table 2. Bivariate Regression of Return Autocorrelation (*AR*) on the Absolute Change in Mean Observed Return Between Two Half-Samples. The sample of available observations for each stock is split into two halves and the explanatory variable ($|\Delta ER|$) is the absolute difference in the sample mean return across the two halves. The dependent variable is the unconditional *AR* with all monthly observations in each stock's entire sample and it is transformed based on the equation in Section II.3. Panel B presents regression results of *AR* on the estimated ρ from equation (5). $|\Delta ER|$ is standardized to have a zero mean and unit variance. All stock prices are the mid-price of bid and ask closing prices to avoid bid-ask bounce. Standard errors are clustered at the industry level. The sample period is 1967-2018, inclusive. ***, **, and * represent significance at the 1%, 5%, and 10% levels, respectively.

Panel A: Regression of *AR* on $|\Delta ER|$

	All firms	Positive book value	Non-financial	Positive book value and non-financial
Intercept	−0.066*	−0.066	−0.091***	−0.102***
t-stat.	−1.923	−1.705	−3.529	−4.322
$\|\Delta ER\|$	0.142***	0.148***	0.137***	0.144***
t-stat.	7.166	6.595	9.725	10.292
R^2	1.27%	1.20%	1.42%	1.43%
# stocks	15,820	13,266	11,607	9,271

Panel B: Regression of *AR* on *AR* bias (ρ) from equation (5)

	All firms	Positive book value	Non-financial	Positive book value and non-financial
Intercept	−0.066**	−0.074**	−0.077***	−0.094***
t-stat.	−2.143	−2.089	−3.088	−4.040
$\|\Delta ER\|$	0.185***	0.183***	0.174***	0.170***
t-stat.	13.704	12.03	12.984	14.41
R^2	2.17%	2.08%	2.00%	1.89%
# stocks	15,820	13,266	11,607	9,271

Table 3. Bivariate Regression of Return Autocorrelation (*AR*) on the Absolute Change In (and Volatility of) Expected Return based on Martin and Wagner (2019). The sample of available observations for each stock is split into two halves. The expected returns *(ER*) are estimated via the Martin and Wagner (2019) approach. The dependent variable is the unconditional *AR* with all monthly observations in each stock's entire sample and it is transformed based on the equation in Section II.3. The explanatory variable is either the absolute difference in the *ER* ($|\Delta ER|$) across the two halves or the monthly volatility of *ER*. Both $|\Delta ER|$ and the volatility of *ER* are standardized to have a zero mean and unit variance. The sample consists of 834 stocks from January 1996 to September 2013. All stock prices are the mid-price of bid and ask closing prices to avoid bid-ask bounce. Standard errors are clustered at the industry level. ***, **, and * represent significance at the 1%, 5%, and 10% levels, respectively.

	Estimate	*t*-stat.	R^2	# stocks
Intercept	−0.096*	−1.656	0.79%	799
$\|\Delta ER\|$	0.108*	1.973		
Intercept	−0.096	−1.597	3.42%	799
Volatility of *ER*	0.226***	3.517		

Table 4. Bivariate Regression of Return Autocorrelation (*AR*) on the Absolute Change In (and Volatility of) Expected Returns Estimated From Fama-French 5- and 6-Factor Models. The sample of available observations for each stock is split into two halves. The expected returns *(ER*) are estimated via the Fama-French 5- and 6-factor models. The dependent variable is the unconditional *AR* with all monthly observations in each stock's entire sample and it is transformed based on the equation in Section II.3 The sample period is 1967-2018 inclusive. The explanatory variable is either the absolute difference in the *ER* ($|\Delta ER|$) across the two halves or the monthly volatility of *ER*. Both $|\Delta ER|$ and volatility of *ER* are standardized to have a zero mean and unit variance. All stock prices are the mid-price of bid and ask closing prices to avoid bid-ask bounce. Standard errors are clustered at the industry level. ***, **, and * represent significance at the 1%, 5%, and 10% levels, respectively.

	Estimate	*t*-stat.	R^2	# stocks
	Fama-French 5 factor model			
Intercept	−0.055*	−1.725	0.38%	13,298
$\|\Delta ER\|$	0.080***	4.904		
Intercept	−0.059	−1.682	0.74%	13,298
Volatility of *ER*	0.106***	5.178		
	Fama-French 6 factor model			
Intercept	−0.056*	−1.752	0.35%	13,298
$\|\Delta ER\|$	0.075***	5.938		
Intercept	−0.060*	−1.728	0.63%	13,298
Volatility of *ER*	0.097***	5.225		

Table 5. Bivariate Regression of Return Autocorrelation (*AR*) on the Absolute Change In (and Volatility of) Expected Return Estimated From Analysts' Price Targets. The sample of available observations for each stock is split into two halves. The expected returns (*ER)* are estimated via analysts' 12-month price targets in IBES. The dependent variable is the unconditional *AR* with all monthly observations in each stock's entire sample and it is transformed based on the equation in Section II.3. All stock prices are measured as the mid-point of bid and ask quotes to avoid bid-ask bounce. The explanatory variable is either the absolute difference in the *ER* ($|\Delta ER|$) across the two halves or the monthly volatility of *ER*. Both $|\Delta ER|$ and volatility of *ER* are standardized to have a zero mean and unit variance. The sample period is 1999-2019, inclusive. . All stock prices are measured as the mid-point of bid and ask quotes to avoid bid-ask bounce. Standard errors are clustered at the industry level. ***, **, and * represent significance at the 1%, 5%, and 10% levels, respectively.

	Estimate	*t*-stat.	R^2	# stocks
Intercept	−0.155***	−6.268	0.57%	5,730
$\lvert\Delta ER\rvert$	0.098***	5.660		
Intercept	−0.155***	−6.231	1.43%	5,730
Volatility of *ER*	0.151***	6.824		

Table 6. Bivariate Regression of Return Autocorrelation (*AR*) on the Absolute Change In Mean Return during Periods before and after Identified Mean Return Breakpoints. The sample of available observations for individual stocks and factors is split based on either all breakpoints or significant breakpoints identified by the Quandt-Andrews test developed by Andrews (1993, 2003), and Quandt (1960) and significant breakpoints suggested by the Bai-Perron (1998) test. Then a cross-sectional regression is computed where the explanatory variable is the absolute difference in the sample mean returns ($|\Delta ER|$) for a stock or a factor before and after the breakpoint. The dependent variable is the unconditional *AR* with all monthly observations in each stock's entire sample and it is transformed based on the equation in Section II.3. $|\Delta ER|$ is standardized to have a zero mean and unit variance. All stock prices are measured as the mid-point of bid and ask quotes to avoid bid-ask bounce. Standard errors are clustered at the industry level. The sample period is 1967-2018, inclusive. ***, **, and * represent significance at the 1%, 5%, and 10% levels, respectively.

	All breakpoints from Quandt-Andrews test	All breakpoints from Bai-Perron test	Significant breakpoints from Quandt-Andrews test	Significant breakpoints from Bai-Perron test
	Observed returns			
$\|\Delta ER\|$	0.126***	0.148***	0.133***	0.160***
t-stat.	3.536	2.782	3.413	2.965
# stocks	13,802	1,415	2,795	1,409
R^2	0.93%	0.82%	0.84%	0.96%
	Martin and Wagner (2019)			
$\|\Delta ER\|$	−0.021	−0.059	−0.044	−0.059
t-stat.	−0.401	−1.177	−0.842	−0.117
# stocks	776	613	688	613
R^2	0.03%	0.23%	0.13%	0.23%
	Fama-French 5 factor model			
$\|\Delta ER\|$	0.099***	0.090***	0.102***	0.092***
t-stat.	5.231	3.755	5.013	3.621
# stocks	13,225	10,976	12,713	10,551
R^2	0.64%	0.51%	0.69%	0.54%
	Fama-French 6 factor model			
$\|\Delta ER\|$	0.094***	0.086***	0.095***	0.085***
t-stat.	5.928	4.062	5.630	3.775
# stocks	13,225	10,970	12,688	10,524
R^2	0.60%	0.47%	0.60%	0.46%
	Analysts' price targets			
$\|\Delta ER\|$	0.115***	0.124***	0.119**	0.124***
t-stat.	5.090	4.942	5.193	4.968
# stocks	5,684	4,536	5,406	4,534
R^2	0.84%	0.97%	0.87%	0.97%

Table 7. Bivariate Regression of Corporate Bond Return Autocorrelation (*AR*) on the Absolute Change in Mean Observed Return and Expected Returns between Two Half-Samples. The sample of available observations for each bond is split into two halves, and the explanatory variable is the absolute difference in the sample mean return in the two halves. The dependent variable is the unconditional *AR* with all monthly observations in each bond's entire sample and it is transformed based on the equation in Section II.3. The explanatory variable is the absolute difference in the *ER* ($|\Delta ER|$) across the two halves or the monthly volatility of *ER*. Both $|\Delta ER|$ and volatility of *ER* are standardized to have a zero mean and unit variance. Panels A and B report results from expected returns (*ER*) estimated by realized mean returns and those imputed from the Bai, Bali, and Wen (2019) (BBW) factor model, respectively. The bond data are from 2002-2019, inclusive. Standard errors are clustered at the industry level. ***, **, and * represent significance at the 1%, 5%, and 10% levels, respectively.

Panel A: Actual bond returns as expected returns

	Intercept	$\|\Delta ER\|$	# bonds	R^2
Estimate	−0.623***	0.214***	21,358	1.36%
t-stat.	−63.153	17.139		
	Intercept	Volatility of *ER*	# bonds	R^2
Estimate	−0.647***	0.059***	21,358	0.17%
t-stat.	−65.789	5.966		

Panel B: Expected bond returns estimated from the BBW factors

	Intercept	$\|\Delta ER\|$	# bonds	R^2
Estimate	−0.152***	0.015**	1,137	0.48%
t-stat.	−24.300	2.346		
	Intercept	Volatility of *ER*	# bonds	R^2
Estimate	−1.070***	0.109**	1,137	0.48%
t-stat.	−22.875	2.331		

Table 8. Cross-Autocorrelation and Market-Model Betas. This table presents cross-autocorrelation among stocks sorted by their market-model beta into quintiles. For those stocks that have at least 30 monthly returns observations, we estimate the market-model beta. We sort stocks into beta quintiles and, within a quintile, randomly draw a pair of stocks and compute their cross-autocorrelations. As a robustness check, we also sort stocks by their PERMNOs and compute cross-autocorrelations of stocks that have adjacent PERMNOs. We then take an average of the cross-autocorrelations across stock pairs within a quintile. For both methods, we do not use any stock twice and estimate cross autocorrelations between $R_{i,t}$ and $R_{j,t-1}$ (and $R_{i,t-1}$ and $R_{j,t}$) where i and j represent stocks i and j, and t is the time index, in units of a month. The autocorrelation is transformed based on the equation in Section II.3. The sample period is from 1967 to 2018, inclusive. All stock prices are measured as the mid-point of bid and ask quotes to avoid bid-ask bounce. Standard errors are clustered at the industry level.

Beta Quintile	Q_1	Q_2	Q_3	Q_4	Q_5
Mean market-model beta	0.014	0.656	1.000	1.381	2.378
	Pairs chosen randomly within each beta quintile (without replacement within each Beta Quintile)				
	$AR(R_{1t}, R_{2t-1})$				
Mean *AR*	0.071	0.111	0.181	0.210	0.244
# of unique pairs	2,011	2,015	2,011	2,016	2,006
	$AR(R_{1t-1}, R_{2t})$				
Mean *AR*	0.083	0.157	0.188	0.203	0.211
# of unique pairs	2,013	2,019	2,014	2,017	2,004
	Pairs with adjacent PERMNOs				
	$AR(R_{1t}, R_{2t-1})$				
Mean *AR*	0.186	0.197	0.238	0.302	0.331
# of unique pairs	1,866	1,852	1,856	1,834	1,915
	$AR(R_{1t-1}, R_{2t})$				
Mean *AR*	0.154	0.240	0.261	0.352	0.330
# of unique pairs	1,866	1,854	1,857	1,838	1,917

Appendix A: Results via Feasible Generalized Least Square (FGLS)

Table A1. Bivariate FGLS Regression of Return Autocorrelation (*AR*) on the Absolute Change In Mean Observed Return between Two Half-Samples. The sample of available observations for each stock is split into two halves and the explanatory variable ($|\Delta ER|$) is the absolute difference in the sample mean return across the two halves. The dependent variable is the unconditional *AR* with all monthly observations in each stock's entire sample and it is transformed based on the equation in Section II.3. Panel B presents regression results of *AR* on the estimated ρ from equation (5). $|\Delta ER|$ is standardized to have a zero mean and unit variance. All stock prices are the mid-price of bid and ask closing prices to avoid bid-ask bounce. Standard errors are clustered at the industry level. The sample period is 1967-2018, inclusive. ***,**, and * represent significance at the 1%, 5%, and 10% levels, respectively.

Panel A: FGLS regression of *AR* on $|\Delta ER|$

	All Firms	Positive book value	Non-financial	Positive book value and non-financial
$\lvert\Delta ER\rvert$	0.142***	0.148***	0.137***	0.144***
t-stat.	14.24	12.70	12.95	11.59
F-Value	202.70***	161.26***	167.66***	134.31***
Pr>*F*	<0.0001	<.0001	<.0001	<.0001
# stocks	15,820	13,266	11,607	9,271

Panel B: FGLS regression of *AR* on *AR* bias (ρ) in equation (5)

	All firms	Positive book value	Non-financial	Positive book value and non-financial
$\lvert\Delta ER\rvert$	0.185***	0.183***	0.174***	0.170***
t-stat.	18.75	16.80	15.39	13.35
F-value	351.66***	282.41***	236.95***	178.28***
Pr>*F*	<.0001	<.0001	<.0001	<.0001
# stocks	15,820	13,266	11,607	9,271

Table A2. Bivariate FGLS Regression of Return Autocorrelation (*AR*) on the Absolute Change In (and Volatility of) Expected Return based on Martin and Wagner (2019). The sample of available observations for each stock is split into two halves. The expected returns *(ER)* are estimated via the Martin and Wagner (2019) approach. The dependent variable is the unconditional *AR* with all monthly observations in each stock's entire sample and it is transformed based on the equation in Section II.3. The sample is split into two halves and the explanatory variable is the absolute difference in the *ER* ($|\Delta ER|$) in the two halves or the volatility of *ER*. The explanatory variable is the absolute difference in the *ER* ($|\Delta ER|$) across the two halves or the monthly volatility of *ER*. Both $|\Delta ER|$ and volatility of *ER* are standardized to have a zero mean and unit variance. The sample consists of 834 stocks from January 1996 to September 2013. All stock prices are the mid-price of bid and ask closing prices to avoid bid-ask bounce. Standard errors are clustered at the industry level. ***, **, and * represent significance at the 1%, 5%, and 10% levels, respectively.

	Estimate	*t*-stat.	*F*-value	Pr>*F*	# stocks
Intercept	−0.096**	−2.21	6.33**	0.012	799
$\lvert\Delta ER\rvert$	0.108**	2.52			
	Estimate	*t*-stat.	*F*-value	Pr>*F*	
Intercept	−0.096***	−2.25	28.33***	<.0001	799
Volatility of *ER*	0.226***	5.32			

Table A3. Bivariate FGLS Regression of Return Autocorrelation (*AR*) on the Absolute Change In (and Volatility of) Expected Returns Estimated From Fama-French 5- and 6-Factor Models. The sample of available observations for each stock is split into two halves. The expected returns *(ER)* are estimated via the Fama-French 5- and 6-factor models. The dependent variable is the unconditional *AR* with all monthly observations in each stock's entire sample and it is transformed based on the equation in Section II.3 The sample period is 1967-2018, inclusive. The explanatory variable is either the absolute difference in the *ER* ($|\Delta ER|$) across the two halves or the monthly volatility of *ER*. Both $|\Delta ER|$ and volatility of *ER* are standardized to have a zero mean and unit variance. All stock prices are the mid-price of bid and ask closing prices to avoid bid-ask bounce. Standard errors are clustered at the industry level. ***, **, and * represent significance at the 1%, 5%, and 10% levels, respectively.

	Estimate	t-stat.	F-value	Pr>F
		Fama-French 5 factor model		
Intercept	−0.055***	−5.01	51.12***	<.0001
$\lvert\Delta ER\rvert$	0.080***	7.15		
Intercept	−0.059***	−5.38	99.57***	<.0001
Volatility of *ER*	0.106***	9.98		
		Fama-French 6 factor model		
Intercept	−0.056***	−5.08	46.30***	<.0001
$\lvert\Delta ER\rvert$	0.075***	6.80		
Intercept	−0.060***	−5.47	83.95****	<.0001
Volatility of *ER*	0.098***	9.16		

Table A4. Bivariate FGLS Regression of Return Autocorrelation (*AR*) on the Absolute Change In (and Volatility of) Expected Return Estimated From Analysts' Price Targets. The sample of available observations for each stock is split into two halves. The expected returns (*ER)* are estimated via analysts' 12-month price targets in IBES. The dependent variable is the unconditional *AR* with all monthly observations in each stock's entire sample and it is transformed based on the equation in Section II.3. All stock prices are measured as the mid-point of bid and ask quotes to avoid bid-ask bounce. The explanatory variable is either the absolute difference in the *ER* ($|\Delta ER|$) across the two halves or the monthly volatility of *ER*. Both $|\Delta ER|$ and volatility of *ER* are standardized to have a zero mean and unit variance. The sample period is 1999-2019, inclusive. Standard errors are clustered at the industry level. ***, **, and * represent significance at the 1%, 5%, and 10% levels, respectively.

	Estimate	t-stat.	F-value	Pr>F
Intercept	−0.155***	−9.85	33.01	<.0001
$|\Delta ER|$	0.098***	5.75		
Intercept	−0.155***	−9.91	82.92	<.0001
Volatility of *ER*	0.151***	9.11		

Table A5. Bivariate FGLS Regression of Corporate Bond Return Autocorrelation (*AR*) on the Absolute Change In Mean Observed Return and Expected Returns between Two Half-Samples. The sample of available observations for each bond is split into two halves, and the explanatory variable is the absolute difference in the sample mean return ($|\Delta ER|$) across the two halves. The dependent variable is the unconditional autocorrelation with all monthly observations in each bond's entire sample and it is transformed based on the equation in Section II.3. $|\Delta ER|$ is standardized to have a zero mean and unit variance. In Panel A, the expected returns are estimated by realized mean returns. Panel B uses monthly averages of yields to maturity (*YTM*) and Macaulay durations as *ER* proxies. The bond data is from 2002-2019, inclusive. ***, **, and * represent significance at the 1%, 5%, and 10% levels, respectively.

Panel A: Different samples

	Intercept	$\|\Delta ER\|$	# of bonds	F-value	Pr>F
			All firms		
Estimate	0.39***	0.78***	20,607	1000.0***	<.0001
t-stat.	16.89	31.62			
			Positive BV		
Estimate	0.40***	0.82***	20,415	1024.6***	<.0001
t-stat.	17.10	32.01			
			Non-finance		
Estimate	0.02	0.63***	10,997	355.2***	<.0001
t-stat.	0.95	18.85			
			Positive BV and non-finance		
Estimate	0.05*	0.74***	10,813	382.3***	<.0001
t-stat.	1.89	19.55			

Panel B: Yields to maturity and durations as ER proxies

	Intercept	$\|\Delta ER\|$	# of bonds	F-value	Pr>F
			$\|\Delta YTM\|$		
Estimate	0.98***	0.63***	21,723	450.7***	<.0001
t-stat.	33.10	21.23			
			$\|\Delta Duration\|$		
Estimate	0.99***	0.22***	20,415	49.7***	<.0001
t-stat.	32.51	7.05			

Appendix B: Biased Cross-Autocorrelations and Betas

If *ER* changes are entirely driven by changes in the market's expected return transmitted through the assets' betas, which are assumed to be constant, then equation (18) can be written as

$$\rho_c = \frac{\alpha(1-\alpha)\beta_i\beta_j[E(R_{M,B})-E(R_{M,A})]^2}{\left\{\left[\bar{\sigma}_i^2+\alpha(1-\alpha)\beta_i^2\left(E(R_{M,B})-E(R_{M,A})\right)^2\right]\left[\bar{\sigma}_j^2+\alpha(1-\alpha)\beta_j^2\left(E(R_{M,B})-E(R_{M,A})^2\right]\right\}^{1/2}}. \quad \text{(B.1)}$$

This is the same as

$$\rho_c = \frac{\beta_i\beta_j\phi}{\left\{\left[\bar{\sigma}_i^2+\beta_i^2\phi\right]\left[\bar{\sigma}_j^2+\beta_j^2\phi\ \right]\right\}^{1/2}}, \quad \text{(B. 2)}$$

where $\phi = (1-\alpha)\left(E(R_{M,B})-E(R_{M,A})\right)^2$.

The weighted average conditional variance for asset i is

$$\sigma_i^2 = \alpha\left[\beta_i^2 Var\left(R_{M,A}\right) + Var(\varepsilon_{i,A})\right] + (1-\alpha)\left[\beta_i^2 Var\left(R_{M,B}\right) + Var(\varepsilon_{i,B})\right],$$

or in simpler notation,

$$\bar{\sigma}_i^2 = \beta_i^2\left[\alpha\sigma_{M,A}^2 + (1-\alpha)\sigma_{M,B}^2\right] + \alpha\sigma_{\varepsilon,A}^2 + (1-\alpha)\sigma_{\varepsilon,B}^2,$$

where $\varepsilon_{i,A}$ is the market model idiosyncratic disturbance for asset i in Regime A and similarly for asset j.

If the market's variance and the idiosyncratic variances are the same in the two regimes, this expression simplifies to

$$\bar{\sigma}_i^2 = \beta_i^2 \sigma_M^2 + \sigma_\varepsilon^2.$$

If the betas of the two assets are the same (and are constants over time) and both weight-averaged variances are also the same, equation (B.2) reduces to

$$\rho_c = \frac{\beta^2 \phi}{\beta^2 \sigma_M^2 + \sigma_\varepsilon^2 + \beta_i^2 \phi} = \frac{1}{\frac{\sigma_M^2}{\phi} + \frac{\sigma_\varepsilon^2}{\beta_i^2 \phi} + 1},$$

so the cross-*AR* increases with beta (assuming that idiosyncratic volatility is non-zero.)

More generally, dividing the numerator and denominator of equation (B.2) by its numerator, it becomes

$$\rho_c = \frac{1}{\phi}\left\{(\phi + \sigma_M^2)^2 + (\phi + \sigma_M^2)\left(\frac{\sigma_{\varepsilon_i}^2}{\beta_i^2} + \frac{\sigma_{\varepsilon_j}^2}{\beta_j^2}\right) + \frac{\sigma_{\varepsilon_i}^2}{\beta_i^2}\frac{\sigma_{\varepsilon_j}^2}{\beta_j^2}\right\}^{-\frac{1}{2}}. \quad \text{(B.3)}$$

A simplified version of equation (B.3) is

$$\rho_c = \phi\left\{\left(\phi + \sigma_M^2 + \frac{\sigma_{\varepsilon_i}^2}{\beta_i^2}\right)^2 + \left(\phi + \sigma_M^2 + \frac{\sigma_{\varepsilon_j}^2}{\beta_j^2}\right)\right\}^{-\frac{1}{2}}. \quad \text{(B.4)}$$

If either or both betas increase, *ceteris paribus*, the denominator decreases and the cross-*AR* increases. Hence, groups of stocks ranked by beta should display higher cross-*AR*s in the higher betas groups. The same would be true for groups ranked by the ratio of beta to idiosyncratic variance, $\frac{\beta_j^2}{\sigma_{\varepsilon_j}^2}$.