# Commodity Dependence and Optimal Asset Allocation

Vianney Dequiedt[1], Mathieu Gomes[2], Kuntara Pukthuanthong[3], and Benjamin Williams-Rambaud[2]

[1]Université Clermont Auvergne, Cerdi, 11 bd Charles de Gaulle, 63000 Clermont-Ferrand, France.

[2]Université Clermont Auvergne, CleRMa, 11 bd Charles de Gaulle, 63000 Clermont-Ferrand, France.

[3]Robert Trulaske Sr. College of Business, University of Missouri, Columbia, MO 65211, USA.

May 24, 2023

**Abstract**

We present a model to explain the diversification benefits of incorporating commodities into a portfolio of traditional assets from the perspective of domestic investors. Utilizing a sample of 39 countries from 2000 to 2020, we show that investors in high-commodity dependence countries generally do not benefit from adding commodities to their portfolios while investors located in low-commodity dependence countries usually do. Commodities may augment a diversified portfolio if investors are not excessively exposed to commodity risk through their country's economic structure. Portfolio management research should consider the diversity of local contexts as it can yield different insights into asset allocation.

**Keywords:** Commodity dependence; Asset allocation; Diversification.

# I. Introduction

Numerous studies have examined the diversification benefits of incorporating commodities in a multi-asset portfolio. For instance, Bodie and Rosansky [1980] analyze 23 individual commodities over the 1950-1976 period and find that switching from an all-stock portfolio to a portfolio with 40% in commodity futures and 60% in US stocks could reduce the overall risk by 30% without sacrificing returns. Gorton and Rouwenhorst [2006] extend these findings by demonstrating that commodity futures, in terms of mean returns and Sharpe ratio, have historically offered equivalent performance to US equities. However, commodity futures negatively correlate with US stocks and bonds due to their distinct behavior over business cycles. Such findings are consistent with research that suggests equities and commodities are segmented markets, with equity pricing factors failing to account for the cross-section of commodity futures returns [Bessembinder and Chan, 1992].

More recent investigations into the matter have produced divergent results. Some studies posit that the financialization of commodities, characterized by institutional funds' unprecedented inflow into commodity futures markets over the past two decades, has heightened their correlation with traditional assets, thereby diminishing their diversification potential [Basak and Pavlova, 2016, Tang and Xiong, 2012].[1] Conversely, other works indicate that correlations between commodity futures and traditional assets remain stable over time and experience only transient increases during severe recessions [Bhardwaj et al., 2015, Levine et al., 2018]. Furthermore, certain studies suggest that the diversification benefits of commodities hinge on the portfolio selection method [Gao and Nardari, 2018] or the specific commodity in question [Bessler and Wolff, 2015]. Notably, most studies examining the role of commodities in a portfolio consider the perspective of a US investor or an investor in US

[1]The financialization of commodities refers to the unprecedented inflow of institutional funds into commodity futures markets over the past two decades. The U.S. Commodity Futures Trading Commission (CFTC) estimates that commodity index investments in US and non-US futures markets increased from $15 billion in 2003 to $160.4 billion in 2015, after reaching a peak of $201.5 billion in 2008. Source: https://www.cftc.gov/

dollar-denominated assets.[2]

This research paper examines the diversification properties of commodities for domestic investors in a broad set of developed and emerging countries. To achieve this, we develop a small-economy model comprising a commodity- and a non-commodity-sector. Through this model, we derive a series of implications that lead us to contend that the extent to which a country is exposed to commodity risk should impact the benefits, or lack thereof, of incorporating commodities into the portfolios of domestic investors.

We define country commodity risk exposure as resulting from the degree of commodity dependence (i.e., the proportion of total merchandise export accounted for by commodities). When a country is highly exposed to commodity risk, both domestic stock and bond returns may be correlated with trends in commodity prices. This is especially true for resource-based economies, or countries that demonstrate a high degree of commodity dependence. Economic growth in such nations is positively linked to the prices of exported commodities [Adams et al., 1979]. The relationship between commodity prices and economic growth, in turn, has significant consequences for the expected relationship between commodities and the returns on domestic stock and bond investments. As a major export experiences increasing demand, the economic growth of the exporting country is expected to rise, thereby driving up stock and bond returns in the market. Consequently, this implies a positive correlation between the returns of major exports and those of traditional assets, thereby diminishing the diversification benefits of the exports for local investors.

Drawing upon a global dataset of stock and bond market data from 39 countries and the Dow Jones Commodity Total Return Index as a proxy for investing in commodity futures, our study examines the diversification benefits of commodities for investors around the world, with particular attention paid to the degree of commodity dependence in their respective countries. Our findings indicate that the diversifying benefits of commodities depend heavily

[2]Exceptions are Cheung and Miu [2010] and Belousova and Dorfleitner [2012], who study the diversification benefits of commodities from the perspective of Canadian and euro investors, respectively.

on the level of commodity dependence in each country, with inclusion in the optimal portfolio improving the Sharpe ratio in 71% of low-commodity dependence countries while providing no diversification benefits in high-commodity dependence countries. Moreover, we observe that the optimal portfolio weight for commodities is substantially greater in low-commodity dependence countries (9.20%) than in high-commodity dependence countries (0.81%).

To test the robustness of our findings, we conduct additional tests using an equally-weighted commodity index and diversification benefit measures based on the proportion of asset returns that can be explained by a set of global factors [Pukthuanthong and Roll, 2009]. Results are consistent with our initial findings, demonstrating the reliability and validity of our conclusions. The implications of our research suggest that investors seeking to diversify their portfolios with commodities must carefully consider their country's degree of commodity dependence to optimize their returns.

Our study makes significant contributions to the literature in several ways. First, we establish a crucial link between the potential diversification benefits of commodities and the degree of commodity dependence at the country level. In doing so, we highlight that an assessment of the relevance of commodities for an investor must account for the inherent risk exposure of the investor's home country. Second, while prior studies on commodities have typically focused on one or a few countries, we expand on this by examining a large sample of 39 countries with varying levels of economic development.[3] This broad cross-section of countries strengthens the generalizability of our results. Third, our study also has implications for international business, and could be extended to other country-dependent exports. Specifically, local investors of countries dependent on exporting a particular product will likely gain less diversification benefits from investing in that product. Future research could explore this avenue. Lastly, we suggest valuable insights into the economic channels through which diversification benefits can decrease. An increase in demand for a significant

[3] A notable exception is Driesprong et al. [2008], who study the links between oil price changes and stock returns across 48 countries.

export can drive up stock and bond returns in the market, reducing the diversification benefits of that export for local investors.

The remainder of this paper is organized as follows. In Section II, we develop our theoretical model and derive implications regarding the link between commodity diversification benefits and a country's commodity dependence. Section III discusses the data and methodology. In section IV, we present our main results. Section V reports the results of additional tests. We discuss the economic rationale underlying our findings in more detail in Section VI. Finally, Section VII concludes.

# II. Model

We consider a two sector, small and open economy. Sector 1 is the commodity sector while Sector 2 is the non-commodity sector. In the commodity sector, production requires capital $K$ and labor $L$ with $K_1$ and $L_1$, the respective quantities used by sector 1. The commodity price $p_1$ is the world price and is taken as given by agents in this small economy. The price of labor is the wage $w$, which is determined in equilibrium. This sector's return on capital is denoted by $r_1$. $r_1$ is sector specific in the short run, as we will explain below, and it is also determined in equilibrium. Finally, the aggregate production function is $f_1(K_1, L_1)$, with $f_1$ being a homogeneous function of degree 1 so that sector profits are equal to zero when all inputs are paid according to their marginal returns. With this notation, we can express the commodity sector profits as

$$\Pi_1 = p_1 f_1(K_1, L_1) - r_1 K_1 - w L_1.$$

In sector 2, production requires capital, labor, and commodities, with quantities denoted $K_2, L_2$, and $q_2$, respectively. The output of sector 1 is an input in sector 2. This aspect is standard in the business cycle literature that analyzes the diffusion of shocks on commodity prices in the economy (e.g., Drechsel and Tenreyro [2018]). Sector 2's output price is a world price denoted $p_2$. Labor moves freely from one sector to the other, and there is a unique wage level $w$ in equilibrium. The return on capital in sector 2 is $r_2$ which can differ from $r_1$ in the short run. The non-commodity sector profits are thus given by

$$\Pi_2 = p_2 f_2(K_2, L_2, q_2) - r_2 K_2 - w L_2 - p_1 q_2.$$

The total amount of capital available in this economy is $\bar{K}$ which is split between the two sectors, i.e., $K_1 + K_2 = \bar{K}$ or equivalently $K_2 = \bar{K} - K_1$. We will also consider that there is no unemployment and that the total available quantity of labor is $\bar{L}$, so that $L_1 + L_2 = \bar{L}$ or, equivalently, that $L_2 = \bar{L} - L_1$.

In the following, we further assume that the production functions $f_1$ and $f_2$ are increasing and concave in each argument. Cross-partial derivatives are positive as is the case for Cobb-Douglas functions.

## A. Long-run Equilibrium Conditions

In the long run, all factors can be adjusted so that their marginal productivity equals their price. This aligns with the standard Hecksher-Ohlin model [Jones and Scheinkman, 1977]. In equilibrium, there is a unique wage level $w^*$ and a unique rate of return on capital $r^*$. Capital and labor can move freely from one sector to the other. The five equilibrium quantities $K_1^*, K_2^*, L_1^*, L_2^*, q_2^*$, together with the two equilibrium prices $w^*$ and $r^*$ are jointly determined by the following first-order conditions. First and second, labor should be paid according to its marginal productivity in sector 1 as well as in sector 2, i.e.,

$$w^* = p_1 \frac{\partial f_1}{\partial L}(K_1^*, L_1^*), \tag{1}$$

$$w^* = p_2 \frac{\partial f_2}{\partial L}(K_2^*, L_2^*, q_2^*). \tag{2}$$

Third, commodities should be used in sector 2 in quantity that equalizes its marginal productivity and its price:

$$p_1 = p_2 \frac{\partial f_2}{\partial q}(K_2^*, L_2^*, q_2^*). \tag{3}$$

Fourth and fifth, returns on capital will be equal to capital's marginal productivity in both sectors, i.e.,

$$r^* = p_1 \frac{\partial f_1}{\partial K}(K_1^*, L_1^*), \tag{4}$$

$$r^* = p_2 \frac{\partial f_2}{\partial K}(K_2^*, L_2^*, q_2^*). \tag{5}$$

To these five first-order conditions, we add the two binding resources constraints:

$$\bar{K} = K_1^* + K_2^*, \tag{6}$$

$$\bar{L} = L_1^* + L_2^*. \tag{7}$$

From these seven equalities we obtain the long-run equilibrium values $K_1^*, K_2^*, L_1^*, L_2^*, q_2^*, w^*$ and $r^*$ which we assume are unique.

In what follows, we further assume that the long run equilibrium is such that

$$p_2 \frac{\partial^2 f_2}{\partial q^2}(K_2^*, L_2^*, q_2^*) \left( p_1 \frac{\partial^2 f_1}{\partial L^2}(K_1^*, L_1^*) + p_2 \frac{\partial^2 f_2}{\partial L^2}(K_2^*, L_2^*, q_2^*) \right) - \left( p_2 \frac{\partial^2 f_2}{\partial L \partial q}(K_2^*, L_2^*, q_2^*) \right)^2 \geq 0. \tag{8}$$

While admittedly restrictive, such inequality is verified for a wide range of Cobb-Douglas production functions.

## B. Short-run Responses to Shocks

Starting from the long-run equilibrium situation, we assume that world prices $p_1$ and $p_2$ are subject to small shocks and become respectively $p_1 + \tilde{p}_1$ and $p_2 + \tilde{p}_2$ where $\tilde{p}_1$ and $\tilde{p}_2$ are zero-mean random variables.

In the short run, we assume that capital in sector $i$ is fixed, i.e., $K_1 = K_1^*$ and $K_2 = K_2^*$, while the other factors, labor, and commodities, can be adjusted. Our model is thus a version of the specific factors model of Jones [1971]. In response to the shock, $L_1^*$ and $L_2^*$ will become $L_1^* + \tilde{L}$ and $L_2^* - \tilde{L}$, respectively, because these should still sum up to $\bar{L}$, and $q_2^*$ will become $q_2^* + \tilde{q}$. Finally, wages will adjust to $w^* + \tilde{w}$ while returns to capital will be such that in each sector, capital is paid according to its marginal productivity (all profits are redistributed as wages, payment for commodity inputs, or returns on capital). Thus, in sector 1, $r$ will become $r + \tilde{r}_1$, while in sector 2, it will become $r + \tilde{r}_2$.

Short run equilibrium is characterized by the five variables $\tilde{L}, \tilde{q}, \tilde{w}, \tilde{r}_1, \tilde{r}_2$ satisfying the following first-order conditions.

First and second, labor should be paid according to its marginal productivity in sector 1 as well as in sector 2, i.e.,

$$w^* + \tilde{w} = (p_1 + \tilde{p}_1)\frac{\partial f_1}{\partial L}(K_1^*, L_1^* + \tilde{L}), \tag{9}$$

$$w^* + \tilde{w} = (p_2 + \tilde{p}_2)\frac{\partial f_2}{\partial L}(K_2^*, L_2^* - \tilde{L}, q_2^* + \tilde{q}). \tag{10}$$

Third, commodities should be used in sector 2 in quantity that equalizes their marginal productivity and price:

$$p_1 + \tilde{p}_1 = (p_2 + \tilde{p}_2)\frac{\partial f_2}{\partial q}(K_2^*, L_2^* - \tilde{L}, q_2^* + \tilde{q}). \tag{11}$$

Fourth and fifth, returns on capital will be equal to capital's marginal productivity in both sectors, i.e.,

$$r^* + \tilde{r}_1 = (p_1 + \tilde{p}_1)\frac{\partial f_1}{\partial K}(K_1^*, L_1^* + \tilde{L}), \tag{12}$$

$$r^* + \tilde{r}_2 = (p_2 + \tilde{p}_2)\frac{\partial f_2}{\partial K}(K_2^*, L_2^* - \tilde{L}, q_2^* + \tilde{q}). \tag{13}$$

## C. Linear Approximations of Short-term Responses

At this stage, we consider that $(\tilde{p}_1, \tilde{p}_2)$ is a vector of small shocks with zero means, small variances and covariances, and negligible higher (cross-)moments so that a first-order expansion of equalities (9) to (13) is an acceptable approximation. Such linear approximations can be simplified using long-term equilibrium conditions and become:

$$\tilde{w} = \tilde{p}_1\frac{\partial f_1}{\partial L}(K_1^*, L_1^*) + \tilde{L}p_1\frac{\partial^2 f_1}{\partial L^2}(K_1^*, L_1^*), \tag{14}$$

$$\tilde{w} = \tilde{p}_2\frac{\partial f_2}{\partial L}(K_2^*, L_2^*, q_2^*) - \tilde{L}p_2\frac{\partial^2 f_2}{\partial L^2}(K_2^*, L_2^*, q_2^*) + \tilde{q}p_2\frac{\partial^2 f_2}{\partial L \partial q}(K_2^*, L_2^*, q_2^*), \tag{15}$$

$$\tilde{p}_1 = \tilde{p}_2\frac{\partial f_2}{\partial q}(K_2^*, L_2^*, q_2^*) - \tilde{L}p_2\frac{\partial^2 f_2}{\partial L \partial q}(K_2^*, L_2^*, q_2^*) + \tilde{q}p_2\frac{\partial^2 f_2}{\partial q^2}(K_2^*, L_2^*, q_2^*), \tag{16}$$

$$\tilde{r}_1 = \tilde{p}_1\frac{\partial f_1}{\partial K}(K_1^*, L_1^*) + \tilde{L}p_1\frac{\partial^2 f_1}{\partial K \partial L}(K_1^*, L_1^*), \tag{17}$$

$$\tilde{r}_2 = \tilde{p}_2\frac{\partial f_2}{\partial K}(K_2^*, L_2^*, q_2^*) - \tilde{L}p_2\frac{\partial^2 f_2}{\partial K \partial L}(K_2^*, L_2^*, q_2^*) + \tilde{q}p_2\frac{\partial^2 f_2}{\partial K \partial q}(K_2^*, L_2^*, q_2^*). \tag{18}$$

We thus have a linear system of five equations with five variables. It can be solved to obtain $\tilde{r}_1$ and $\tilde{r}_2$ as linear combinations of $\tilde{p}_1$ and $\tilde{p}_2$.

**Proposition 1.** *The short-run movements of returns on capital in both sectors in response to shocks on international commodity and non-commodity prices can be approximated by the linear formulae*

$$\begin{cases} \tilde{r}_1 = \alpha_1^1 \tilde{p}_1 + \alpha_2^1 \tilde{p}_2, \\ \tilde{r}_2 = \alpha_1^2 \tilde{p}_1 + \alpha_2^2 \tilde{p}_2, \end{cases} \tag{19}$$

where $\alpha_1^1 \geq 0$, $\alpha_2^2 \geq 0$, $\alpha_2^1 \leq 0$ and $\alpha_1^2 \leq 0$.

*Proof.* To simplify notations let us denote

$$\frac{\partial f_1}{\partial L}(K_1^*, L_1^*) \equiv a \ \text{ and } \ p_1 \frac{\partial^2 f_1}{\partial L^2}(K_1^*, L_1^*) \equiv b,$$

$$\frac{\partial f_2}{\partial L}(K_2^*, L_2^*, q_2^*) \equiv c \ , \ p_2 \frac{\partial^2 f_2}{\partial L^2}(K_2^*, L_2^*, q_2^*) \equiv d \ \text{ and } \ p_2 \frac{\partial^2 f_2}{\partial L \partial q}(K_2^*, L_2^*, q_2^*) \equiv e,$$

$$\frac{\partial f_2}{\partial q}(K_2^*, L_2^*, q_2^*) \equiv f \ \text{ and } \ p_2 \frac{\partial^2 f_2}{\partial q^2}(K_2^*, L_2^*, q_2^*) \equiv g,$$

$$\frac{\partial f_1}{\partial K}(K_1^*, L_1^*) \equiv h \ \text{ and } \ p_1 \frac{\partial^2 f_1}{\partial K \partial L}(K_1^*, L_1^*) \equiv j,$$

$$\frac{\partial f_2}{\partial K}(K_2^*, L_2^*, q_2^*) \equiv k \ , \ p_2 \frac{\partial^2 f_2}{\partial K \partial L}(K_2^*, L_2^*, q_2^*) \equiv l \ \text{ and } \ p_2 \frac{\partial^2 f_2}{\partial K \partial q}(K_2^*, L_2^*, q_2^*) \equiv m.$$

From the monotonicity, concavity and positivity of the cross-derivative hypothesis stated at the end of section II for $f_1$ and $f_2$ we deduce that $a, c, e, f, h, j, k, l$ and $m$ are positive. At the same time, $b, d$ and $g$ are negative. We could further remark that long-run equilibrium conditions impose $p_1 a = p_2 c$, $f = \frac{p_1}{p_2}$ and $p_1 h = p_2 k$ but we will not use those three latter equalities.

Our linear system becomes:

$$
(20) \qquad \begin{cases} -b\tilde{L} + \tilde{w} = a\tilde{p}_1, \\ d\tilde{L} - e\tilde{q} + \tilde{w} = c\tilde{p}_2, \\ e\tilde{L} - g\tilde{q} = -\tilde{p}_1 + f\tilde{p}_2, \\ -j\tilde{L} + \tilde{r}_1 = h\tilde{p}_1, \\ l\tilde{L} - m\tilde{q} + \tilde{r}_2 = k\tilde{p}_2. \end{cases}
$$

Solving it for $\tilde{r}_1$ and $\tilde{r}_2$ gives:

$$
(21) \qquad \begin{cases} \tilde{r}_1 = \tilde{p}_1 \left( \frac{-agj+ej}{gb+gd-e^2} + h \right) + \tilde{p}_2 \left( \frac{cgj-fej}{gb+gd-e^2} \right), \\ \tilde{r}_2 = \tilde{p}_1 \left( \frac{-aem+agl+bm+dm-el}{gb+gd-e^2} \right) + \tilde{p}_2 \left( \frac{cem-cgl-fbm-fdm+fel}{gb+gd-e^2} + k \right). \end{cases}
$$

Equation (8) guarantees that the denominator $gb + gd - e^2$ is positive and by exploiting the signs of the diverse derivatives, we obtain that

$$
\alpha_1^1 \equiv \left( \frac{-agj + ej}{gb + gd - e^2} + h \right) \geq 0,
$$

$$
\alpha_2^1 \equiv \left( \frac{cgj - fej}{gb + gd - e^2} \right) \leq 0,
$$

$$
\alpha_1^2 \equiv \left( \frac{-aem + agl + bm + dm - el}{gb + gd - e^2} \right) \leq 0,
$$

$$
\alpha_2^2 \equiv \left( \frac{cem - cgl - fbm - fdm + fel}{gb + gd - e^2} + k \right) \geq 0.
$$

## D. Covariances

In this section, we analyze the covariances of diverse price variables. We restrict our attention to independent shocks on world prices so that $\tilde{p}_1$ and $\tilde{p}_2$ are independently distributed random variables with zero means so that $Cov(\tilde{p}_1, \tilde{p}_2) = 0$. By doing so, we isolate the correlation linked to the structure of our small open economy from the correlation from comovements in world prices.

In what follows, we are interested, *inter alia*, in the returns of a representative portfolio of assets

$$r_r(s) = sr_1 + (1-s)r_2,$$

where $s$ is related to the size of the commodity sector in the economy, $s$ increasing with that size. The short-run response to shocks on $p_1$ and $p_2$ of this average return will be denoted by

$$\tilde{r}_r(s) \equiv s\tilde{r}_1 + (1-s)\tilde{r}_2,$$

where weights $s$ are constant in the short run, consistent with our assumption that capital in each sector is fixed in the short run.

**Proposition 2.** *When world prices $p_1$ and $p_2$ are subject to small independent shocks $\tilde{p}_1$ and $\tilde{p}_2$ with zero mean, small variance and negligible higher moments, the short-run price responses $\tilde{r}_1, \tilde{r}_2$ and $\tilde{r}_r(s)$ are such that:*

1. $Cov(\tilde{p}_i, \tilde{r}_i) \geq 0$, for $i = 1, 2$,
2. $Cov(\tilde{p}_i, \tilde{r}_j) \leq 0$, for $i \neq j$,
3. $Cov(\tilde{r}_i, \tilde{r}_j) \leq 0$, for $i \neq j$,
4. $Cov(p_1, \tilde{r}_r(s))$ is positive for $s$ close to 1, is negative for $s$ close to 0, and is increasing in $s$ for $0 \leq s \leq 1$.

*Proof.* When $\tilde{p}_1$ and $\tilde{p}_2$ are independently distributed, $Cov(\tilde{p}_1, \tilde{r}_1) = \alpha_1^1 Var(\tilde{p}_1) \geq 0$, while $Cov(\tilde{p}_2, \tilde{r}_1) = \alpha_2^1 Var(\tilde{p}_2) \leq 0$. Similarly, $Cov(\tilde{p}_2, \tilde{r}_2) = \alpha_2^2 Var(\tilde{p}_2) \geq 0$, while $Cov(\tilde{p}_1, \tilde{r}_2) = \alpha_1^2 Var(\tilde{p}_1) \leq 0$. Next,

$$Cov(\tilde{r}_i, \tilde{r}_j) = \alpha_1^1 \alpha_1^2 Var(\tilde{p}_1) + \alpha_2^1 \alpha_2^2 Var(\tilde{p}_2) \leq 0.$$

Finally, $Cov(p_1, \tilde{r}_r(s)) = (\alpha_1^1 s + \alpha_1^2(1-s))Var(\tilde{p}_1)$, which is positive for $s$ close enough to 1, is negative for $s$ close enough to 0 (since $\alpha_1^1 \geq 0$ and $\alpha_1^2 \leq 0$) and is increasing with $s$ since $\alpha_1^1 - \alpha_1^2 \geq 0$.

Item 4 in Proposition 2 relates the benefits of diversifying a portfolio of domestic assets with commodities to the size of the commodity sector in the economy: the larger the commodity sector, the lower the risk diversification benefits.

# III. Empirical Analysis

## A. Sample Selection

To evaluate the potential benefits of including commodities in a portfolio, we focus on portfolios consisting of domestic stocks and bonds. To ensure a comprehensive assessment of the impact of commodity allocation for investors worldwide, we initially limit our analysis to countries with an equity market index available through MSCI. This narrows our scope to 78 countries categorized as developed, emerging, or frontier markets. We employ Aswath Damodaran's approach –detailed in Appendix A– to estimating domestic bond returns by assuming a ten-year constant maturity, using historical ten-year yields on government bonds from Thomson Reuters Refinitiv where possible.[4] This method reduces the number of countries in our sample from 78 to 52. Subsequently, we require at least ten years of uninterrupted data for MSCI equity indexes so as to ensure we have enough observations to obtain statistically meaningful results. This further reduces the number of countries to 39.

[4] http://pages.stern.nyu.edu/~adamodar/

## B. Commodity Dependence and Country Classification

We retrieve export trade data from the United Nations Conference on Trade and Development (UNCTAD) statistics website. To measure commodity dependence at the country level, we define export commodity dependence as the percentage ratio of commodity exports to total merchandise exports.[5] We compute the overall commodity dependence for each country by dividing the total value of commodity exports by the total value of merchandise exports. Additionally, we calculate the dependence ratio for each sector and each commodity where an investable commodity index is available.[6] The UNCTAD nomenclature categorizes primary commodities into eight groups: Food and live animals (0), Beverages and tobacco (1), Crude materials, inedible, except fuels (2), Mineral fuels, lubricants and related materials (3), Animal and vegetable oils, fats and waxes (4), Non-ferrous metals (68), Pearls, precious and semi-precious stones (667), and Gold, non-monetary excluding ores and concentrates (97).

Table 1 presents the levels of commodity dependence for the countries included in our sample, along with the two primary commodity export groups for each country. Given that the Standard International Trade Classification (SITC) classification does not align precisely with the main commodity index components, we establish a matching protocol between the two, as detailed in Table 2. Not all commodities, however, are investable. As such, we also report the degree of dependence accounted for by investable commodities only.

[INSERT TABLE 1 ABOUT HERE]

[INSERT TABLE 2 ABOUT HERE]

To understand the distribution of commodity risk exposure across countries, we classify them *a priori* based on their level of commodity dependence, utilizing the sample mean commodity dependence (32.2%) as the threshold for differentiation. This approach yields

[5] https://unctadstat.unctad.org/

[6] We adopt the UNCTAD nomenclature and utilize sectors as classified by the Standard International Trade Classification (SITC).

two distinct groups of countries. Group 1 encompasses low-commodity dependence countries with low to moderate commodity risk exposure. Group 2, in contrast, comprises high-commodity dependence countries, where commodity risk exposure ranges from moderate to high.

## C. Commodities Data

To proxy for an investment in commodity futures, we use the Dow Jones Commodity Total Return Index (DJCTRI), available since December 1999. This index reflects a fully collateralized investment in nearby commodity futures, with positions rolled over five days (20% of the position is rolled each day into the next futures contract), assuming equal weighting of three major sectors: energy, agriculture/livestock, and metals. Commodities are weighted by relative liquidity based on the five-year average total dollar value traded, and the index is rebalanced quarterly. See Appendix B for the descriptive statistics of equity, bond, and commodity returns across the countries in our sample.

## D. Methodology

We employ the classic mean-variance optimization framework to assess the diversifying potential of commodities within a portfolio of traditional assets [Markowitz, 1952].[7] Within this framework, the benefits of diversification are related to the mean correlation observed among the assets: the lower the mean correlation, the higher the expected benefits of diversification.

According to our assumptions, countries exhibiting a high degree of commodity dependence should feature a higher positive correlation between commodities and domestic stocks and bonds, reducing the potential diversification benefits of commodities. It follows that domestic investors in countries with a high degree of commodity dependence –Group 2– should

[7]Criticisms against the mean-variance framework stress that it applies precisely only when distributions are normal or utility functions quadratic. However, Levy and Markowitz [1979], Pulley [1981], and Kroll et al. [1984] show that mean-variance portfolio results are very similar to those obtained from a direct optimization of expected utility for various utility functions and historical distributions of returns, suggesting that higher moments in practice play a secondary role.

hold a smaller allocation to commodities in their portfolios than those in countries with a low degree of commodity dependence –Group 1.

To study the diversification benefits of commodities, we assess the shift in the optimal portfolio resulting from adding commodities to the asset allocation. Commodities are considered here as the $n$-th asset class, which is added to $n-1$ assets domestic investors already hold. A comparison of the optimal Sharpe ratios [Sharpe, 1966] is conducted in-sample to evaluate the shift in the optimal portfolio when adding commodities. Assuming that short positions and leverage are not allowed, the weights of assets $i$ –denoted by $w_i$– are subject to the following conditions: $0 \leq w_i \leq 1$ (no short sales) and $\sum_i w_i = 1$ (no leverage).

To evaluate the diversification properties of commodities, we assess the weight assigned to commodities in the $n$-asset optimal portfolio –denoted by $w_n$– and the shift in the said optimal portfolio resulting from their inclusion. We expect a higher value for $w_n$ in Group 1 than in Group 2, in which $w_n$ should theoretically be close to zero due to the increased exposure of domestic assets to commodity risk.

In the mean-variance framework, the tangency portfolio (i.e., the portfolio offering the best possible combination of portfolio risk and expected return) is of particular relevance: All investors, depending on their respective risk aversion, are expected to hold a given mixture of the risk-free asset and the tangency (or optimal) portfolio. With this in mind, we focus on the shift in the optimal portfolio by statistically assessing the difference in Sharpe ratios between portfolios without commodities and portfolios including commodities. If the Sharpe ratio of the optimal portfolio increases with the inclusion of commodities, then all the investors will benefit regardless of their risk aversion, as their expected utilities will increase.

We use a simple mean comparison test to assess the statistical significance of the improvement in Sharpe ratios. To this end, we account for the Sharpe ratio mean, standard deviation, and sample size $T$.

Under the assumption of normal independent, identically-distributed returns [Jobson and Korkie, 1981, Lo, 2002], the standard deviation of the estimated Sharpe ratio $\hat{SR}$ is as follows: $SD(\hat{SR}) = \sqrt{(1 + \frac{1}{2}SR^2)/T}$. We use a Z-test to compare the Sharpe ratios of optimal portfolios with and without commodities. Under the null hypothesis, the Z-stat follows a standard normal distribution:

$$Z = \frac{SR_{\text{with}} - SR_{\text{without}}}{\sqrt{SD(SR_{\text{with}})/T + SD(SR_{\text{without}})/T}} \sim N(0,1).$$

The rejection of the null hypothesis indicates that adding commodities to the asset mix significantly improves the Sharpe ratio of the optimal portfolio. The test is run for each country so that we can identify cases where domestic investors benefit from adding commodities to their portfolios. Following our research hypothesis, we expect the rejection of the null for the majority of low-commodity dependence countries –Group 1– and the non-rejection for the majority of high-commodity dependence countries –Group 2.

# IV. Main Results

Table 3 reports the individual country results. It contains the main characteristics of optimal portfolios with and without commodities for low-commodity dependence countries –Group 1– and high-commodity dependence countries –Group 2. The Z-stat column indicates whether the null hypothesis of no commodity diversification benefits can be rejected for each country. Results show that in most low-commodity dependence countries (17 out of 24, or 71%), adding commodities to the asset mix improves the Sharpe ratio of the optimal portfolio. Looking at the high-commodity dependence countries, we see that adding commodities to the asset mix only enhances the Sharpe ratio of the optimal portfolio in the case of the Netherlands (1 out of 14, or 7%). We believe this is due to two main reasons: First, the Netherlands appears at the lower end of the commodity dependence spectrum (as indicated in Table 1) with a commodity dependence of 33%, just above the sample average of 32.2% used as the cut-off point to discriminate between low-commodity dependence countries and high-commodity dependence countries (and way below the 60% cut-off most commonly used by the United Nations).[8] Second, the main export category in the Netherlands is Food and live animals. Specifically, the most exported products include dairy and eggs, meat, and vegetables. In addition, flowers also account for a significant proportion of the Netherlands' exports.[9] Most of these commodities are not traded in organized futures markets, and it, therefore appears logical that investors in the Netherlands still enjoy diversification benefits resulting from commodity futures.

[INSERT TABLE 3 ABOUT HERE]

The last column of Table 3 reports the weight assigned to commodities in the optimal portfolio for all countries. We see that the average weight of commodities in optimal portfolios is much higher for low-commodity dependence countries (9.20%) than for high-commodity dependence countries (0.81%). To make things more transparent, we provide scatter plots of commodity weights versus commodity dependence for all countries in Figure 1. This figure shows a clear negative relationship between the weight assigned to commodities in the optimal portfolio and the country's degree of commodity dependence. Calculating Spearman's rank correlation coefficient confirms this: we find a negative value $\rho = -0.5004$, which is highly significant ($p\text{-}value < .01$).[10] The dispersion of countries along the Y-axis is higher for low-commodity dependence countries than for high-commodity dependence countries. Indeed, in Group 1 the optimal commodity allocation ranges from 0% (for China, Denmark, and Slovenia) to 18.84% for Portugal. In contrast, Group 2 features commodity allocations ranging from 0% (for 10 out of 14 countries, or 71%) to 6.61% for the Netherlands.

[INSERT FIGURE 1 ABOUT HERE]

[8] https://unctad.org/publication/state-commodity-dependence-2021

[9] According to the Netherlands' Ministry of Agriculture, Nature and Food Quality, flowers, dairy and eggs, meat, and vegetables accounted for €33 billion of exports in 2017, representing circa 33% of the country's total exports. Source: https://www.government.nl/latest/news/2018/01/19/agricultural-exports-worth-nearly-%E2%82%AC92-billion-in-2017

[10] The Pearson's linear correlation coefficient equals −0.5534 and is also clearly significant ($p\text{-}value < .01$).

# V. Additional Tests

## A. Accounting for the DJCTRI Individual Commodity Bias

Although the DJCTRI applies equal weights at the sector level (energy, metals, and agriculture/livestock sectors each account for 33.33% of the index), the weights assigned to individual commodities can differ significantly.[11] As a result, some people could argue that the observed diversification benefits of commodities may result from extensive exposure to one or a few commodities. To address this concern, we build a yearly-rebalanced equally-weighted commodity index using the indexes representing the individual commodities present within the DJCTRI. Results are reported in Table 4 and mostly confirm our previous findings. Specifically, adding commodities to the asset mix improves the Sharpe ratio of the optimal portfolio in 15 low-commodity dependence countries out of 24 (63%). In contrast, high-commodity dependence countries never feature a significant commodity allocation. The Sharpe ratios of optimal portfolios are lower than when using the DJCTRI. This finding suggests that a few specific commodities (e.g., crude oil) contribute significantly to the diversifying benefits of commodities.

[INSERT TABLE 4 ABOUT HERE]

## B. Computing Commodity Dependence Using Investable Commodities Only

So far, we have looked at overall commodity dependence and linked it to the diversifying properties of commodities for local investors. However, as we mentioned earlier, not all commodities are investable. As such, it is interesting to look at the relationship between commodity dependence and the benefits of commodities in a portfolio when considering only

[11] For example, as of 2014, WTI crude oil accounted for 12.2% of the index while cotton only represented 0.9%. Source: S&P Dow Jones Indices Indexing 101: Weighing in on the S&P GSCI and the Dow Jones Commodity Index. August 2014.

investable commodities. We therefore use the degree of commodity dependence accounted for by investable commodities only, as reported in Table 1. The average commodity dependence based on investable commodities only is 14.9%. We proceed as before by classifying countries as high-commodity dependence or low-commodity dependence depending on whether they stand above or below that threshold. Compared to our primary analysis, India, the Netherlands and New Zealand now appear as low-commodity dependence countries, while Singapore is now part of the high-commodity dependence group. Scatter plots of commodity weights versus commodity dependence using investable commodities only for all countries are shown in Figure 2. Although slightly different, our main results are primarily confirmed as there appears to be a clear negative relationship between the degree of commodity dependence and the weight of commodities in optimal portfolios.

[INSERT FIGURE 2 ABOUT HERE]

## C. Assessing Commodity Diversification Benefits with $R^2$

We use the $R^2$ methodology developed by Pukthuanthong and Roll [2009] to complement our main results. The authors measure the global market integration of country $i$ as the $R^2$ from regressing equity market returns of country $i$ on ten principal components extracted from equity market returns of countries in a global market. The ten principal components (PCs) are pervasive factors that explain about 90% of eigenvectors' variation. We apply their approach to commodity returns by regressing commodity returns on the PCs extracted from stocks and bonds return of each country $i$. Once the eigenvectors are computed and sorted from the largest to smallest eigenvalue, principal components are estimated from returns in the subsequent calendar year. In other words, the weightings (eigenvectors) computed from the 2000 covariance matrix are applied to the returns of the same countries during 2001. This is repeated in each calendar year; weightings from 2001 are used with returns from 2002, and so on until the 2019 weightings are applied to the 2020 returns that comprise the final available full sample year. This produces 20 calendar years with out-of-sample principal components.

We run a regression of commodities returns on equity, bond, and equity plus bond returns. For the first two series, the index is included. For the latter, we extract global factors or the principal component (PC) for equities and bond return indexes. If the first PC explains at least 75% of the eigenvalue (or, intuitively, 75% of the total volatility in the covariance matrix), we include it. If the first PC explains less than 75%, we include the first two PCs. We then regress the commodities index on principal component(s).

The $R^2$ from these regressions captures the integration of commodities with stock and bond markets. A higher $R^2$ implies lower diversification benefits. Pukthuanthong and Roll [2009] argue that $R^2$ is a better measure of market integration than correlation. To illustrate, $R^2$ indirectly regards country-specific residual variance in a factor model as an indicator of imperfect integration. When multiple factors drive returns, markets may be imperfectly correlated but perfectly integrated.

Pukthuanthong and Roll [2009] show that while perfect integration implies that identical global factors explain index returns across countries, some countries may differ in their sensitivities to those factors and, thus, are not perfectly correlated. In a world of multiple factors, the simple correlation between index returns could be a flawed measure of integration. Papers that have directly examined diversification and are complementary to our analysis include Christoffersen et al. [2012]. The authors present a dynamic diversification measure based on expected shortfall and tail values. Unlike their measure, Pukthuanthong and Roll [2009]'s diversification indexes do not require a specific portfolio allocation nor an estimation of the full covariance matrix.

Table 5 displays the findings using the $R^2$ methodology. In this experiment, we use two series of commodity indexes, including commodity total returns (DJCTRI) and index returns with equal weighting. We demonstrate the integration of the commodities indexes with equities, bonds, and equities plus bonds. The outcomes support our premise. Investors in countries heavily dependent on commodities get lower diversification benefits from holding commodities in their portfolios. The 1-$R^2$ of commodity-dependent countries is lower for all asset combinations, regardless of how we compute commodity dependence (using all commodities or investable commodities only).[12]

[INSERT TABLE 5 ABOUT HERE]

# VI. Discussion

The present study has constructed a theoretical model that establishes a connection between the diversification advantages of commodities vis-à-vis domestic assets and the weight of the commodity sector in the economy. Our empirical examination of the model's implications has confirmed that investors in countries that depend heavily on commodities typically do not gain from incorporating commodities into their portfolio of domestic assets. In contrast, those in countries with low commodity dependence usually do.

We offer suggestions regarding the precise mechanisms that may explain these empirical results. Determining which channels prevail will constitute an interesting avenue for future research.

First, because economic growth is a crucial driver of public debt sustainability, the price of exported commodities should impact the returns of bonds issued by commodity-dependent countries as changes in commodity prices translate into shifts in fiscal performance [Kumah and Matovu, 2007]. Presbitero et al. [2015] have demonstrated that spreads on sovereign bonds tend to be lower for nations with strong external and fiscal positions and robust economic growth. Hence, a rise or fall in commodity prices should result in a decrease or an increase in government bond yields, respectively, through a reduction or an expansion of the sovereign default risk premium. Given a constant-maturity government bond portfolio, the fluctuation in commodity prices should be positively correlated with government bond re-

[12] Results remain intact when including both stock and bond indexes rather than the global factor in the regression.

turns.[13,14] Consequently, a more significant commodity sector in the economy would decrease the diversification benefits of commodities for a local bond portfolio.

Second, commodity prices affect stock returns through the earnings of commodity firms. Assuming constant production costs, the profits of commodity firms will be a direct positive function of commodity prices. Since commodity firms are typically over-represented in the major indexes of commodity-dependent countries, the stock returns of such countries should be positively correlated with commodity price changes.[15] Again, a larger commodity sector in the economy would decrease the diversification benefits of commodities for a local stock portfolio.

Third, high-commodity-dependent countries often have currencies strongly correlated with commodity prices [Chen and Rogoff, 2003, Chen et al., 2010]. Domestic investors in these countries might already face currency risks linked to commodity price fluctuations, reducing the diversification benefits of adding commodities to their portfolios.

Fourth, high-commodity-dependent countries tend to have less diversified economies, which could make them more susceptible to commodity price shocks. In contrast, investors in low-commodity-dependence countries with more diversified economies might benefit from including commodities in their portfolios since these investments could hedge against inflation or other macroeconomic risks unrelated to their domestic economy.

Finally, the financial markets in high-commodity-dependent countries might be more susceptible to volatility spillover from commodity markets, making the potential diversification benefits of adding commodities to domestic investors' portfolios less pronounced. We leave these issues for future research to examine.

[13] Although the standard analyses of bond behavior are based on either short-term returns or a hold-to-maturity model, the vast majority of bond portfolios follow a constant-maturity (or constant-duration) approach in which the portfolio manager maintains an approximately constant duration by selling bonds as they approach maturity and replacing them with longer-dated issues [Leibowitz et al., 2014].

[14] Bond returns have two main components: the coupon yield and the price change. Under the assumption of a long-term maturity (e.g., ten years), the price effect will dominate the coupon effect between two rolling periods.

[15] Taking Russia as an example, energy companies Gazprom, Lukoil, Novatek, Rosneft, Surgutneftegas and Tatneft represent circa 40% of the MOEX index as of December 2022. Source: https://www.moex.com/

# VII. Conclusion

This study investigates the potential diversification benefits of commodities for domestic investors across the globe. The research advances a theoretical model comprising two sectors: a commodity sector and a non-commodity sector. It derives implications for the association between the weight of the commodity sector in the local economy and the diversifying advantages of commodities in a portfolio of domestic assets. The principal contribution of this study is to provide empirical evidence for the proposition that a nation's degree of commodity dependence affects the diversification benefits of commodity futures in a portfolio of traditional assets. Specifically, we establish that investors in high-commodity dependence countries generally do not accrue benefits from adding commodities to their portfolios. In contrast, those located in low-commodity dependence countries typically do. Our findings are robust to an alternative weighting scheme for the commodity index, and our conclusions are corroborated via the utilization of the $R^2$ methodology.

From a broader asset allocation standpoint, our results indicate that commodities may augment a diversified portfolio if investors are not excessively exposed to commodity risk through their country's economic structure. These findings highlight that portfolio management research may profit from emphasising the economic environment. The diversity of local contexts concerning both economic and financial development and conditions can yield different insights on asset allocation issues. Nevertheless, future research should contemplate including more assets. However, we must acknowledge a caveat: most countries heavily relying on commodities are emerging markets, and the requisite granular data for those countries is often unavailable.

# Appendix A. Calculation of Bond Returns Based on Aswath Damodaran's Method

We model an implicit constant $n$-year maturity bond denoted by $\mathbb{B}_n$ assuming $i_t$ is the long-term interest rate at time $t$. $r_t$ is the bond return between $t-1$ and $t$ and results from the rolling-over of $\mathbb{B}_n$. For a yearly roll-over, we have:

$$r_t = \frac{i_{t-1} + P_t - P_{t-1}}{P_{t-1}}$$

.

By definition, we have $P_{t-1} = 1$ since $\mathbb{B}_n$ pays a constant coupon equal to $i_{t-1}$:

$$P_{t-1} = \sum_{k=1}^{n} \frac{i_{t-1}}{(1+i_{t-1})^k} + \frac{1}{(1+i_{t-1})^n} = 1$$

.

It follows that $r_t$ can be rewritten as:

$$r_t = i_{t-1} + P_t - 1$$

. The price at $t$ of the bond $\mathbb{B}_n$ notationally issued at $t-1$ equals:

$$P_t = \sum_{k=1}^{n} \frac{i_{t-1}}{(1+i_t)^k} + \frac{1}{(1+i_t)^n}$$

$P_t$ can be rewritten as follows:

$$P_t = i_{t-1} \sum_{k=1}^{n} \frac{1}{(1+i_t)^k} + \frac{1}{(1+i_t)^n} = i_{t-1} \frac{1-(1+i_t)^{-n}}{i_t} + \left(\frac{1}{1+i_t}\right)^n$$

By substituting the right-hand part of the above equation for $P_t$, $r_t = i_{t-1} + P_t - 1$ becomes:

$$r_t = i_{t-1} + i_{t-1} \frac{1-(1+i_t)^{-n}}{i_t} + \left(\frac{1}{1+i_t}\right)^n - 1$$

This equation gives the yearly return for an investor holding a constant-maturity portfolio. The bond returns thus obtained are chained over time: the bond $\mathbb{B}_n$ notionally issued at $t-1$ is sold at $t$ while simultaneously, a new bond $\mathbb{B}_n$ issued at $t$ is bought. The process is then repeated at each following period.

For a higher frequency, denoted by $f = \frac{1}{m}$, the bond return is defined as follows:

$$r_t = \frac{i_{t-1}}{m} + i_{t-1} \frac{1-(1+i_t)^{-n}}{i_t} + \left( \frac{1}{1+i_t} \right)^n - 1$$

where $m$ is the number of sub-periods within a year (e.g. $m = 12$ for monthly data).

# Appendix B. Descriptive Statistics of Used Times Series

### Table B1. Descriptive statistics of domestic equity returns

This table reports descriptive statistics covering the country-specific equity indexes within the sample. The availability of data determines the starting date for each series. All series end in December 2020.

| Country | Starting date | Observations | Mean | Standard deviation | Kurtosis | Skewness |
|---|---|---|---|---|---|---|
| **Group 1. Low commodity dependence countries** | | | | | | |
| Austria | 2000-01 | 252 | 0.005 | 0.069 | 4.868 | -0.806 |
| Belgium | 2000-01 | 252 | 0.003 | 0.057 | 4.734 | -1.177 |
| China | 2005-02 | 191 | 0.011 | 0.069 | 1.414 | -0.51 |
| Denmark | 2000-01 | 252 | 0.010 | 0.051 | 1.589 | -0.415 |
| Finland | 2000-01 | 252 | 0.003 | 0.077 | 3.191 | -0.096 |
| France | 2000-01 | 252 | 0.004 | 0.051 | 1.538 | -0.335 |
| Germany | 2000-01 | 252 | 0.005 | 0.06 | 2.055 | -0.516 |
| Hong Kong | 2000-01 | 252 | 0.007 | 0.059 | 1.078 | -0.309 |
| Hungary | 2003-09 | 208 | 0.014 | 0.066 | 2.853 | -0.246 |
| Ireland | 2000-01 | 252 | 0.001 | 0.061 | 1.104 | -0.662 |
| Italy | 2000-01 | 252 | 0.002 | 0.059 | 1.673 | -0.134 |
| Japan | 2000-01 | 252 | 0.003 | 0.051 | 0.789 | -0.31 |
| Malaysia | 2001-12 | 229 | 0.007 | 0.038 | 1.919 | -0.117 |
| Mexico | 2001-08 | 233 | 0.010 | 0.048 | 1.436 | -0.47 |
| Poland | 2000-11 | 242 | 0.006 | 0.068 | 1.096 | 0.084 |
| Portugal | 2000-01 | 252 | 0.001 | 0.052 | 1.184 | -0.49 |
| Singapore | 2000-01 | 252 | 0.004 | 0.056 | 3.772 | -0.642 |
| Slovenia | 2008-09 | 148 | 0.002 | 0.055 | 2.507 | -0.178 |
| South Korea | 2000-11 | 242 | 0.008 | 0.062 | 2.448 | 0.485 |
| Spain | 2000-01 | 252 | 0.004 | 0.06 | 2.231 | 0.012 |
| Sweden | 2000-01 | 252 | 0.006 | 0.06 | 1.841 | -0.155 |
| Switzerland | 2000-01 | 252 | 0.004 | 0.039 | 0.487 | -0.451 |
| United Kingdom | 2000-01 | 252 | 0.004 | 0.04 | 0.997 | -0.49 |
| United States | 2000-01 | 252 | 0.006 | 0.044 | 1.13 | -0.491 |
| **Group 2. High commodity dependence countries** | | | | | | |
| Australia | 2000-01 | 252 | 0.012 | 0.094 | 1.55 | -0.287 |
| Brazil | 2008-01 | 156 | 0.007 | 0.067 | 2.091 | -0.625 |
| Canada | 2000-01 | 252 | 0.006 | 0.042 | 2.838 | -0.866 |
| Chile | 2007-05 | 164 | 0.003 | 0.049 | 0.457 | 0.214 |
| Colombia | 2008-08 | 149 | 0.008 | 0.058 | 7.193 | -1.281 |
| India | 2002-04 | 225 | 0.014 | 0.066 | 2.499 | -0.207 |
| Israel | 2003-01 | 216 | 0.015 | 0.067 | 2.608 | -0.227 |
| Kenya | 2008-09 | 148 | 0.012 | 0.066 | 2.142 | -0.377 |
| Netherlands | 2000-01 | 252 | 0.006 | 0.051 | 1.588 | -0.757 |
| New Zealand | 2000-01 | 252 | 0.006 | 0.057 | 25.991 | -2.821 |
| Norway | 2000-01 | 252 | 0.008 | 0.058 | 2.778 | -0.832 |
| Russia | 2009-01 | 144 | 0.006 | 0.05 | 3.015 | 0.232 |
| South Africa | 2000-01 | 252 | 0.011 | 0.051 | 0.513 | 0.019 |
| Vietnam | 2008-04 | 153 | 0.008 | 0.079 | 1.49 | -0.121 |

**Table B2. Descriptive statistics of domestic bond returns**

This table reports descriptive statistics covering the country-specific bond indexes within the sample. The availability of data determines the starting date for each series. All series end in December 2020.

| Country | Starting date | Observations | Mean | Standard deviation | Kurtosis | Skewness |
|---|---|---|---|---|---|---|
| **Group 1. Low commodity dependence countries** | | | | | | |
| Austria | 2000-01 | 252 | 0.004 | 0.017 | 0.554 | 0.004 |
| Belgium | 2000-01 | 252 | 0.005 | 0.018 | 1.613 | -0.246 |
| China | 2005-02 | 191 | 0.004 | 0.014 | 2.215 | 0.088 |
| Denmark | 2000-01 | 252 | 0.004 | 0.018 | 0.322 | 0.097 |
| Finland | 2000-01 | 252 | 0.004 | 0.017 | 0.151 | 0.070 |
| France | 2000-01 | 252 | 0.004 | 0.017 | 0.337 | -0.014 |
| Germany | 2000-01 | 252 | 0.004 | 0.016 | -0.016 | 0.060 |
| Hong Kong | 2000-01 | 252 | 0.005 | 0.023 | 0.618 | 0.058 |
| Hungary | 2003-09 | 208 | 0.006 | 0.021 | 10.803 | 1.380 |
| Ireland | 2000-01 | 252 | 0.005 | 0.027 | 9.715 | -0.033 |
| Italy | 2000-01 | 252 | 0.005 | 0.022 | 2.444 | -0.274 |
| Japan | 2000-01 | 252 | 0.001 | 0.009 | 3.906 | -0.924 |
| Malaysia | 2001-12 | 229 | 0.004 | 0.017 | 4.442 | -1.007 |
| Mexico | 2001-08 | 233 | 0.008 | 0.025 | 0.653 | -0.125 |
| Poland | 2000-11 | 242 | 0.008 | 0.024 | 1.341 | 0.264 |
| Portugal | 2000-01 | 252 | 0.006 | 0.032 | 5.093 | 0.376 |
| Singapore | 2000-01 | 252 | 0.004 | 0.019 | 3.04 | -0.813 |
| Slovenia | 2008-09 | 148 | 0.006 | 0.027 | 1.095 | -0.250 |
| South Korea | 2000-11 | 242 | 0.006 | 0.020 | 4.604 | 0.284 |
| Spain | 2000-01 | 252 | 0.005 | 0.021 | 4.658 | 0.013 |
| Sweden | 2000-01 | 252 | 0.004 | 0.018 | 0.366 | -0.033 |
| Switzerland | 2000-01 | 252 | 0.003 | 0.015 | 0.74 | -0.064 |
| United Kingdom | 2000-01 | 252 | 0.005 | 0.018 | 1.094 | 0.131 |
| United States | 2000-01 | 252 | 0.005 | 0.021 | 1.372 | 0.118 |
| **Group 2. High commodity dependence countries** | | | | | | |
| Australia | 2000-01 | 252 | 0.006 | 0.018 | 0.289 | 0.183 |
| Brazil | 2008-01 | 156 | 0.012 | 0.040 | 9.139 | 1.069 |
| Canada | 2000-01 | 252 | 0.005 | 0.017 | 0.050 | 0.177 |
| Chile | 2007-05 | 164 | 0.006 | 0.022 | 3.277 | 0.463 |
| Colombia | 2008-08 | 149 | 0.010 | 0.028 | 1.966 | 0.123 |
| India | 2002-04 | 225 | 0.007 | 0.020 | 10.190 | 1.270 |
| Israel | 2003-01 | 216 | 0.006 | 0.020 | 10.600 | 1.336 |
| Kenya | 2008-09 | 148 | 0.011 | 0.058 | 16.301 | 1.130 |
| Netherlands | 2000-01 | 252 | 0.004 | 0.017 | 0.631 | 0.227 |
| New Zealand | 2000-01 | 252 | 0.006 | 0.017 | 1.408 | -0.102 |
| Norway | 2000-01 | 252 | 0.005 | 0.016 | 0.254 | 0.042 |
| Russia | 2009-01 | 144 | 0.006 | 0.028 | 1.051 | -0.263 |
| South Africa | 2000-01 | 252 | 0.009 | 0.026 | 2.635 | -0.185 |
| Vietnam | 2008-04 | 153 | 0.011 | 0.044 | 42.748 | -1.780 |

## Table B3. Descriptive statistics of DJCTRI returns expressed in local currencies

This table reports descriptive statistics covering the country-specific commodity indexes (i.e., expressed in local currencies) within the sample. The availability of data determines the starting date for each series. All series end in December 2020.

| Country | Starting date | Observations | Mean | Standard deviation | Kurtosis | Skewness |
|---|---|---|---|---|---|---|
| **Group 1. Low commodity dependence countries** | | | | | | |
| Austria | 2000-01 | 252 | 0.003 | 0.042 | 1.523 | -0.561 |
| Belgium | 2000-01 | 252 | 0.003 | 0.042 | 1.523 | -0.561 |
| China | 2005-02 | 191 | 0.000 | 0.049 | 2.923 | -0.759 |
| Denmark | 2000-01 | 252 | 0.003 | 0.042 | 1.545 | -0.567 |
| Finland | 2000-01 | 252 | 0.003 | 0.042 | 1.523 | -0.561 |
| France | 2000-01 | 252 | 0.003 | 0.042 | 1.523 | -0.561 |
| Germany | 2000-01 | 252 | 0.003 | 0.042 | 1.523 | -0.561 |
| Hong Kong | 2000-01 | 252 | 0.004 | 0.047 | 2.977 | -0.758 |
| Hungary | 2003-09 | 208 | 0.005 | 0.046 | 1.392 | -0.374 |
| Ireland | 2000-01 | 252 | 0.003 | 0.042 | 1.523 | -0.561 |
| Italy | 2000-01 | 252 | 0.003 | 0.042 | 1.523 | -0.561 |
| Japan | 2000-01 | 252 | 0.004 | 0.053 | 3.479 | -1.019 |
| Malaysia | 2001-12 | 229 | 0.004 | 0.044 | 2.353 | -0.614 |
| Mexico | 2001-08 | 233 | 0.007 | 0.044 | 0.002 | 0.045 |
| Poland | 2000-11 | 242 | 0.002 | 0.041 | 0.659 | -0.108 |
| Portugal | 2000-01 | 252 | 0.003 | 0.042 | 1.523 | -0.561 |
| Singapore | 2000-01 | 252 | 0.003 | 0.041 | 2.76 | -0.671 |
| Slovenia | 2008-09 | 148 | -0.002 | 0.042 | 2.334 | -0.825 |
| South Korea | 2000-11 | 242 | 0.003 | 0.045 | 2.214 | -0.537 |
| Spain | 2000-01 | 252 | 0.003 | 0.042 | 1.523 | -0.561 |
| Sweden | 2000-01 | 252 | 0.004 | 0.04 | 0.681 | -0.334 |
| Switzerland | 2000-01 | 252 | 0.002 | 0.045 | 2.404 | -0.771 |
| United Kingdom | 2000-01 | 252 | 0.004 | 0.043 | 1.292 | -0.152 |
| United States | 2000-01 | 252 | 0.004 | 0.047 | 2.816 | -0.719 |
| **Group 2. High commodity dependence countries** | | | | | | |
| Australia | 2000-01 | 252 | 0.006 | 0.075 | 2.461 | -0.477 |
| Brazil | 2008-01 | 156 | 0.004 | 0.042 | 0.408 | -0.085 |
| Canada | 2000-01 | 252 | 0.003 | 0.038 | 0.827 | -0.346 |
| Chile | 2007-05 | 164 | 0.000 | 0.044 | 1.552 | 0.193 |
| Colombia | 2008-08 | 149 | 0.001 | 0.042 | 0.939 | -0.534 |
| India | 2002-04 | 225 | 0.005 | 0.045 | 1.444 | -0.395 |
| Israel | 2003-01 | 216 | 0.005 | 0.046 | 1.307 | -0.381 |
| Kenya | 2008-09 | 148 | 0.000 | 0.047 | 0.907 | -0.320 |
| Netherlands | 2000-01 | 252 | 0.003 | 0.042 | 1.523 | -0.561 |
| New Zealand | 2000-01 | 252 | 0.001 | 0.058 | 23.783 | -2.674 |
| Norway | 2000-01 | 252 | 0.004 | 0.037 | 0.092 | -0.050 |
| Russia | 2009-01 | 144 | 0.001 | 0.038 | 2.553 | -0.581 |
| South Africa | 2000-01 | 252 | 0.008 | 0.053 | 0.667 | 0.380 |
| Vietnam | 2008-04 | 153 | -0.001 | 0.052 | 2.160 | -0.612 |

**Figure 1.**

The figure plots for each country the relationship between the degree of commodity dependence (featured on the horizontal axis) and the weight assigned to commodities (featured on the vertical axis) in the optimal portfolio. High-dependence countries are featured in red. Low-dependence countries are featured in blue.

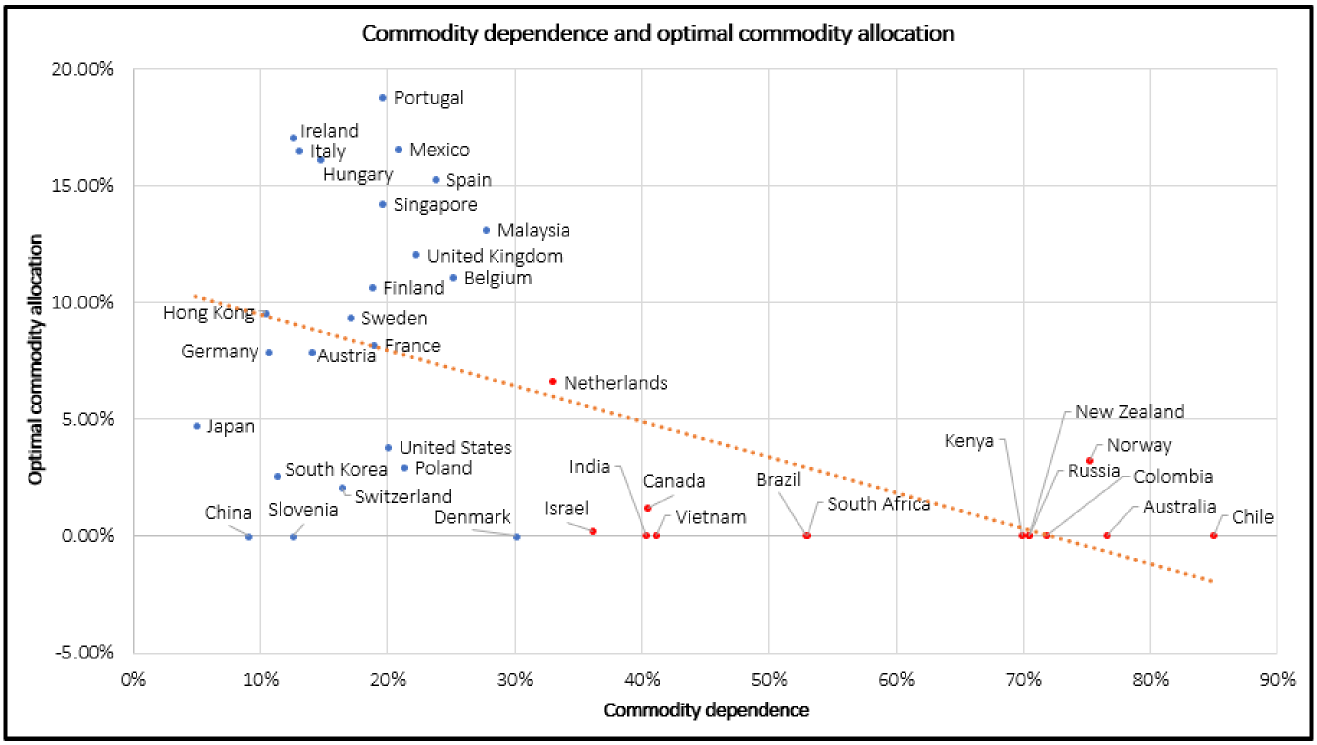
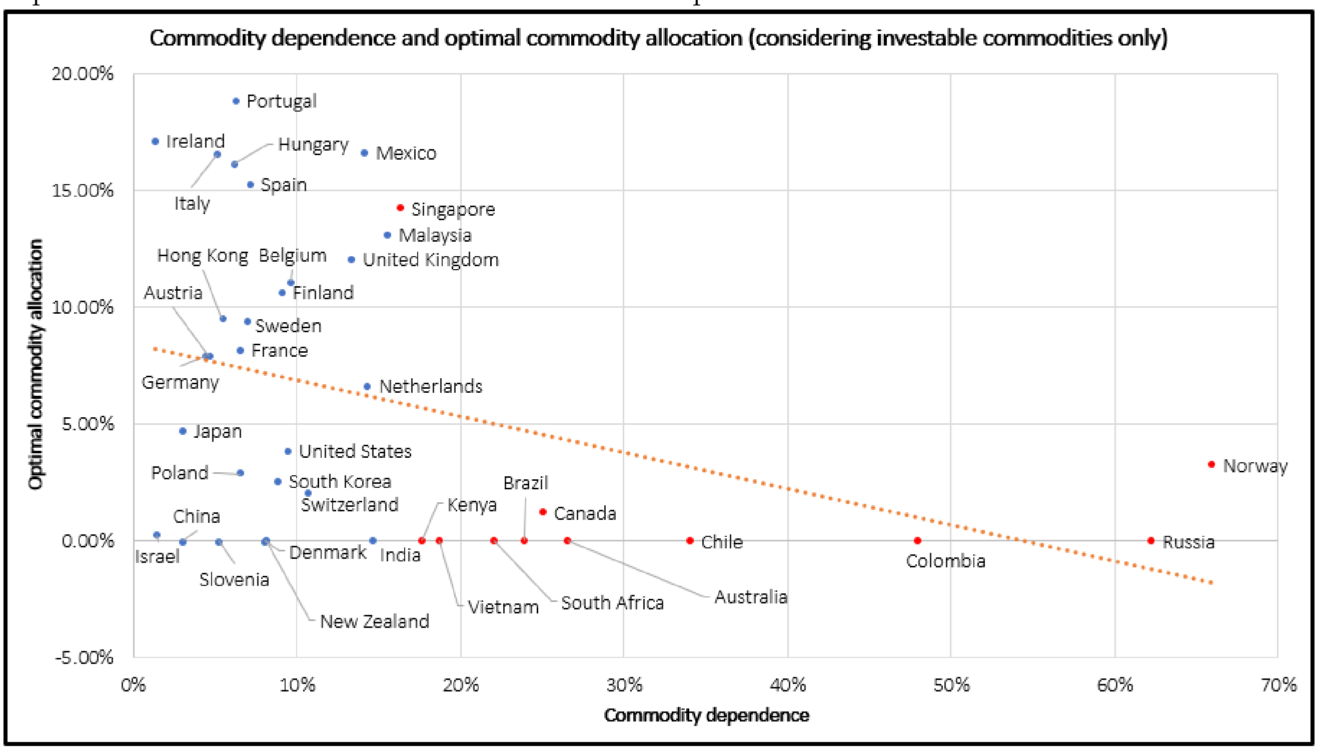


**Figure 2.**

The figure plots for each country the relationship between the degree of commodity dependence (featured on the horizontal axis) considering only investable commodities and the weight assigned to commodities (featured on the vertical axis) in the optimal portfolio. High-dependence countries are featured in red. Low-dependence countries are featured in blue.



## Table 1. Commodity dependence

This table reports the degree of commodity dependence for all countries in our sample. It also features the two leading commodity export groups for each country. The last column reports the degree of commodity dependence accounted for by investable commodities only (i.e., commodities for which futures are available).

| Country | Commodity dependence | Two-leading groups (SITC) | % investable |
|---|---|---|---|
| Australia | 76.6 | Crude materials, inedible, ex fuels / Mineral fuels, lub. and related mat. | 26.5 |
| Austria | 14.0 | Food and live animals / Crude materials, inedible, ex fuels | 4.6 |
| Belgium | 25.1 | Food and live animals / Mineral fuels, lub. and related mat. | 9.6 |
| Brazil | 52.9 | Food and live animals / Crude materials, inedible, ex fuels | 23.9 |
| Canada | 40.4 | Mineral fuels, lub. and related mat. / Crude materials, inedible, ex fuels | 25.0 |
| Chile | 85.0 | Non-ferrous metals / Crude materials, inedible, ex fuels | 34.0 |
| China | 9.1 | Food and live animals / Mineral fuels, lub. and related mat. | 3.0 |
| Colombia | 71.9 | Mineral fuels, lub. and related mat. / Food and live animals | 47.9 |
| Denmark | 30.0 | Food and live animals / Mineral fuels, lub. and related mat. | 8.0 |
| Finland | 18.7 | Crude materials, inedible, ex fuels / Mineral fuels, lub. and related mat. | 9.0 |
| France | 18.8 | Food and live animals / Mineral fuels, lub. and related mat. | 6.5 |
| Germany | 10.5 | Food and live animals / Non-ferrous metals | 4.4 |
| Hong Kong | 10.3 | Gold, non-monetary ex ores, and conc. / Pearls, prec. and semi-prec. stones | 5.4 |
| Hungary | 14.7 | Food and live animals / Mineral fuels, lub. and related mat. | 6.1 |
| India | 40.3 | Pearls, prec. and semi-prec. stones / Food and live animals | 14.6 |
| Ireland | 12.5 | Food and live animals / Crude materials, inedible, ex fuels | 1.3 |
| Israel | 36.2 | Pearls, prec. and semi-prec. stones / Food and live animals | 1.4 |
| Italy | 13.0 | Food and live animals / Mineral fuels, lub. and related mat. | 5.1 |
| Japan | 4.9 | Non-ferrous metals / Crude materials, inedible, ex fuels | 3.0 |
| Kenya | 69.9 | Food and live animals / Crude materials, inedible, ex fuels | 17.6 |
| Malaysia | 27.7 | Mineral fuels, lub. and related mat. / Animal and vegetable oils, fats and waxes | 15.5 |
| Mexico | 20.8 | Mineral fuels, lub. and related mat. / Food and live animals | 14.1 |
| Netherlands | 33.0 | Food and live animals / Mineral fuels, lub. and related mat. | 14.3 |
| New Zealand | 70.5 | Food and live animals / Crude materials, inedible, ex fuels | 8.1 |
| Norway | 75.2 | Mineral fuels, lub. and related mat. / Food and live animals | 65.9 |
| Poland | 21.2 | Food and live animals / Mineral fuels, lub. and related mat. | 6.5 |
| Portugal | 19.6 | Food and live animals / Mineral fuels, lub. and related mat. | 6.2 |
| Russia | 70.4 | Mineral fuels, lub. and related mat. / Non-ferrous metals | 62.2 |
| Singapore | 19.6 | Mineral fuels, lub. and related mat. / Food and live animals | 16.3 |
| Slovenia | 12.5 | Non-ferrous metals / Food and live animals | 5.2 |
| South Africa | 53.0 | Non-ferrous metals / Crude materials, inedible, ex fuels | 22.0 |
| South Korea | 11.3 | Mineral fuels, lub. and related mat. / Non-ferrous metals | 8.8 |
| Spain | 23.7 | Food and live animals / Mineral fuels, lub. and related mat. | 7.1 |
| Sweden | 17.0 | Crude materials, inedible, ex fuels / Mineral fuels, lub. and related mat. | 6.9 |
| Switzerland | 16.4 | Gold, non-monetary ex ores and conc. / Non-ferrous metals | 10.6 |
| Thailand | 26.3 | Food and live animals / Crude materials, inedible, ex fuels | 7.2 |
| United Kingdom | 22.2 | Mineral fuels, lub. and related mat. / Food and live animals | 13.3 |
| United States | 20.0 | Food and live animals / Crude materials, inedible, ex fuels | 9.4 |
| Vietnam | 41.1 | Food and live animals / Mineral fuels, lub. and related mat. | 18.7 |
| Mean | 32.2 | | |

**Table 2. Matching between SITC classification and investable commodities**

This table links the Standard International Trade Classification (SITC) items with investable commodities (i.e., commodities for which futures are available) and usually accepted commodity market sectors.

| SITC Code | Investable commodities | Sector |
|---|---|---|
| **0 - Food and live animals** | | |
| 001 - Live animals other than animals of division 03 | Feeder cattle, Lean hogs, Live cattle | Livestock |
| 041 - Wheat (including spelt) and meslin, unmilled<br>046 - Meal and flour of wheat and meslin | Wheat (Chicago), Wheat (Kansas) | Agriculture |
| 044 - Maize (not including sweet corn) unmilled | Corn | Agriculture |
| 06 - Sugar, sugar preparation and honey | Sugar | Agriculture |
| 071 - Coffee and coffee substitutes | Coffee | Agriculture |
| 072 - Cocoa | Cocoa | Agriculture |
| **1 - Beverages and tobacco** | | |
| **2 - Crude materials, inedible, except fuels** | | |
| 22 - Oil seeds and oleaginous fruits | Soybeans | Agriculture |
| 263 - Cotton | Cotton | Agriculture |
| **3 - Mineral fuels, lubricants and related materials** | | |
| 33 - Petroleum, petroleum products and related materials | Crude oil (Brent), Crude oil (WTI), Heating oil, Unleaded gasoline | Energy |
| 34 - Gas, natural and manufactured | Natural gas | Energy |
| **4 - Animal and vegetable oils, fats and waxes** | | |
| **68 - Non-ferrous metals** | | |
| 681 - Silver, platinum, other metals of the platinum group | Silver | Industrial metals |
| 682 - Copper | Copper | Industrial metals |
| 683 - Nickel | Nickel | Industrial metals |
| 684 - Aluminium | Aluminium | Industrial metals |
| 685 - Lead | Lead | Industrial metals |
| 686 - Zinc | Zinc | Industrial metals |
| **667 - Pearls, precious and semi-precious stones** | | |
| **97 - Gold, non-monetary (excluding ores and concentrates)** | | |
| 971 - Gold, non-monetary (excluding ores and concentrates) | Gold | Precious metals |

## Table 3. Diversifying benefits of commodities

The table presents test statistics (Z-stat) for the null hypothesis that commodities do not improve the Sharpe ratio of the optimal portfolio. Commodity exposure is proxied by the DJCTRI. All series end in December 2020. *, **, and *** indicate rejection of the null hypothesis at the 10%, 5%, and 1% level, respectively. The columns report the portfolio's Sharpe ratio (SR), the portfolio's volatily (Volatility), the portfolio's return over the risk-free rate (ER) and the standard deviation of the Sharpe ratio (SD of SR). The last column reports the weight assigned to commodities in the optimal portfolio.

| | | **Without commodities** | | | | **With commodities** | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Country | Obs | SR | Volatility | ER | SD of SR | SR | Volatility | ER | SD of SR | Z-stat | Commodity weight (%) |
| **Group 1. Low-commodity dependence countries** | | | | | | | | | | | |
| Austria | 252 | 0.214 | 0.016 | 0.003 | 0.004 | 0.219 | 0.015 | 0.003 | 0.004 | 4.885*** | 7.91 |
| Belgium | 252 | 0.199 | 0.016 | 0.003 | 0.004 | 0.209 | 0.015 | 0.003 | 0.004 | 10.166*** | 11.1 |
| China | 191 | 0.188 | 0.016 | 0.003 | 0.005 | 0.188 | 0.016 | 0.003 | 0.005 | 0.000 | 0.00 |
| Denmark | 252 | 0.167 | 0.05 | 0.008 | 0.004 | 0.167 | 0.05 | 0.008 | 0.004 | 0.000 | 0.00 |
| Finland | 252 | 0.183 | 0.016 | 0.003 | 0.004 | 0.191 | 0.015 | 0.003 | 0.004 | 9.042*** | 10.66 |
| France | 252 | 0.213 | 0.015 | 0.003 | 0.004 | 0.219 | 0.014 | 0.003 | 0.004 | 6.793*** | 8.17 |
| Germany | 252 | 0.234 | 0.014 | 0.003 | 0.004 | 0.241 | 0.013 | 0.003 | 0.004 | 8.779*** | 7.92 |
| Hong Kong | 228 | 0.214 | 0.022 | 0.005 | 0.004 | 0.219 | 0.02 | 0.004 | 0.004 | 3.207*** | 9.53 |
| Hungary | 205 | 0.122 | 0.035 | 0.004 | 0.005 | 0.125 | 0.028 | 0.003 | 0.005 | 1.759** | 16.13 |
| Ireland | 252 | 0.136 | 0.027 | 0.004 | 0.004 | 0.143 | 0.023 | 0.003 | 0.004 | 4.997*** | 17.1 |
| Italy | 252 | 0.173 | 0.022 | 0.004 | 0.004 | 0.185 | 0.019 | 0.004 | 0.004 | 10.148*** | 16.55 |
| Japan | 252 | 0.186 | 0.008 | 0.002 | 0.004 | 0.192 | 0.008 | 0.002 | 0.004 | 11.235*** | 4.74 |
| Malaysia | 229 | 0.101 | 0.018 | 0.002 | 0.004 | 0.108 | 0.016 | 0.002 | 0.004 | 6.320*** | 13.12 |
| Mexico | 222 | 0.14 | 0.027 | 0.004 | 0.005 | 0.147 | 0.022 | 0.003 | 0.005 | 4.841*** | 16.61 |
| Poland | 242 | 0.176 | 0.024 | 0.004 | 0.004 | 0.176 | 0.023 | 0.004 | 0.004 | 0.153 | 2.93 |
| Portugal | 252 | 0.125 | 0.032 | 0.004 | 0.004 | 0.13 | 0.026 | 0.003 | 0.004 | 3.486*** | 18.84 |
| Singapore | 252 | 0.158 | 0.017 | 0.003 | 0.004 | 0.169 | 0.015 | 0.003 | 0.004 | 11.746*** | 14.25 |
| Slovenia | 148 | 0.163 | 0.027 | 0.004 | 0.007 | 0.163 | 0.027 | 0.004 | 0.007 | 0.000 | 0.00 |
| South Korea | 242 | 0.185 | 0.018 | 0.003 | 0.004 | 0.185 | 0.017 | 0.003 | 0.004 | 0.319 | 2.56 |
| Spain | 252 | 0.166 | 0.021 | 0.003 | 0.004 | 0.178 | 0.018 | 0.003 | 0.004 | 9.960*** | 15.27 |
| Sweden | 252 | 0.207 | 0.016 | 0.003 | 0.004 | 0.214 | 0.015 | 0.003 | 0.004 | 7.100*** | 9.38 |
| Switzerland | 252 | 0.217 | 0.013 | 0.003 | 0.004 | 0.218 | 0.012 | 0.003 | 0.004 | 0.656 | 2.07 |
| United Kingdom | 252 | 0.183 | 0.016 | 0.003 | 0.004 | 0.193 | 0.015 | 0.003 | 0.004 | 9.448*** | 12.06 |
| United States | 252 | 0.226 | 0.016 | 0.004 | 0.004 | 0.228 | 0.016 | 0.004 | 0.004 | 1.209 | 3.84 |
| Mean | | | | | | | | | | | 9.20 |
| **Group 2. High-commodity dependence countries** | | | | | | | | | | | |
| Australia | 252 | 0.203 | 0.017 | 0.003 | 0.004 | 0.203 | 0.017 | 0.003 | 0.004 | 0.000 | 0.00 |
| Brazil | 156 | 0.108 | 0.04 | 0.004 | 0.006 | 0.108 | 0.04 | 0.004 | 0.006 | 0.000 | 0.00 |
| Canada | 252 | 0.205 | 0.014 | 0.003 | 0.004 | 0.205 | 0.014 | 0.003 | 0.004 | 0.097 | 1.21 |
| Chile | 164 | 0.115 | 0.021 | 0.002 | 0.006 | 0.115 | 0.021 | 0.002 | 0.006 | 0.000 | 0.00 |
| Colombia | 149 | 0.189 | 0.028 | 0.005 | 0.007 | 0.189 | 0.028 | 0.005 | 0.007 | 0.000 | 0.00 |
| India | 225 | 0.146 | 0.028 | 0.004 | 0.004 | 0.146 | 0.028 | 0.004 | 0.004 | 0.000 | 0.00 |
| Israel | 216 | 0.246 | 0.021 | 0.005 | 0.005 | 0.246 | 0.021 | 0.005 | 0.005 | 0.002 | 0.24 |
| Kenya | 148 | 0.07 | 0.048 | 0.003 | 0.007 | 0.07 | 0.048 | 0.003 | 0.007 | 0.000 | 0.00 |
| Netherlands | 252 | 0.225 | 0.015 | 0.003 | 0.004 | 0.229 | 0.014 | 0.003 | 0.004 | 4.335*** | 6.61 |
| New Zealand | 252 | 0.167 | 0.016 | 0.003 | 0.004 | 0.167 | 0.016 | 0.003 | 0.004 | 0.000 | 0.00 |
| Norway | 252 | 0.217 | 0.014 | 0.003 | 0.004 | 0.218 | 0.013 | 0.003 | 0.004 | 0.851 | 3.24 |
| Russia | 144 | 0.145 | 0.052 | 0.008 | 0.007 | 0.145 | 0.052 | 0.008 | 0.007 | 0.000 | 0.00 |
| South Africa | 240 | 0.108 | 0.028 | 0.003 | 0.004 | 0.108 | 0.028 | 0.003 | 0.004 | 0.000 | 0.00 |
| Vietnam | 145 | 0.292 | 0.032 | 0.009 | 0.007 | 0.292 | 0.032 | 0.009 | 0.007 | 0.000 | 0.00 |
| Mean | | | | | | | | | | | 0.81 |

# Table 4. Diversifying benefits of commodities (with equally-weighted commodity index)

The table presents test statistics (Z-stat) for the null hypothesis that commodities do not improve the Sharpe ratio of the optimal portfolio. Commodity exposure is proxied by an equally-weighted index of commodity futures included in the DJCITR. All series end in December 2020. *, **, and *** indicate rejection of the null hypothesis at the 10%, 5%, and 1% level, respectively. The columns report the portfolio's Sharpe ratio (SR), the portfolio's volatily (Volatility), the portfolio's return above the risk-free rate (ER) and the standard deviation of the Sharpe ratio (SD of SR). The last column reports the weight assigned to commodities in the optimal portfolio.

| | | **Without commodities** | | | | **With commodities** | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Country | Obs | SR | Volatility | ER | SD of SR | SR | Volatility | ER | SD of SR | Z-stat | Commodity weight (%) |
| **Group 1. Low-commodity dependence countries** | | | | | | | | | | | |
| Austria | 252 | 0.214 | 0.016 | 0.003 | 0.004 | 0.216 | 0.015 | 0.003 | 0.004 | 1.291* | 4.71 |
| Belgium | 252 | 0.191 | 0.017 | 0.003 | 0.004 | 0.198 | 0.016 | 0.003 | 0.004 | 6.684*** | 11.45 |
| China | 191 | 0.188 | 0.016 | 0.003 | 0.005 | 0.188 | 0.016 | 0.003 | 0.005 | 0.000 | 0.00 |
| Denmark | 252 | 0.262 | 0.017 | 0.004 | 0.004 | 0.262 | 0.017 | 0.004 | 0.004 | 0.000 | 0.00 |
| Finland | 252 | 0.194 | 0.016 | 0.003 | 0.004 | 0.198 | 0.015 | 0.003 | 0.004 | 4.099*** | 8.2 |
| France | 252 | 0.213 | 0.015 | 0.003 | 0.004 | 0.215 | 0.014 | 0.003 | 0.004 | 2.213** | 5.41 |
| Germany | 252 | 0.234 | 0.014 | 0.003 | 0.004 | 0.236 | 0.013 | 0.003 | 0.004 | 3.447*** | 5.75 |
| Hong Kong | 228 | 0.214 | 0.022 | 0.005 | 0.004 | 0.216 | 0.015 | 0.003 | 0.004 | 1.426* | 6.69 |
| Hungary | 205 | 0.122 | 0.035 | 0.004 | 0.005 | 0.125 | 0.027 | 0.003 | 0.005 | 1.67** | 16.81 |
| Ireland | 252 | 0.136 | 0.027 | 0.004 | 0.004 | 0.138 | 0.023 | 0.003 | 0.004 | 1.304* | 10.74 |
| Italy | 252 | 0.147 | 0.022 | 0.003 | 0.004 | 0.149 | 0.02 | 0.003 | 0.004 | 2.283** | 10.6 |
| Japan | 252 | 0.186 | 0.008 | 0.002 | 0.004 | 0.189 | 0.008 | 0.002 | 0.004 | 5.793*** | 3.8 |
| Malaysia | 229 | 0.136 | 0.019 | 0.003 | 0.004 | 0.136 | 0.019 | 0.003 | 0.004 | 0.409 | 4.37 |
| Mexico | 222 | 0.14 | 0.027 | 0.004 | 0.005 | 0.146 | 0.022 | 0.003 | 0.005 | 4.228*** | 16.79 |
| Poland | 242 | 0.176 | 0.024 | 0.004 | 0.004 | 0.176 | 0.024 | 0.004 | 0.004 | 0.000 | 0.00 |
| Portugal | 252 | 0.125 | 0.032 | 0.004 | 0.004 | 0.126 | 0.028 | 0.004 | 0.004 | 0.897 | 12.08 |
| Singapore | 252 | 0.158 | 0.017 | 0.003 | 0.004 | 0.162 | 0.015 | 0.002 | 0.004 | 4.181*** | 10.5 |
| Slovenia | 148 | 0.163 | 0.027 | 0.004 | 0.007 | 0.163 | 0.027 | 0.004 | 0.007 | 0.000 | 0.00 |
| South Korea | 242 | 0.185 | 0.018 | 0.003 | 0.004 | 0.185 | 0.018 | 0.003 | 0.004 | 0.000 | 0.00 |
| Spain | 252 | 0.166 | 0.021 | 0.003 | 0.004 | 0.171 | 0.018 | 0.003 | 0.004 | 4.596*** | 12.47 |
| Sweden | 252 | 0.207 | 0.016 | 0.003 | 0.004 | 0.209 | 0.015 | 0.003 | 0.004 | 2.327*** | 6.34 |
| Switzerland | 252 | 0.217 | 0.013 | 0.003 | 0.004 | 0.217 | 0.013 | 0.003 | 0.004 | 0.000 | 0.00 |
| United Kingdom | 252 | 0.162 | 0.016 | 0.003 | 0.004 | 0.165 | 0.015 | 0.002 | 0.004 | 3.449*** | 8.89 |
| United States | 252 | 0.226 | 0.016 | 0.004 | 0.004 | 0.226 | 0.016 | 0.004 | 0.004 | 0.000 | 0.00 |
| Mean | | | | | | | | | | | 6.48 |
| **Group 2. High-commodity dependence countries** | | | | | | | | | | | |
| Australia | 252 | 0.203 | 0.017 | 0.003 | 0.004 | 0.203 | 0.017 | 0.003 | 0.004 | 0.000 | 0.00 |
| Brazil | 156 | 0.108 | 0.04 | 0.004 | 0.006 | 0.108 | 0.04 | 0.004 | 0.006 | 0.000 | 0.00 |
| Canada | 252 | 0.205 | 0.014 | 0.003 | 0.004 | 0.205 | 0.014 | 0.003 | 0.004 | 0.000 | 0.00 |
| Chile | 164 | 0.115 | 0.021 | 0.002 | 0.006 | 0.115 | 0.021 | 0.002 | 0.006 | 0.000 | 0.00 |
| Colombia | 149 | 0.189 | 0.028 | 0.005 | 0.007 | 0.189 | 0.028 | 0.005 | 0.007 | 0.000 | 0.00 |
| India | 225 | 0.146 | 0.028 | 0.004 | 0.004 | 0.146 | 0.028 | 0.004 | 0.004 | 0.000 | 0.00 |
| Israel | 216 | 0.246 | 0.021 | 0.005 | 0.005 | 0.246 | 0.021 | 0.005 | 0.005 | 0.000 | 0.00 |
| Kenya | 148 | 0.07 | 0.048 | 0.003 | 0.007 | 0.07 | 0.048 | 0.003 | 0.007 | 0.000 | 0.00 |
| Netherlands | 252 | 0.225 | 0.015 | 0.003 | 0.004 | 0.226 | 0.014 | 0.003 | 0.004 | 0.814 | 3.36 |
| New Zealand | 252 | 0.167 | 0.016 | 0.003 | 0.004 | 0.167 | 0.016 | 0.003 | 0.004 | 0.000 | 0.00 |
| Norway | 252 | 0.217 | 0.014 | 0.003 | 0.004 | 0.217 | 0.014 | 0.003 | 0.004 | 0.000 | 0.00 |
| Russia | 144 | 0.145 | 0.052 | 0.008 | 0.007 | 0.145 | 0.052 | 0.008 | 0.007 | 0.000 | 0.00 |
| South Africa | 240 | 0.108 | 0.028 | 0.003 | 0.004 | 0.108 | 0.028 | 0.003 | 0.004 | 0.000 | 0.00 |
| Vietnam | 145 | 0.292 | 0.032 | 0.009 | 0.007 | 0.292 | 0.032 | 0.009 | 0.007 | 0.000 | 0.00 |
| Mean | | | | | | | | | | | 0.24 |

**Table 5. Diversifying benefits of commodities - $R^2$ analysis**

The table presents 1 - $R^2$ where $R^2$ comes from regressing commodity returns on the returns of bonds, stocks, and PCs constructed from bond and stock returns. PCs included in the regressions account for 75% or more of the cumulative eigenvalue.

| **1 - $R^2$** | | | |
|---|---|---|---|
| **Overall commodity dependence** | | | |
| Commodity dependence | Bonds | Equities | Bonds & Equities |
| DJCTRI | | | |
| Low (Group 1) | 0.956 | 0.924 | 0.924 |
| High (Group 2) | 0.953 | 0.905 | 0.901 |
| Equally-weighted commodity index | | | |
| Low (Group 1) | 0.962 | 0.916 | 0.915 |
| High (Group 2) | 0.960 | 0.906 | 0.901 |
| **Commodity dependence considering investable commodities only** | | | |
| DJCTRI | | | |
| Low (Group 1) | 0.960 | 0.927 | 0.926 |
| High (Group 2) | 0.952 | 0.901 | 0.897 |
| Equally-weighted commodity index | | | |
| Low (Group 1) | 0.966 | 0.922 | 0.921 |
| High (Group 2) | 0.961 | 0.892 | 0.884 |

# References

F Gerard Adams, Jere R Behrman, and Romualdo A Roldan. Measuring the impact of primary commodity fluctuations on economic development: coffee and Brazil. *American Economic Review*, 69(2):164–168, 1979.

Suleyman Basak and Anna Pavlova. A model of financialization of commodities. *Journal of Finance*, 71(4):1511–1556, 2016.

Julia Belousova and Gregor Dorfleitner. On the diversification benefits of commodities from the perspective of euro investors. *Journal of Banking and Finance*, 36(9):2455–2472, 2012.

Hendrik Bessembinder and Kalok Chan. Time-varying risk premia and forecastable returns in futures markets. *Journal of Financial Economics*, 32(2):169–193, 1992.

Wolfgang Bessler and Dominik Wolff. Do commodities add value in multi-asset portfolios? an out-of-sample analysis for different investment strategies. *Journal of Banking and Finance*, 60:1–20, 2015.

Geetesh Bhardwaj, Gary Gorton, and Geert Rouwenhorst. Facts and fantasies about commodity futures ten years later. Technical report, National Bureau of Economic Research, 2015.

Zvi Bodie and Victor I Rosansky. Risk and return in commodity futures. *Financial Analysts Journal*, 36(3):27–39, 1980.

Yu-chin Chen and Kenneth Rogoff. Commodity currencies. *Journal of International Economics*, 60(1):133–160, 2003.

Yu-Chin Chen, Kenneth S Rogoff, and Barbara Rossi. Can exchange rates forecast commodity prices? *Quarterly Journal of Economics*, 125(3):1145–1194, 2010.

C Sherman Cheung and Peter Miu. Diversification benefits of commodity futures. *Journal of International Financial Markets, Institutions and Money*, 20(5):451–474, 2010.

Peter Christoffersen, Vihang Errunza, Kris Jacobs, and Hugues Langlois. Is the potential for international diversification disappearing? A dynamic copula approach. *The Review of Financial Studies*, 25(12):3711–3751, 2012.

Thomas Drechsel and Silvana Tenreyro. Commodity booms and busts in emerging economies. *Journal of International Economics*, 112:200–218, 2018.

Gerben Driesprong, Ben Jacobsen, and Benjamin Maat. Striking oil: another puzzle? *Journal of Financial Economics*, 89(2):307–327, 2008.

Xin Gao and Federico Nardari. Do commodities add economic value in asset allocation? new evidence from time-varying moments. *Journal of Financial and Quantitative Analysis*, 53 (1):365–393, 2018.

Gary Gorton and K Geert Rouwenhorst. Facts and fantasies about commodity futures. *Financial Analysts Journal*, 62(2):47–68, 2006.

Gur Huberman and Shmuel Kandel. Mean-variance spanning. *Journal of Finance*, 42(4): 873–888, 1987.

J Dave Jobson and Bob M Korkie. Performance hypothesis testing with the sharpe and treynor measures. *Journal of Finance*, 36(4):889–908, 1981.

Ronald W Jones and Jose A Scheinkman. The relevance of the two-sector production model in trade theory. *Journal of Political Economy*, 85(5):909–935, 1977.

WR Jones. A three factor model in theory, trade, and history. *Trade, balance of payments and growth*, pages 3–21, 1971.

Raymond Kan and Guofu Zhou. Tests of mean-variance spanning. 2008. URL `https://papers.ssrn.com/sol3/papers.cfm?abstract_id=231522&download=yes`.

Yoram Kroll, Haim Levy, and Harry M Markowitz. Mean-variance versus direct utility maximization. *Journal of Finance*, 39(1):47–61, 1984.

Francis Y Kumah and John M Matovu. Commodity price shocks and the odds on fiscal performance: A structural vector autoregression approach. *IMF Staff Papers*, 54(1):91–112, 2007.

Martin L Leibowitz, Anthony Bova, and Stanley Kogelman. Long-term bond returns under duration targeting. *Financial Analysts Journal*, 70(1):31–51, 2014.

Ari Levine, Yao Hua Ooi, Matthew Richardson, and Caroline Sasseville. Commodities for the long run. *Financial Analysts Journal*, 74(2):55–68, 2018.

Haim Levy and Harry M Markowitz. Approximating expected utility by a function of mean and variance. *American Economic Review*, 69(3):308–317, 1979.

Andrew W Lo. The statistics of sharpe ratios. *Financial Analysts Journal*, 58(4):36–52, 2002.

Harry Markowitz. Portfolio selection. *Journal of Finance*, 7(1):77–91, 1952.

Andrea Filippo Presbitero, Dhaneshwar Ghura, Olumuyiwa Adedeji, and Lamin Njie. International sovereign bonds by emerging markets and developing economies: Drivers of issuance and spreads. *IMF Working Paper*, 15/275, 2015.

Kuntara Pukthuanthong and Richard Roll. Global market integration: An alternative measure and its application. *Journal of Financial Economics*, 94(2):214–232, 2009.

Lawrence B Pulley. A general mean-variance approximation to expected utility for short holding periods. *Journal of Financial and Quantitative Analysis*, 16(3):361–373, 1981.

William F Sharpe. Mutual fund performance. *Journal of Business*, 39(1):119–138, 1966.

Ke Tang and Wei Xiong. Index investment and the financialization of commodities. *Financial Analysts Journal*, 68(6):54–74, 2012.