

# Cojumps and Asset Allocation in International Equity Markets<sup>☆</sup>

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## Abstract

This paper examines the patterns of intraday cojumps between international equity markets as well as their impact on international asset holdings and portfolio diversification benefits. Using intraday index-based data for exchange-traded funds as proxies for international equity markets, we document evidence of significant cojumps, with the intensity increasing during the global financial crisis of 2008-2009. The application of the Hawkes process also shows that jumps propagate from the US and other developed markets to emerging markets. Correlated jumps are found to reduce diversification benefits and foreign asset holdings in minimum risk portfolios, whereas idiosyncratic jumps increase the diversification benefits of international equity portfolios. In contrast, the impact of higher-order moments induced by idiosyncratic and systematic jumps on the optimal composition of international portfolios is not significant.

*Keywords:* Cojumps; Foreign asset holdings; International diversification.

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## 1. Introduction

It is now well established in the finance literature that price discontinuities or jumps should be taken into account when studying asset price dynamics and allocating funds across assets (Bekaert et al., 1998; Das and Uppal, 2004; Guidolin and Ossola, 2009; Branger et al., 2017). In this regard, the recent development of non-parametric jump identification tests has enabled jump detection in financial asset prices. The seminal works in this area include Barndorff-Nielsen and Shephard (2004; 2006a) who test for the presence of jumps at the daily level using measures of bipower variation. The same family of intraday jump identification procedures includes the tests developed by, among others, Jiang and Oomen (2008), Andersen et al. (2012), Corsi et al. (2010), Podolskij and Ziggel (2010), and Christensen et al. (2014). Andersen et al. (2007) and Lee and Mykland (2008) have developed techniques to identify intraday jumps using high frequency data. All of these jump detection techniques provide empirical evidence in favor of the presence of asset price discontinuities or jumps.

More recently, researchers have been interested in studying cojumps between assets (Dungey et al., 2009; Lahaye et al., 2010; Dungey and Hvozdyk, 2012; Pukthuanthong and Roll, 2015; Ait-Sahalia and Xiu, 2016). For instance, Gilder et al. (2012) examine the frequency of cojumps between individual stocks and the market portfolio. They find a tendency for a relatively large number of stocks to be involved in systematic cojumps, which are defined as cojumps between a stock and market portfolio. Lahaye et al. (2010) show that asset cojumps are partially associated with macroeconomic news announcements. Ait-Sahalia et al. (2015) develop a multivariate Hawkes jump-diffusion model to capture jumps propagation over time and across markets. They provide strong evidence for jumps to arrive in clusters within the same market and to propagate to other international markets. Borretti et al. (2015) find that Hawkes one-factor model is more suitable to capture the high synchronization of jumps across assets than the multivariate Hawkes model.

Our study furthers the above-mentioned literature in two ways. First, we empirically investigate intraday cojumps between international equity markets. Second, we show their impact on international asset allocation and portfolio diversification benefits. To the best of our knowledge, we are the first study that examines the impact of intraday cojumps on portfolio allocation decisions in an international setting. Past studies focus more on the impact of return correlation without separating between continuous and jump parts. Modern portfolio theory suggests that international diversification is an effective way to minimize portfolio risks given that international assets are often less correlated and driven by different economic

factors. However, one might expect that cojumps can lead to an increase in the correlation between these international assets and thus reduce the benefit from international diversification. Inversely, if price jumps of different assets do not occur simultaneously, they are categorized as idiosyncratic jumps and will not affect portfolio allocation decisions in an international setting. Choi et al. (2017) show, in contrast to traditional asset pricing theory and in support of information advantage theory, concentrated investment strategies in international markets are associated with higher risk-adjusted returns. Our study complements their study by showing investors prefer concentrated portfolios tilted toward home market because cojumps between home and foreign stock markets significantly reduce diversification benefits.

Accordingly, a risk-averse investor who holds an international portfolio is exposed to two types of jump risks: cojump or systematic jump risk (jumps common to all markets) and idiosyncratic jump risk (jumps specific to one market). If an investor's portfolio is well diversified, the idiosyncratic jump risk will be reduced or even eliminated. On the other hand, the cojump risk cannot be eliminated through diversification, thus making its identification central to asset pricing, asset allocation and portfolio risk hedging. Identifying cojumps is also important for policymakers attempting to propose the policies that stabilize financial markets.

Our empirical tests rely on the use of intraday returns for three dedicated international exchange-traded funds (ETFs) – SPY, EFA, and EEM – which respectively aim to replicate the performance of three international equity market indices: S&P 500, MSCI EAFE (Europe, Australasia and Far East), and MSCI Emerging Markets.<sup>1</sup> We use the technique proposed by Andersen et al. (2007) and Lee and Mykland (2008) to empirically identify all intraday jumps and cojumps of the three funds from January 2008 to October 2013. Lee and Mykland (2008) show that the power of their non-parametric jump identification test increases with the sampling frequency and that spurious detection of jumps is negligible when high frequency data are used. Unlike Ait-Sahalia et al. (2015) who use low frequency data to study the dynamics of jumps, we employ a bivariate Hawkes model to reproduce the time clustering features of intraday jumps and the dynamics of their propagation across markets. The application of the Hawkes process allows us to capture the dependence between the occurrences of jumps which cannot be reproduced by, for example, the standard Poisson process, owing to the hypothesis of independence of the increments (i.e., the numbers of jumps on disjoint time intervals should be independent). Under this analysis, we find jumps from the US

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<sup>1</sup>S&P 500 index is used as a proxy for the US market. MSCI EAFE index is the benchmark for developed markets excluding the US and Canada, whereas the MSCI Emerging Markets is used to capture the performance of emerging equity markets.

propagate to other developed markets and emerging markets. However, the evidence of jump spillover from emerging markets to developed markets is weak.

Finally, we assess the impact of cojumps on international portfolio allocation by considering a domestic risk-averse investor who selects the portfolio composition based on one domestic asset and two foreign assets in a way to maximize his expected utility.<sup>2</sup>As investors are concerned about negative movements of asset returns, we take the risk of extreme events into account using the Conditional Value-at-Risk or CVaR (Rockafellar and Uryasev, 2000) as a risk measure in our portfolio allocation problem. Unlike the standard mean-variance approach, which typically underestimates the risk of large movements of asset returns, the mean-CVaR approach allows us to provide a fairly accurate estimate of the downside risk induced by negative cojumps of asset returns. As to cojumps, we apply two approaches to assess how assets jumps are linked to each other. The first one is cojump intensity measure obtained from the co-exceedance rule (Bae et al., 2003) and univariate jump identification tests proposed by Andersen et al. (2007) and Lee and Mykland (2008). The second one is based on the realized jump correlation measure proposed by Jacod and Todorov (2009). Contrary to the first approach that only measures the frequency of simultaneous jumps, the second approach captures both the intensity and size effects of cojumps. It has also the advantage to be robust to jump identification tests.

Once the optimal portfolio composition is determined, we analyze how jumps and cojumps affect investor demand for domestic and foreign assets. Our results show evidence of a negative and significant link between the demand for foreign assets and the jump correlation between the domestic and foreign markets. We also find a negative effect of cross-market cojumps on diversification benefits. In contrast, we find that idiosyncratic jumps have a positive effect on foreign asset holding and diversification benefits.

We also examine how higher-order moments induced by idiosyncratic and systematic jumps affect the optimal portfolio composition. For this purpose, we consider an investor who recognizes idiosyncratic and systematic jump risks and assumes that asset returns are given by a multivariate jump-diffusion process as well as another investor who ignores jumps and assumes a pure-diffusion process for asset returns. Both investors have a CRRA utility function and select their respective portfolios composition in a way to maximize their respective expected utilities. Our results show that both investors have almost the same portfolio composition, which typically indicates that the impact of jump higher-order moments on optimal portfolio composition is not significant.

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<sup>2</sup>The domestic country is defined as a reference country considered to be the home country for our investors.

The remainder of the paper is organized as follows. Section 2 introduces the jump and cojump identification techniques used in our study. Section 3 presents the portfolio allocation problem. Section 4 describes the data. Section 5 discusses our main empirical findings. Section 6 concludes the paper.

## 2. Jump and cojump identification

This section briefly introduces the methodology that we follow to detect intraday jumps and cojumps. We first begin with the univariate jump identification tests proposed by Andersen et al. (2007, henceforth ABD) and Lee and Mykland (2008, henceforth LM).<sup>3</sup> The LM and ABD procedures use the same test statistic, but differ on the choice of the critical value. ABD assumes that the test statistic is asymptotically normal, whereas LM provides critical value from the limit distribution of the maximum of the test statistic.

The LM test statistic compares the current asset return with the bipower variation calculated over a moving window with a given number of preceding observations. It tests on day  $t$  at time  $k$  whether there was a jump from  $k - 1$  to  $k$  and is defined as:

$$L_{t,k} = \frac{|r_{t,k}|}{\hat{\sigma}_{t,k}} \quad (1)$$

where

$$(\hat{\sigma}_{t,k})^2 = \frac{1}{K-2} \sum_{j=k-K+2}^{k-1} |r_{t,j-1}| |r_{t,j}| \quad (2)$$

$r_{t,k}$  is the  $k^{th}$  intraday return.  $\hat{\sigma}_{t,k}$  refers to the realized bipower variation calculated for a window of  $K$  observations and provides a jump robust estimator of the instantaneous volatility. A jump is detected with LM test on day  $t$  in intraday interval  $k$  if the following condition is satisfied:

$$|L_{t,k}| > -\log(-\log(1-\alpha)) \times S_M + C_M \quad (3)$$

where  $\alpha$  is the test significance level.  $S_M$  and  $C_M$  are function of the number of observations in a day  $M$ , introduced in Lee and Mykland (2008).

On the other hand, the ABD test statistic is assumed to be normally distributed in the absence of jumps. A jump is detected with the ABD test on day  $t$  in intraday interval  $k$  if the following condition is satisfied:

$$\frac{|r_{t,k}|}{\sqrt{\frac{1}{M} BV_t}} > \Phi_{1-\frac{\beta}{2}}^{-1} \quad (4)$$

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<sup>3</sup>Dumitru and Urga (2012) show that intraday jump tests of LM and ABD outperform other test procedures especially when price volatility is not high.

where  $BV_t$  is the bipower variation (Barndorff-Nielsen and Shephard 2004) defined as follows:

$$BV_t = \frac{\pi}{2} \frac{M}{M-1} \sum_{k=2}^M |r_{t,k-1}| |r_{t,k}| \quad (5)$$

$\Phi_{1-\frac{\beta}{2}}^{-1}$  represents the inverse of the standard normal cumulative distribution function evaluated at a cumulative probability of  $1 - \frac{\beta}{2}$  and  $(1 - \beta)^M = 1 - \alpha$ , where  $\alpha$  represents the daily significance level of the test.

In our study, we identify intraday jumps by relying on the intraday procedure of LM-ABD. A jump is detected with the LM-ABD test on day  $t$  in intraday interval  $k$  when:

$$\frac{|r_{t,k}|}{\hat{\sigma}_{t,k}} > \theta \quad (6)$$

The threshold value  $\theta$  is calculated for different significance levels. For a daily significance level of 5% and a sampling frequency of 5 minutes (which corresponds to 77 intraday returns per day in our study), we obtain a threshold value of 3.40 and 4.40 using ABD and LM methods, respectively. In our study, we combine both procedures by taking an intermediate threshold value equal to 4.<sup>4</sup>

Once all intraday jumps are identified using the univariate jump detection test of LM-ABD, we apply the following co-exceedance rule to verify if a cojump occurs between assets  $i$  and  $j$  at intraday time  $k$  on day  $t$  (Bae et al., 2003):<sup>5</sup>

$$1 \left\{ \frac{|r_{i,t,k}|}{\hat{\sigma}_{i,t,k}} > \theta \right\} \times 1 \left\{ \frac{|r_{j,t,k}|}{\hat{\sigma}_{j,t,k}} > \theta \right\} = \begin{cases} 1 : & \text{a cojump between assets } i \text{ and } j \\ 0 : & \text{no cojump} \end{cases} \quad (7)$$

Thus, a cojump exists when asset returns jump simultaneously. We distinguish between an idiosyncratic jump defined as jump of a single asset or jump that occurs independently of the market movement and cojump defined as jumps of two or more assets that occur simultaneously.

Other techniques have recently been proposed to identify cojumps in the multivariate context using a single cojump test statistic such as those proposed by Barndorff-Nielsen and Shephard (2006b), Bollerslev et al. (2008) and Jacod and Todorov (2009). For instance, Bollerslev et al. (2008) uses the mean of cross products of returns of a large number of stocks as a test statistic to detect common arrival of jumps

<sup>4</sup>This threshold value is also employed by Bormetti and al. (2015). We also consider different threshold values (3 and 5); however, the results remain intact.

<sup>5</sup>See Lahaye et al. (2010) and Dungey et al. (2009) for applications.

at portfolio level. Their test statistic is sensitive to the number of the stocks considered in the portfolio. Indeed, a large number of stocks is required to diversify away asset idiosyncratic jumps. Jacod and Todorov (2009) develop a bivariate jump identification test using the ratio of power variation estimators. However, their approach can only be applied to detect if a particular day contains cojumps. Gobbi and Mancini (2012) also propose a daily cojump test by applying thresholding techniques.

The cojump test based on co-exceedance rule is appropriate for our context because it presents simple estimates of precisely timed cojumps with a relatively narrow range of intraday data. Moreover, Gnabo et al. (2014) show that univariate tests we use are satisfactory and best-suited for detecting jumps and cojumps as long as the jumps sizes are sufficiently large and have the same sign as the assets' correlation. This is effective in our case where the intraday jump return is greater than four times the estimate of the local volatility and assets are jumping in the same direction of the correlation.

### 3. Portfolio allocation problem

In this section, we present two different approaches for addressing the portfolio allocation problem and derive the optimal portfolio composition when there are domestic and foreign assets. First, we consider a representative domestic investor with a quadratic utility and show that the optimal weight of foreign asset holdings is a decreasing function of the correlation between assets, provided that the variability of the domestic asset is lower than the foreign asset. Second, we consider the standard CRRA utility approach and examine how higher-order moments induced by systematic and idiosyncratic jumps affect the optimal portfolio composition.

#### 3.1. Optimal portfolio composition and jump correlation

##### 3.1.1. Two-fund case

We consider a risk-averse investor who selects his portfolio composition based on two assets: a domestic risky asset and a foreign risky asset. Both asset returns are expressed in the investor's domestic currency. We consider the standard mean-variance approach initially formulated by Markowitz (1952). The approach defines the risk as the variance of the portfolio return. The domestic investor chooses the proportion of his wealth portfolio to invest in foreign asset ( $w_f$ ) and domestic asset ( $1 - w_f$ ) to maximize his objective utility function given by:

$$U(w_f) = \mu_P(w_f) - \frac{\gamma}{2} v_P(w_f) \quad (8)$$

where  $\mu_P$  and  $v_P$  are respectively the portfolio's mean return and variance.  $\gamma$  is the investor's risk aversion coefficient. The investor's objective function increases with the portfolio mean return and decreases with its variability.  $\mu_d(\sigma_d)$  and  $\mu_f(\sigma_f)$  denote as the expected returns (volatilities) of the domestic and foreign assets, respectively. The proportion of the foreign asset that maximizes the investor's objective function is given by:

$$w_f^* = \frac{1}{\gamma} \frac{(\mu_f - \mu_d)}{\sigma_f^2 - 2\rho\sigma_f\sigma_d + \sigma_d^2} + \frac{\sigma_d(\sigma_d - \rho\sigma_f)}{\sigma_f^2 - 2\rho\sigma_f\sigma_d + \sigma_d^2} \quad (9)$$

where  $\rho$  is the correlation coefficient between domestic and foreign assets.

The optimal proportion of foreign asset is composed of two terms. The first one represents the demand stemming from a higher potential return of the foreign asset. This term decreases with the investor's risk aversion. The second term represents the demand of foreign asset that minimizes the portfolio variance. The first order derivative of the optimal proportion of the foreign asset per correlation is:

$$\frac{dw_f^*}{d\rho} = \frac{1}{\gamma} \frac{2\sigma_f\sigma_d(\mu_f - \mu_d)}{(\sigma_f^2 - 2\rho\sigma_f\sigma_d + \sigma_d^2)^2} + \frac{\sigma_f\sigma_d(\sigma_d^2 - \sigma_f^2)}{(\sigma_f^2 - 2\rho\sigma_f\sigma_d + \sigma_d^2)^2} \quad (10)$$

The optimal proportion of the foreign asset in Eq. 9 is thus a decreasing function of the correlation if :

$$\sigma_d^2 + \frac{2}{\gamma}(\mu_f - \mu_d) < \sigma_f^2 \quad (11)$$

This condition is verified if the domestic asset has a higher expected return and a lower variability than the foreign asset.

The optimal weight of the foreign asset can be approximated by the second term in Eq. 9 (minimum variance portfolio) for conservative investors with high risk aversion levels.<sup>6</sup> In that case, an increase in the correlation between assets will lead to a decrease on the demand of foreign asset provided that the volatility of the foreign asset is greater than the domestic one. In practice, it is likely that the foreign asset's volatility is higher than the domestic asset one, given that the variability of the domestic asset only depends on the stock market whereas the variability of foreign asset depends on the foreign market and the variability of the domestic investor's exchange rate against foreign currency.

The correlation of two assets can be seen as the sum of two components. The first component represents the correlation arising from comovement of smooth returns of two assets whereas the second one represents

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<sup>6</sup>The demand of the foreign asset is also given by the minimum variance portfolio if the difference between domestic and foreign expected returns is not significantly different from zero.

the correlation stemming from simultaneous jumps. Indeed, the expression of the realized correlation (RC) of assets  $i$  and  $j$  over a time period  $[0, T]$  is given by:

$$\begin{aligned}
 RC &= \frac{\sum_{k=1}^N r_{i,k} r_{j,k}}{\sqrt{\sum_{k=1}^N r_{i,k}^2 \sum_{k=1}^N r_{j,k}^2}} \\
 &= \frac{\sum_{k=1}^N r_{i,k} r_{j,k} \mathbb{1}\left\{\frac{|r_{i,k}|}{\hat{\sigma}_{i,k}} \leq \theta\right\} \mathbb{1}\left\{\frac{|r_{j,k}|}{\hat{\sigma}_{j,k}} \leq \theta\right\}}{\sqrt{\sum_{k=1}^N r_{i,k}^2 \sum_{k=1}^N r_{j,k}^2}} + \frac{\sum_{k=1}^N r_{i,k} r_{j,k} \mathbb{1}\left\{\frac{|r_{i,k}|}{\hat{\sigma}_{i,k}} > \theta\right\} \mathbb{1}\left\{\frac{|r_{j,k}|}{\hat{\sigma}_{j,k}} > \theta\right\}}{\sqrt{\sum_{k=1}^N r_{i,k}^2 \sum_{k=1}^N r_{j,k}^2}}
 \end{aligned}$$

where  $r_{i,k}$  and  $r_{j,k}$  are the intraday returns of respectively the  $i^{\text{th}}$  and  $j^{\text{th}}$  assets over the  $k^{\text{th}}$  intraday time interval.  $N$  is the number of intraday returns over the time period  $[0, T]$ . The indicator functions are introduced to disentangle jumps from smooth intraday returns. We assume that the diffusive and jump parts of the return are independent.<sup>7</sup> We use the estimator provided by Jacod and Todorov (2009) to estimate the realized correlation between assets jumps. This estimator has the advantage to be robust to the jump identification procedure and it is given by:

$$\rho_{i,j}^{\text{jump}} = \frac{\sum_{k=1}^N r_{i,k}^2 r_{j,k}^2}{\sqrt{\sum_{k=1}^N r_{i,k}^4 \sum_{k=1}^N r_{j,k}^4}} \quad (12)$$

The jump correlation increases with the intensity and the size of cojumps. The squared returns are introduced to filter out smooth returns. As a result, the numerator in the above equation only takes into account simultaneous jumps whereas the denominator is calculated using assets jumps that occurs simultaneously or not. We also introduce the formula that we will use to estimate the correlation of respectively positive

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<sup>7</sup>This assumption is supported by the fact that the two return components are not determined by the same sources of risk. While fundamental factors such as firm-specific characteristics (e.g., size, earnings, leverage, dividend, and momentum) and macroeconomic variables (e.g., economic growth, interest rate, inflation, and exchange rates) derive smooth price movements, infrequent and large price changes (jumps) are generated by the arrival of important news. For instance, Bollerslev and Todorov (2011) show that jump risk requires a different premium.

and negative jumps.

$$\rho_{i,j}^{jump,up} = \frac{\sum_{k=1}^N r_{i,k}^2 r_{j,k}^2 1_{\{r_{i,k}>0\}} 1_{\{r_{j,k}>0\}}}{\sqrt{\sum_{k=1}^N r_{i,k}^4 1_{\{r_{i,k}>0\}} \sum_{k=1}^N r_{j,k}^4 1_{\{r_{j,k}>0\}}}} \quad (13)$$

$$\rho_{i,j}^{jump,down} = \frac{\sum_{k=1}^N r_{i,k}^2 r_{j,k}^2 1_{\{r_{i,k}<0\}} 1_{\{r_{j,k}<0\}}}{\sqrt{\sum_{k=1}^N r_{i,k}^4 1_{\{r_{i,k}<0\}} \sum_{k=1}^N r_{j,k}^4 1_{\{r_{j,k}<0\}}}} \quad (14)$$

In our study, we examine how cojumps between domestic and foreign assets affect the demand of foreign assets. We hypothesize the correlation of jumps between US stock market and foreign stock markets decreases the demand of foreign assets of an US representative investor.

### 3.1.2. General case

We now consider the general case where the investor selects his portfolio composition based on  $n$  assets: one domestic risky asset and  $n - 1$  foreign risky assets. We suppose that all asset returns are expressed in the investor's domestic currency. The investor allocates funds across  $n$  assets in a way to maximize his utility function as follows:

$$\begin{aligned} \max_w \left( \mu' w - \frac{\gamma}{2} w' \Sigma w \right) \\ \text{subject to } e' w = 1 \end{aligned} \quad (15)$$

where  $w = (w_1, w_2, \dots, w_n)'$  is the vector of portfolio weights and  $\mu = (\mu_1, \mu_2, \dots, \mu_n)'$  is the vector of expected returns.  $\Sigma = \text{cov}(r_i, r_k)_{1 \leq i, k \leq n}$  the variance-covariance matrix of returns.  $e = (1, 1, \dots, 1)'$  denotes the vector of ones. The optimal weights that maximize the investor's utility are given by:

$$w^* = \frac{1}{\gamma} \Sigma^{-1} \mu + \left( 1 - \frac{e' \Sigma^{-1} \mu}{\gamma} \right) \frac{\Sigma^{-1} e}{e' \Sigma^{-1} e}$$

where  $\Sigma^{-1}$  is the inverse of the returns covariance matrix.

If the coefficient of the risk aversion  $\gamma$  goes to infinity, we get the optimal weights that minimize the portfolio variance.<sup>8</sup>

$$w^* = \frac{\Sigma^{-1} e}{e' \Sigma^{-1} e}$$

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<sup>8</sup>The minimum variance portfolio has the advantage to be robust to the estimation error of expected returns.

It is established that if the investors have a quadratic utility or asset returns are normally distributed, the mean-variance framework is sufficient to obtain the optimal portfolio weights. To the extent that asset returns are non-normal in the presence of jumps, higher-order moments of the return distribution should be considered in the portfolio optimization problem.

Moreover, the variance, as a symmetric risk measure, fails to differentiate between the upside and downside risks, and often leads to an overestimation of the risk for positively skewed distribution and an underestimation of the risk for negatively skewed distribution. It is also unable to capture the risk of extreme events (large losses and large gains) when returns follow a fat-tailed distribution. Since investors are more concerned about extremely negative movements of asset returns, they pay a particular attention to the downside risk when selecting portfolio assets. The issue of the portfolio allocation under the non-normality of asset returns has been widely studied and several alternatives to the standard mean-variance framework have been proposed by, among others, Jondeau and Rockinger (2006) and Guidolin and Timmermann (2008). These two studies have extended the mean-variance framework to cover higher moments of asset returns by approximating the expected utility using Taylor series expansions. Other studies have considered the downside risk in portfolio optimization and allocation, and proposed several percentile risk measures as an alternative to the variance such as Value-at-Risk or VaR (Basak and Shapiro, 2001; Gaivoronski and Pflug, 1999) and Conditional Value-at-Risk or CVaR (Rockafellar and Uryasev, 2000 and 2002; Krokmal et al., 2002). The CVaR is also known as mean excess loss, mean shortfall, or tail VaR.

The VaR is an estimate of the upper percentile of loss distribution. It is calculated for specified confidence level over a certain period of time. The VaR is widely used by financial practitioners to manage and control risks. On other hand, the CVaR of a portfolio represents the conditional expectation of losses that exceeds the VaR. This definition ensures that VaR is never higher than the CVaR. In portfolio optimization, the CVaR has more attractive financial and mathematical properties than the VaR. Indeed, the CVaR is sub-additive and convex (Rockafellar and Uryasev, 2000) which can provide stable and efficient estimates, and is also considered as a coherent risk measure (Artzner et al., 1997; 1999; Pflug, 2000).<sup>9</sup>

The mean-CVaR portfolio problem under budget constraint (weights summing to 1) and a target portfo-

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<sup>9</sup>The lack of sub-additivity implies that VaR of a portfolio with two instruments may be greater than the sum of the individual VaRs of these two instruments (Artzner et al., 1997; 1999). Additionally, since the VaR is non-convex and non-smooth, the portfolio optimization may become very unstable and lead to multiple local extrema.

lios expected return  $\bar{\mu}$  is formulated as follows:<sup>10</sup>

$$\begin{aligned} \min_{\alpha, w, u} \quad & \alpha + \frac{1}{q(1-\beta)} \sum_{i=1}^q u_i \\ \text{subject to} \quad & \begin{cases} e'w = 1, u_i \geq 0 \\ \mu'w = \bar{\mu} \\ u_i + w'r^{(i)} + \alpha \geq 0, i = 1, \dots, q \end{cases} \end{aligned} \quad (16)$$

where  $\alpha$  is the VaR of the portfolio loss function.  $\beta$  is the confidence level of the VaR and CVaR and  $(r^{(1)}, r^{(2)}, \dots, r^{(q)})$  is a random collection of the vector of returns  $r = (r_1, r_2, \dots, r_n)'$ .  $u = (u_1, r_2, \dots, u_q)'$  is an auxiliary variable.

The mean-CVaR optimization problem in Eq. 16 can be solved using linear programming techniques. We note that if asset returns are normally distributed and  $\beta \geq 0.5$ , the values of the mean-variance and mean-CVaR approaches are equivalent and give the same optimal portfolio weights (Rockafellar and Uryasev, 2000). In this paper, we consider both approaches to determine the optimal portfolio composition and examine how the departure from the normality caused by the presence of jumps affects the optimal portfolio composition.

### 3.2. Optimal portfolio composition and jump higher-order moments

#### 3.2.1. Asset returns dynamics

In what follows, we introduce a jump-diffusion model that accommodates both systematic and idiosyncratic jumps in asset prices. Systematic jumps are defined as jumps that occur simultaneously across all assets whereas idiosyncratic jumps are asset-specific. Our model also allows for asymmetric effects between positive and negative jumps. The price dynamics of the  $i^{th}$  asset is given by:<sup>11</sup>

$$\frac{dS_{t,i}}{S_{t,i}} = \mu_i dt + \sigma_i dZ_{t,i} + \sum_{x \in (up, down)} J_i^{sys,x} dQ^{sys,x} + \sum_{x \in (up, down)} J_i^{id,x} dQ_i^{id,x} \quad (17)$$

where  $i = 1, \dots, n$  and  $0 \leq t \leq T$ .

The price process is composed of two components : a diffusion component with a drift  $\mu_i$  and a volatility  $\sigma_i$  and a jump component that covers the upside and downside changes of both idiosyncratic and systematic

<sup>10</sup>The details of the mean-CVaR optimization problem are in the Internet Appendix posted in one of the authors' websites at <https://kuntara.weebly.com>.

<sup>11</sup>This process is an extension of the multivariate jump-diffusion model proposed by Das and Uppal 2004.

price jumps. The upside (downside) jump is a jump with positive (negative) return. The upside systematic jump (respectively downside systematic, upside idiosyncratic and downside idiosyncratic) is characterized with a jump amplitude  $J_i^{sys,up}$  (respectively  $J_i^{sys,down}$ ,  $J_i^{id,up}$  and  $J_i^{id,down}$ ) and a Poisson process  $Q^{sys,up}$  (respectively  $Q^{sys,down}$ ,  $Q_i^{id,up}$  and  $Q_i^{id,down}$ ) with intensity  $\lambda^{sys,up}$  (respectively  $\lambda^{sys,down}$ ,  $\lambda_i^{id,up}$  and  $\lambda_i^{id,down}$ ). We assume that the Brownian motion  $Z_{t,i}$ , the jump amplitudes and their corresponding Poisson processes are independent and that the jump size  $\tilde{J}_i^x = \log(1 + J_i^x)$  has a normal distribution with mean  $\mu_i^x$  and variance  $(\nu_i^x)^2$ , where  $x \in (sys, up; sys, down; id, up; id, down)$ . We also assume that, conditional on a systematic (either positive or negative) jump, the systematic jump sizes are perfectly correlated across assets. We note  $\rho_{i,j}$  the correlation coefficient between the Brownian motions of the  $i^{th}$  and  $j^{th}$  assets. All model parameters are assumed to be constant over time.<sup>12</sup>

By applying Ito's lemma to the jump-diffusion process, we obtain:

$$d \log(S_{t,i}) = \left(\mu_i - \frac{\sigma_i^2}{2}\right)dt + \sigma_i dZ_{t,i} + \sum_{x \in (up, down)} J_i^{\tilde{J}_i^{sys,x}} dQ^{sys,x} + \sum_{x \in (up, down)} J_i^{\tilde{J}_i^{id,x}} dQ_i^{id,x}$$

The above stochastic process is useful for model parameters estimation, which is essentially based on the results of jump identification and the method of moments. We first identify all intraday jumps for each asset by applying the LM-ABD technique. We then classify the detected jumps into systematic and idiosyncratic jumps using the co-exceedance rule. The estimate of the intensity of systematic upside (respectively systematic downside, idiosyncratic upside and idiosyncratic downside) jumps is given by the ratio of the number of occurrences of systematic upside (respectively systematic downside, idiosyncratic upside and idiosyncratic downside) jumps to the total number of intraday time intervals over the estimation period. As jump sizes are normally distributed, we estimate the mean and the variance of each distribution from the empirical mean and variance of its corresponding detected jump returns. Finally, we use the method of moments to estimate the diffusive drift vector and the diffusive variance-covariance matrix of the multivariate price processes. For each asset  $i$ , the diffusive drift  $\mu_i$  is set so that the sum of the diffusive and jump mean returns are equal to the mean of total price return:

$$\mu_i - \frac{\sigma_i^2}{2} + \sum_{x \in (up, down)} \lambda^{sys,x} \mu_i^{sys,x} + \sum_{x \in (up, down)} \lambda_i^{id,x} \mu_i^{id,x} = \frac{1}{N} \sum_{k=1}^N r_{i,k}$$

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<sup>12</sup>These assumptions aim to simplify the model calibration and will thus enable us to focus on studying the effects of jump higher-order moments on optimal portfolio composition.

Similarly, the diffusive covariance between two assets  $i$  and  $j$ ,  $\{\rho_{i,j}\sigma_i\sigma_j\}_{i,j}$ , is set so that the sum of the diffusive and jump covariance components are equal to the total covariance between the two asset returns:<sup>13</sup>

$$\begin{aligned} \rho_{i,j}\sigma_i\sigma_j + & \sum_{x \in (up, down)} \lambda^{sys,x} (\mu_i^{sys,x} \mu_j^{sys,x} + \nu_i^{sys,x} \nu_j^{sys,x}) + \sum_{x \in (up, down)} \lambda_i^{id,x} (\mu_i^{id,x} \mu_j^{id,x} + \nu_i^{id,x} \nu_j^{id,x}) \\ = & \frac{1}{N} \sum_{k=1}^N r_{i,k} r_{j,k} \end{aligned}$$

The method of moments is appropriate for our context because we want to choose the parameters of the multivariate jump-diffusion processes in such a way that the first two moments of the jump-diffusion returns match exactly the first two moments of the pure-diffusion returns. This will then allow us to compare the optimal portfolio weights for an investor who recognizes idiosyncratic and systematic jumps and another investor who ignores them. By matching the first two moments of the pure-diffusion and jump-diffusion returns, we are able to disentangle the difference between two portfolio compositions that is attributed to jump higher-order moments while keeping the impact of the first two moments of returns on the optimal portfolio composition of both investors the same.

### 3.2.2. Optimal portfolio weights

We now derive the optimal portfolio weights for a representative domestic investor when returns are given by the multivariate jump diffusion process described in Eq. 17. The investor selects his portfolio composition based on one riskless asset with an instantaneous riskless rate  $r$ , one domestic risky asset and  $n - 1$  foreign risky assets. All asset returns are expressed in the investor's domestic currency. The investor wants to maximize the expected utility from terminal wealth  $W_T$  under his budget constraint (weights summing to 1). The investor's problem at time  $t$  is given by :

$$\begin{aligned} V(t, W_t) = \max_w & E[U(W_T)] \\ \text{subject to } & e'w + w_0 = 1 \end{aligned} \tag{18}$$

where  $w = (w_1, w_2, \dots, w_n)'$  is the vector of portfolio weights of  $n$  risky assets.  $w_0$  is the weight of the riskless asset and  $e = (1, 1, \dots, 1)'$  denotes the vector of ones.  $U$  is the CRRA utility function with a

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<sup>13</sup>In our experiment, we have three assets, corresponding to eight diffusive parameters to estimate (three diffusive drifts, three diffusive variances and two diffusive correlations) from eight moment conditions. Each moment condition is then used to estimate its corresponding parameter. The estimation problem thus leads to a unique solution.

constant relative risk aversion (CRRA) coefficient  $\gamma$  :

$$U(W_T) = \begin{cases} \frac{W_T^{1-\gamma}}{1-\gamma} & \text{if } \gamma \neq 1 \\ \log(W_T) & \text{if } \gamma = 1 \end{cases}$$

Using stochastic dynamic programming techniques, we can express the vector of optimal portfolio weights  $w$  as a solution of a system of  $n$  nonlinear equations as follows :<sup>14</sup>

$$\mathbf{0} = R - \gamma \Sigma w + \sum_{x \in (up, down)} \lambda^{sys,x} E[J^{sys,x} (1 + w' J^{sys,x})^{-\gamma}] + \sum_{x \in (up, down)} \lambda^{id,x} \cdot E[J^{id,x} \cdot (e + w \cdot J^{id,x})^{-\gamma}] \quad (19)$$

where  $\mathbf{0} = (0, 0, \dots, 0)'$  is the vector of zeros.  $R = (\mu_1 - r, \mu_2 - r, \dots, \mu_n - r)'$  is the diffusive excess-returns vector.  $\Sigma = (\rho_{i,j} \sigma_i \sigma_j)_{1 \leq i, j \leq n}$  is the covariance matrix of the diffusive returns.  $\lambda^{id,x} = (\lambda_1^{id,x}, \lambda_2^{id,x}, \dots, \lambda_n^{id,x})'$  is the vector of idiosyncratic (up or down) jump intensities.  $J^{id,x} = (J_1^{id,x}, J_2^{id,x}, \dots, J_n^{id,x})'$  is the vector of idiosyncratic (up or down) jump amplitudes whereas  $J^{sys,x} = (J_1^{sys,x}, J_2^{sys,x}, \dots, J_n^{sys,x})'$  denotes the vector of systematic (up or down) jump amplitudes. The  $\cdot$  operator denotes the element- by-element multiplication of two equally sized vectors.

The above system of non-linear equations can be solved numerically, which we do in Section 5.3. In the case of a pure-diffusion investor ( $\lambda^{sys,x} = 0; \lambda^{id,x} = \mathbf{0}$ ), Eq. 19 leads to the same solution as the quadratic utility maximization problem presented in Section 3.1:

$$w^* = \frac{1}{\gamma} \Sigma^{-1} R$$

In what follows, we try to provide some insights on how higher-order moments of returns affect the optimal portfolio composition by considering the case when there is one risky asset and approximate the non-linear term of Eq. 19 using a second-order Taylor approximation:<sup>15 16</sup>

$$(1 + w_1 J_1^{sys,x})^{-\gamma} \approx 1 - \gamma (w_1 J_1^{sys,x}) + \frac{\gamma(\gamma + 1)}{2} (w_1 J_1^{sys,x})^2$$

<sup>14</sup>The detailed resolution of the CRRA utility maximization problem is in the Internet Appendix posted in one of the authors' website at <https://kuntara.weebly.com>.

<sup>15</sup>We also employ a Taylor series expansion to approximate the jump amplitude  $J_1^{sys,x} = e^{J_1^{sys,x}} - 1 \approx J_1^{sys,x}$ .

<sup>16</sup>We choose to study the case of a portfolio composed of one risky asset in order to simplify the resolution of Eq. 19. By using a second-order Taylor approximation, we only consider the role of the skewness in determining the optimal portfolio composition. The impact of all higher-order moments is considered by resolving numerically Eq. 19 in Section 5.3.

$$(1 + w_1 J_1^{id,x})^{-\gamma} \approx 1 - \gamma(w_1 J_1^{id,x}) + \frac{\gamma(\gamma + 1)}{2}(w_1 J_1^{id,x})^2$$

Using these approximations, the optimal weight  $w_1$  of the risky asset is the solution of the following quadratic equation:

$$0 = \mu_1^t - \gamma v_1^t w_1 + \frac{\gamma(\gamma + 1)}{2} s_1^t w_1^2$$

where :

$$\begin{aligned} \mu_1^t &= \mu_1 + \sum_{x \in (up, down)} \lambda^{sys,x} \mu_1^{sys,x} + \sum_{x \in (up, down)} \lambda_1^{id,x} \mu_1^{id,x} - r \\ v_1^t &= \sigma_1^2 + \sum_{x \in (up, down)} \lambda^{sys,x} [(\mu_1^{sys,x})^2 + (\nu_1^{sys,x})^2] + \sum_{x \in (up, down)} \lambda_1^{id,x} [(\mu_1^{id,x})^2 + (\nu_1^{id,x})^2] \\ s_1^t &= \sum_{x \in (up, down)} \lambda^{sys,x} \mu_1^{sys,x} [(\mu_1^{sys,x})^2 + 3(\nu_1^{sys,x})^2] + \sum_{x \in (up, down)} \lambda_1^{id,x} \mu_1^{id,x} [(\mu_1^{id,x})^2 + 3(\nu_1^{id,x})^2] \end{aligned}$$

$\mu_1^t$ ,  $v_1^t$  and  $s_1^t$  are respectively the total (the sum of both the diffusive and jump components) mean excess return, variance and skewness of the risky asset. Note that the skewness depends only on the jump component of the price process.

The explicit expression of the optimal risky asset weight is given by:

$$w_1^* \approx \frac{2\mu_1^t}{\gamma v_1^t + \sqrt{\gamma^2 (v_1^t)^2 - 2\gamma(\gamma + 1)\mu_1^t s_1^t}}$$

Note that the optimal weight only depends on the first three moments of the return process because we only consider the first three terms in the Taylor series approximation.

If we consider an investor who ignores the idiosyncratic and systematic jumps and assumes a pure-diffusion model for the price process, the optimal risky asset weight can be written as:<sup>17</sup>

$$w_1^* \approx \frac{\mu_1^t}{\gamma v_1^t}$$

The difference between the optimal portfolio composition of the investor who considers idiosyncratic and systematic jumps and the investor who ignores jumps depends on the sign of the jump skewness  $s_1^t$ . If the skewness is negative, the investor who accounts for jumps will invest less in risky asset than the

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<sup>17</sup>  $\mu_1^t$  and  $v_1^t$  estimates are the same for both the pure-diffusion and jump-diffusion processes.

investor who ignores jumps. This result can be generalized to the case with several risky assets. The jump-diffusion investor will invest less in risky assets than the diffusion investor if the skewness and co-skewness of asset returns are negative. Also, the investor who considers jump risks will prefer risky assets with better higher-order moments (higher skewness and co-skewness and lower kurtosis and co-kurtosis).<sup>18</sup>

#### **4. Data**

We use intraday data of three international exchange-traded funds: SPDR S&P 500 (SPY), iShares MSCI EAFE (EFA) and iShares MSCI Emerging Markets (EEM). The SPDR S&P 500 ETF aims to replicate the performance of S&P 500 index by holding a portfolio of the common stocks that are included in the index, with the weight of each stock in the portfolio substantially corresponding to the weight of such stock in the index. The iShares MSCI EAFE ETF aims to replicate the performance of the MSCI EAFE index, which captures the stock market performance of developed markets outside of the US and Canada and thus a proxy for Europe, Australia and Far East equity markets. The iShares Emerging Markets ETF seeks to replicate the performance of the MSCI Emerging Markets index. The latter captures the stock market performance of emerging markets, currently covers over 800 securities across 21 markets, and represents approximately 11% of world market cap. Our empirical analysis is based on intraday prices of the three funds from January 2008 to October 2013. Prices are sampled every five minutes from 9:30 am to 15:55 pm (UTC-4 time zone) to smooth the impact of market microstructure noise.

#### **5. Empirical findings**

##### *5.1. Intraday jump identification*

This section summarizes the results from applying LM-ABD intraday jump detection test. A particular attention is given to the intraday volatility pattern (Dumitru and Urga, 2012), which can lead to spurious jump detection. We correct the intraday volatility pattern using a jump robust corrector proposed by Bollerslev et al. (2008) to improve the robustness of our jump detection procedure.<sup>19</sup>

We estimate the realized bipower variation using a window of 155 intraday returns, which corresponds to two days of intraday returns sampled at a frequency of five minutes. Jumps are detected with a threshold

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<sup>18</sup>Higher-order co-moments are only induced by systematic jumps.

<sup>19</sup>The details of the volatility pattern corrector used in our study are in the Internet Appendix posted in one of the authors' website at <https://kuntara.weebly.com>.

value  $\theta = 4$ , which means that the intraday jump return size is at least four times greater than the estimate of the local volatility. We also apply threshold values of 3 and 5 to study the robustness of our results.<sup>20</sup>

Table 1 provides the number of total, positive and negative intraday jumps detected over the study period. We identify 1119, 1114 and 1024 intraday jumps for SPY, EFA and EEM funds respectively, or 0.989%, 0.986% and 0.900% of the total number of intraday returns. The number of detected intraday jumps is higher in developed markets (US and EFA) than in emerging markets, suggesting a higher degree of asset comovement within developed markets. A positive (negative) jump is a jump with positive (negative) return. The results show that the number of negative jumps is more than 56% of total number of detected jumps for each fund. Stock markets thus tend to experience more price jumps when markets are bearish. Table 1 also reveals that the mean of intraday jump returns of SPY ( $-4.3e-04$ ) in absolute value is two times higher than the one for EFA and EEM ( $-2.5e-04$  and  $-2.1e-04$ , respectively). This result indicates that negative intraday price movements are larger for the US market. However, the intraday jump return volatility is higher for emerging markets (0.0058) than for developed ones (around 0.0047). At a daily level, Table 2 shows that the percentage number of days with at least one intraday jump is around 40% of the total number of days of the study period (1468 days). The high proportion of jump days might be explained by a higher jump activity during the financial crisis period. It is also related to the high level of sampling frequency (five minutes) used in our paper.

\*\*\* Insert Tables 1 and 2 here \*\*\*

Table 3 shows some statistics of detected cojumps. Over the study period, we find 585 cojumps between SPY and EFA funds, 509 cojumps between EEM and SPY funds, and 458 cojumps between EEM and EFA funds. This finding indicates that developed equity markets are more linked to the US market than emerging markets. The three funds are involved in 365 cojumps, with the number of negative cojumps (61%) being higher than positive cojumps. Table 4 shows the probability to have at least one cojump between SPY and EFA is 0.27 at the daily level. This probability is lower for SPY and EEM (0.23) and for EFA and EEM (0.22). The probability that the three funds simultaneously experience a cojump is 0.18.

\*\*\* Insert Tables 3 and 4 here \*\*\*

We also examine if detected jumps and cojumps can be explained by exchange rates movements. Table 5 reports the number of cojumps that involve EUR/USD exchange rate with one or more of the three funds.

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<sup>20</sup>Detailed results for the threshold values of 3 and 5 are available upon request.

We apply EUR and USD as they are the two major currencies in the world. The probability of cojumps between the EUR/USD and EFA is the highest (24%) whereas the probability of cojumps with SPY and EEM is respectively 14% and 16%. The exposure to currency risk is relatively higher for EFA given that it is composed largely of stocks that are quoted in EUR whereas the whole fund is traded in USD. The probability that two or three funds jump at the same time as the EUR/USD exchange rate remains less than 12%. These results suggest that only 12% of detected cojumps of three funds are induced by exchange rate movements.

\*\*\* Insert Table 5 here \*\*\*

To examine the variation of the jump and cojump intensities over time, we calculate the time-varying daily intensities of jumps (JI) and cojumps (CJI) using a rolling 6-month window of observations as follows:

$$JI = \frac{\sum_k 1_{\{Jump_k^i\}}}{N_{days}} \quad \text{and} \quad CJI = \frac{\sum_k 1_{\{Jump_k^i \cap Jump_k^j\}}}{N_{days}}$$

where  $N_{days}$  is the number of days of the observation period (120 business days in our case).  $1_{\{Jump_k^i\}}$  is an indicator function of jump occurrence for the  $i^{\text{th}}$  asset at the intraday interval  $k$ .  $1_{\{Jump_k^i \cap Jump_k^j\}}$  is an indicator function of cojump occurrence for the  $i^{\text{th}}$  and  $j^{\text{th}}$  assets at the intraday interval  $k$ .

Figure 1 uncovers that the daily jump and cojump intensities have significantly increased during the financial crisis of 2008-2009 for the three funds. There is a pattern that the US market was the first to reach the peak of the jump intensity during the crisis followed by other developed markets and then emerging markets, particularly during a peak in January 2010, a drop in December 2010, and a jump in December 2012. The results support the evidence in Ait-Sahalia et al. (2015). The cojump intensity is highest between funds of the US and other developed markets, followed by funds of the US and emerging markets, and finally the funds of other developed markets and emerging markets. The intensity of simultaneous jumps of three funds is lowest. Overall, there is an evidence of the lead/lag interaction of jumps in that the intraday jumps seems to be triggered in the US market and then propagated to the rest of the world.

\*\*\* Insert Figure 1 here \*\*\*

## 5.2. Time and space clustering of intraday jumps

This section examines the dependencies between intraday jumps both within and across markets. Figure 2 shows jumps in SPY tend to occur simultaneously with jumps in EFA and EEM and their cojumps are

clustered during the periods from June 21, 2010 to June 24, 2010 and August 23, 2010 to August 27, 2010. Thus, international intraday jumps are likely to propagate both in time (in the same market) and in space (across markets). We formally test jump propagation using the Hawkes process (Hawkes, 1971).<sup>21</sup> This process is a self-excited point process whose intensity depends on the path followed by the point process and has been extensively used in different domains such as seismology and neurology, but only recently in finance to model the dynamics of microstructure prices (Lee and Seo, 2017), trading activity, and asset jumps in financial markets (Ait-Sahalia et al., 2015; Bormetti et al., 2015). For example, Ait-Sahalia et al. (2015) use a multivariate Hawkes jump-diffusion model to investigate the financial contagion in the international equity markets at daily level. Bormetti et al. (2015) employ high frequency data and the Hawkes processes to reproduce the time clustering of jumps for 20 high-cap Italian stocks.

\*\*\* Insert Figure 2 here \*\*\*

The univariate Hawkes process we use to capture the time clustering of intraday jumps for each of the ETFs is given by:

$$d\lambda_t = \beta(\lambda_\infty - \lambda_t)dt + \alpha dN_t \quad (20)$$

where  $N_t$  is the number of jumps occurring in the time interval  $[0, t]$ . A jump occurrence at a given time will increase the intensity or the probability of another jump (self excitation). The intensity increases by  $\alpha$  whenever a jump occurs, and then decays back towards a level  $\lambda_\infty$  at a speed  $\beta$ . These parameters can be estimated using the maximum likelihood method. Given the jump arrival times  $t_1, t_2, \dots, t_q$ , the likelihood function is written as:<sup>22</sup>

$$L(t_1, t_2, \dots, t_q) = -\lambda_\infty t_q + \sum_{i=1}^q \frac{\alpha}{\beta} \left( e^{-\beta(t_q - t_i)} \right) + \sum_{i=1}^q \log(\lambda_\infty + \alpha A(i)) \quad (21)$$

where  $A(i) = \sum_{t_j < t_i} e^{-\beta(t_i - t_j)}$  for  $i \geq 2$  and  $A(1) = 0$ .

The univariate Hawkes process can be extended so that it captures the time and space clustering of intraday jumps between  $n$  markets, such as:

$$d\lambda_{i,t} = \beta_i(\lambda_{i,\infty} - \lambda_{i,t})dt + \sum_{j=1}^n \alpha_{i,j} dN_{j,t}, \quad i = 1, \dots, n \quad (22)$$

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<sup>21</sup>Note that this dependence pattern cannot be reproduced by the standard Poisson process, which assumes the hypothesis of independence of the increments (jumps in our case) on disjoint time intervals.

<sup>22</sup>See Ogata (1978) and Ozaki (1979) for the details of the maximum likelihood estimation.

Under this model, a jump in market  $j$  increases not only the jump intensity within the same market through  $\alpha_{j,j}$  (self excitation) but also the cross-market jump intensity through  $\alpha_{i,j}$  (cross excitation).  $\alpha_{j,j}$  implies a degree where jumps are re-created within the same market whereas  $\alpha_{i,j}$  suggests the propagation rate of jumps from market  $j$  to market  $i$ . The jump intensity of market  $i$  reverts exponentially to its average level  $\lambda_{i,\infty}$  at a speed  $\beta_i$ . Since the numerical resolution of the trivariate Hawkes model for three ETFs is problematic owing to the large number of parameters to be estimated, we limit the calibration procedure to the bivariate model whereby the vector of unknown parameters only contains 8 parameters,  $\Theta = (\lambda_{1,\infty}, \lambda_{2,\infty}, \beta_1, \beta_2, \alpha_{1,1}, \alpha_{1,2}, \alpha_{2,1}, \alpha_{2,2})$ , as follows:

$$\begin{cases} d\lambda_{1,t} = \beta_1(\lambda_{1,\infty} - \lambda_{1,t})dt + \alpha_{1,1}dN_{1,t} + \alpha_{1,2}dN_{2,t} \\ d\lambda_{2,t} = \beta_2(\lambda_{2,\infty} - \lambda_{2,t})dt + \alpha_{2,1}dN_{1,t} + \alpha_{2,2}dN_{2,t} \end{cases} \quad (23)$$

Panels A, B and C of Table 6 show the estimation results of the bivariate Hawkes model for SPY/EFA, SPY/EEM and EFA/EEM, respectively. All the model parameters are significant at conventional levels, implying that the bivariate Hawkes model satisfactorily fits the data of intraday jump occurrences of three funds. The large value of the parameters  $\alpha_{1,1}$  and  $\alpha_{2,2}$  provides clear evidence that intraday jumps of the US market, other developed markets and emerging markets are strongly recreated within individual market. Panel A indicates that the self-excitation activity for the US market is higher than that of other developed markets. Compared to emerging markets, the self-excitation activity for the US (Panel B) and for the other developed market (Panel C) is greater. The higher degree of market efficiency in the US and other developed market might explain their high level of self-excitation activity. On the other hand, the value of the parameters  $\alpha_{1,2}$  and  $\alpha_{2,1}$ , which measure the degree of jump transmission between markets, is smaller than the self-excitation parameters. The degree of jump transmission between markets is asymmetric with a stronger transmission from the US market to other developed markets ( $\alpha_{efa,spy} = 2.40 \text{ e-}03$ ) than from the US market to emerging markets ( $\alpha_{em,spy} = 2.23 \text{ e-}03$ ). The transmission of jumps in the other way around is also significant but the strength is weaker. The emerging markets receive more jump spillover from the other developed markets than what they transmit to other markets.

\*\*\* Insert Table 6 here \*\*\*

### 5.3. Cojumps and optimal portfolio composition

We now examine the effect of cojumps on the optimal portfolio composition within the mean-variance and mean-CVaR frameworks from the US investor perspective. More precisely, we study how the demand

of foreign assets varies with cojumps between domestic and foreign assets. The portfolio we consider is composed of one domestic asset (represented by the SPY fund) and two foreign assets (represented by EFA and EEM funds). All fund returns are expressed from the US investor's perspective and thus in USD. Assets weights can be negative, meaning that the domestic investor can take short position on the domestic and foreign assets. The demand of foreign assets is defined as the sum of optimal allocation weights of EFA and EEM funds resulting from our portfolio optimization procedure based on daily historical returns (1469 observations for each fund) and the variance and CVaR approaches. The portfolio optimization is performed each week using a rolling window of about 120 daily returns (6 months) that immediately precede the optimization day. We also consider different rolling window sizes (3, 9, 12, 15 and 18 months), and the results remain intact.<sup>23</sup>

Our optimization problem consists of minimizing the portfolio risk (standard deviation or CVaR) under the budget's constraint of weights summing to one. Figure 3 shows the dynamic changes in the optimal proportion of the foreign assets (EFA and EEM) for variance and CVaR approaches. Both approaches lead to similar portfolio compositions over the study period, but the minimum CVaR portfolio composition is more volatile than the minimum variance portfolio. This might be because CVaR is determined using few extreme observations in the lower tail of the returns distribution whereas the variance takes into account all available observations. Panels A and B of Figure 4 show that the standard deviation and CVaR are varying in a similar fashion over time. Moreover, the risk of the domestic market (SPY fund) is often lower than that of foreign markets (EFA and EEM funds), regardless of the risk measures. This might be because from domestic market perspective (here is the US), the foreign market is subject to the variability of both foreign stock market and exchange rate. Panels C, D and E of Figure 4 show respectively the variation of the daily return correlation, the realized correlation of intraday returns and realized correlation of SPY/EFA jumps, SPY/EEM jumps and EFA/EEM jumps.<sup>24</sup> The graphs suggest that three correlation measures are closely linked to each other. In other words, correlated jumps have similar pattern of comovement to both correlated daily and intraday returns. The correlation of jumps is relatively high over the period of study. It varies between 0.6 and 1 for three pairs of funds. The jump correlation is thus significant and positively high.<sup>25</sup>

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<sup>23</sup>Detailed results for different rolling window sizes (3, 9, 12, 15 and 18 months) are available upon request.

<sup>24</sup>The daily correlation is the standard correlation coefficient calculated monthly using a rolling six-month window of daily returns. The realized correlation of intraday returns and jump correlations are estimated monthly using a rolling six-month window of intraday returns.

<sup>25</sup>Refer to Jacod and Todorov (2009) for more details about the accuracy of the jump correlation estimator.

On average, the correlation between SPY/EFA jumps is the highest. It is then followed by SPY/EMM and EFA/EEM jumps.

\*\*\* Insert Figures 3 and 4 here \*\*\*

As the composition of the optimal portfolio becomes available, we are able to study how cojumps between domestic and foreign markets affect the portfolio composition. We hypothesize that a high intensity of cojumps between domestic and foreign markets leads to a decrease in benefit from diversifying internationally and thus foreign asset holding. We begin our analysis with the calculation of the correlation between the daily intensity of cojumps and the optimal proportion of foreign assets that we obtain from variance and CVaR approaches. Although our exercise onward has the US as home market and the rest as foreign markets, our analysis can be generalized to any home and host markets with respect to our theoretical framework in Section 3.1, provided that the variability of home market is lower than the variability of host markets.

The main results for the variance approach, summarized in Table 7, show a negative correlation between the demand of foreign assets and the daily intensity of cojumps between the domestic market (SPY fund) and each of foreign markets (EFA and EEM funds).<sup>26</sup>We find a correlation of -0.10 for SPY/EFA cojumps intensity and -0.20 for SPY/EEM cojumps intensity with foreign asset holding. When we consider the cojumps that involve the three funds, we find a negative correlation of -0.19. We get similar results for positive and negative cojumps except for positive cojump between SPY/EFA where the correlation with foreign asset holding is not significant. This result implies that, regardless of the sign of returns, investors do not like cojumps between domestic and foreign assets.

\*\*\*Insert Table 7 here\*\*\*

In addition to cojumps intensity, we find that the negative relation between correlated jumps and foreign holdings is more pronounced. The measure of jump correlation is shown analytically in Section 3.1. It is more appropriate in practice to use the jump correlation to measure the degree of linkage between assets jumps. Contrary to cojump intensity measure, which only takes into account the frequency effect, the jump correlation captures both the jump frequency and size effects. It thus provides a more accurate measure

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<sup>26</sup>We find similar results for the CVaR approach. Detailed results are not presented due to limited space but are available upon request.

of the degree of jump comovement than the cojump intensity. We find a strong negative correlation of -0.56 for SPY/EFA correlated jumps and -0.39 for SPY/EEM correlated jumps. Negative and positive jump correlations are also negatively correlated to foreign asset holdings. The correlation between foreign assets holdings and correlated jumps is higher for the portfolio of SPY and EFA funds than for the portfolio of SPY and EEM funds. This is expected given that the US market has a higher correlation with the other developed ones. We also examine if our results are sensitive to exchange rate jump risk. We exclude all assets jumps that occur simultaneously with EUR/USD exchange rate jumps and repeat the analysis. The significant negative link between jump correlation and the demand of foreign assets remains intact. We find a correlation of -0.50 for SPY/EFA and -0.38 for SPY/EEM.<sup>27</sup>

In Table 8, we examine the correlation between foreign asset holdings and idiosyncratic jumps where idiosyncratic jumps are defined earlier as jumps that occur in a single market. Specifically, we measure the correlation between the daily intensity of idiosyncratic jumps (of respectively SPY, EFA and EEM funds) and the optimal proportion of foreign assets and find a positive correlation for SPY (0.45), EFA (0.35) and EEM (0.40). In contrast to systematic jumps, idiosyncratic jumps increase foreign holdings. That is, investors seem to be aware that holding more foreign assets in their portfolios diversify idiosyncratic jump risks.

\*\*\* Insert Table 8 here \*\*\*

Finally, we examine how higher-order moments induced by idiosyncratic and systematic jumps affect the optimal portfolio composition. For this purpose, we compare the optimal portfolio weights for an investor who recognizes idiosyncratic and systematic jumps and another investor who ignores them and assumes a pure multivariate diffusion process for asset returns. Both investors have the same CRRA utility function and select their portfolio compositions based on one domestic asset (SPY) and two foreign assets (EFA and EEM). The portfolio optimization is performed yearly following the methodology described in Section 3.2. The optimal weights are obtained by resolving numerically Eq. 19. The risk aversion coefficient is set to three and the risk-free rate to zero. The parameters of the multivariate jump-diffusion process are estimated on a yearly basis using intraday returns of the preceding year. Table 9 reports the detail of jump-diffusion model estimation over the whole period. The results in Table 10 show that optimal domestic

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<sup>27</sup>Detailed results of the correlation between foreign asset holdings and correlated jumps are not presented for brevity. They are available upon request.

asset weights are almost the same for the pure-diffusion and jump-diffusion investors. The impact of jump higher-order moments on the optimal portfolio composition is thus insignificant.<sup>28</sup> This finding is consistent with Das and Uppal (2004) who find similar results for systemic jumps.<sup>29</sup> That is, the difference between the portfolio composition of an investor who cares about systemic jumps and another investor who ignores them is small.

Taken together, the findings in this section show that the jump correlation, which is the normalized second joint moment of jump returns, reduces foreign asset holdings in international portfolios, whereas higher-order moments have no significant effect on portfolio composition.

\*\*\* Insert Tables 9 and 10 here \*\*\*

#### 5.4. Cojumps and the benefits of international portfolio diversification

The results in Section 5.3 show that systematic cojumps in international equity markets are an indirect barrier to the holding of foreign assets. To the extent that they amplify the level of equity market comovement, it is expected that they negatively affect the benefits of portfolio diversification. We confirm this by examining the link between the number of cojumps and the measure of international diversification benefits in the spirit of Christoffersen et al. (2012) who propose to measure the conditional diversification benefit (CDB) as follows:

$$CDB_{\beta}(w) = \frac{\bar{\phi}_{\beta} - \phi_{\beta}(w)}{\bar{\phi}_{\beta} - \alpha_{\beta}(w)} \quad (24)$$

where  $\alpha_{\beta}(w)$  and  $\phi_{\beta}(w)$  are the values of the VaR and the CVaR of the portfolio loss function associated with the vector of weights  $w$  and the confidence level  $\beta$ . By construction, the  $\alpha_{\beta}(w)$  refers to the lower bound of the portfolio's expected shortfall, while  $\bar{\phi}_{\beta}$  is the upper bound of the portfolio's expected shortfall and is defined as the weighted average of the individual assets' CVaRs ( $\phi_{\beta,i}$ ):

$$\bar{\phi}_{\beta} = \sum_{i=1}^n w_i \phi_{\beta,i} \quad (25)$$

Thus, the CDB measure is a positive function with values ranging between 0 and 1, and increases with the level of diversification benefit. The CDB measure does not depend on the expected returns. Furthermore,

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<sup>28</sup>The ratio  $\frac{\mu_1^t s_1^t}{(v_1^t)^2}$ , defined in Section 3.2, is in the order of 10e-4 ( $\ll 1$ ) for three funds, which explains the insignificant impact of higher-order moments.

<sup>29</sup>Systemic jumps are defined as infrequent, large and highly correlated jumps.

it takes into account the nonlinearity of asset returns and the potential of their nonlinear dependence (i.e., jumps, cojumps, and extreme movements).<sup>30</sup>

We calculate the monthly value of the CDB for our portfolio of three exchange-traded funds based on the intraday data from the previous month (about 1617 intraday returns for each fund per month). The confidence level  $\beta$  is set at 5%. The weights allocated to three funds are chosen monthly in a way to maximize the CDB. Figure 5 depicts the variation of the optimal level of the CDB for the portfolio composed of three funds SPY, EFA and EEM. The CDB measure fluctuates between 0.06 (August 2011) and 0.30 (August 2008 and January 2013). It is high at the beginning and the end of the study period and relatively low during the global financial crisis.

\*\*\* Insert Figure 5 here \*\*\*

To assess the effects of jumps and cojumps on the diversification benefit, we compute the correlation between the optimal level of the CDB and jump comovement, the daily intensity of cojumps and idiosyncratic jumps, both calculated on a monthly basis from the cojumps and jumps detected from the previous month. The daily intensity of cojumps refers to the daily average number of cojumps involving three funds, while the daily intensity of idiosyncratic jumps refers to the daily average number of jumps involving only one fund. We also consider the correlation between the optimal level of CDB and jump comovement.<sup>31</sup> The latter is the monthly average jump correlation of three funds, computed from the previous month intraday data.

\*\*\* Insert Table 11 here \*\*\*

The findings in Table 11 show that the CDB is negatively correlated with cojumps intensity (-0.37) and correlated jumps (-0.65). In contrast, the CDB is positively correlated with idiosyncratic jumps intensity (0.40). The strong negative dependence between the diversification benefit and jump comovement is also shown in Figure 6. Taken together, these results indicate that the international diversification benefit increases with the intensity of idiosyncratic jumps and decreases with the level and intensity of cojumps observed in the international markets. They further confirm our previous findings that domestic (US) investors allocate more money towards home assets in the presence of cojumps between US and foreign markets.

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<sup>30</sup>The high correlation of large down moves in international markets is documented by Longin and Solnik (2001) and Ang and Bekaert (2002).

<sup>31</sup>The jump correlation measure has been introduced in Section 3.1.

\*\*\* Insert Figure 6 here \*\*\*

## 6. Conclusion

In this paper, we investigate how jumps and cojumps in international equity markets affect international asset allocation and diversification benefits. Using a nonparametric intraday jump detection technique developed by Lee and Mykland (2008) and Anderson et al. (2007) and intraday data from three international exchange-traded funds as proxies for international equity markets, we find that intraday jumps are transmitted both in time (in the same market) and in space (across markets). The markets under consideration also have tendency to be involved in cojumps. The high and significant degree of jumps synchronization in international equity markets suggests that the jump risk is rather systematic and thus could not be eliminated through the diversification.

We study the impact of cojumps on optimal international portfolio based on portfolio variance and CVaR minimisation approaches and find a negative link between the demand for foreign assets and cojumps between domestic and foreign markets. This result implies that a domestic investor will invest less in foreign markets when the frequency and size of cojumps between domestic and foreign assets increase. We also find the negative link between the intensity of cojumps and the conditional diversification benefit measure suggested by Christoffersen et al. (2012). Putting differently, the high jump correlation increases the cross-market comovement, and therefore reduces the international diversification benefit and leads investors to prefer home assets. In contrast, we find that idiosyncratic jumps have a positive effect on foreign asset holding and diversification benefits. Finally, the impact of higher-order moments induced by idiosyncratic and systematic jumps on optimal international portfolio composition is insignificant.

This work opens interesting avenues for future research. It would be interesting to broaden the scope of this study by including a larger number of international equity indices and examining the impact of asset cojumps on the demand for foreign assets for a larger panel of countries. Studying the underlying mechanisms of cojumps in international equity markets should also be of interest.

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Table 1: Summary statistics of jump occurrences, jump sizes and intraday returns.

This table shows summary statistics of total and jump returns in Panels A and B, respectively. Jumps statistics include the total number of jumps, positive and negative jumps, mean, standard deviation, skewness and kurtosis of jump returns of the three ETFs including SPY, ETA, and EEM. The percentage of positive jumps and negative jumps from all jumps are reported in brackets next to the number of positive and negative jumps. Jumps are detected using the LM-ABD procedure with a critical value  $\theta = 4$ . See Section 2 for the detail of jump test statistics. The intraday prices of the three ETFs from January 2008 to October 2013 are included. Prices are sampled every five minutes from 9:30 am to 15:55 pm.

	SPY	EFA	EEM
Panel A: Jumps statistics			
Intraday jumps	1119	1114	1024
Positive jumps	475 (42%)	495 (44%)	455 (44%)
Negative jumps	644 (58%)	619 (56%)	569 (56%)
Mean of jump returns	-4.3e-04	-2.5e-04	-2.1e-04
Std of jump returns	0.0048	0.0047	0.0058
Skewness of jump returns	-0.59	0.27	-0.006
Kurtosis of jump returns	14.00	8.40	15.60
Panel B: Returns statistics			
Mean of total returns	2.4e-06	4.7e-06	3.5e-06
Std of total returns	0.0014	0.0013	0.0017
Skewness of total returns	-0.12	0.11	0.09
Kurtosis of total returns	36.66	24.40	38.33

Table 2: Summary statistics of jump occurrences at day level.

This table presents the number of days with no jumps, one jump, and two jumps up to more than 5 jumps of the price of the three ETFs (SPY, EFA and EEM). The last row shows the percentage of days with at least one jump. Jumps are detected using LM-ABD procedure with a critical value  $\theta = 4$ . See Section 2 for the detail of jump test statistics. The intraday prices of the three ETFs from January 2008 to October 2013 are included. Prices are sampled every five minutes from 9:30 am to 15:55 pm.

	SPY	EFA	EEM
0	843	843	879
1	357	365	353
2	139	147	128
3	75	50	60
4	30	37	26
5	12	13	9
More than 5	12	13	13
At least one jump	42%	42%	40%

Table 3: Summary statistics of cojump occurrences.

This table reports the number of total positive and negative detected cojumps among ETFs including SPY and EFA (column 1), SPY and EEM (column 2), EFA and EEM (column 3) and SPY, EFA and EEM (column 4). The percentage of cojumps compared to the total number of detected jumps is shown in brackets next to intraday cojumps. Jumps are detected using the LM-ABD procedure with a critical value  $\theta = 4$ . See Section 2 for the detail of jump and cojump identification procedure. The intraday prices of the three ETFs from January 2008 to October 2013 are included. Prices are sampled every five minutes from 9:30 am to 15:55 pm.

	SPY / EFA	SPY/EEM	EFA/EEM	SPY/EFA/EEM
Intraday cojumps	585 (53%)	509 (50%)	458 (45%)	365 (36%)
Positive cojumps	242	203	193	144
Negative cojumps	343	306	265	221

Table 4: Summary statistics of cojump occurrences at day level.

This table presents the number of days with no cojumps, one cojump, two cojumps up to more than four cojumps among ETFs including SPY and EFA in column 2, SPY and EEM in column 3, EFA and EEM in column 4, and SPY, EFA, and EEM in column 5. The last row shows the percentage of days with at least one cojump. Jumps are detected using the LM-ABD procedure with a critical value  $\theta = 4$ . See Section 2 for jump and cojump identification procedure. The intraday prices of the three funds are from January 2008 to October 2013. Prices are sampled every five minutes from 9:30 am to 15:55 pm.

	SPY/EFA	SPY/EEM	EFA/EEM	SPY/EFA/EEM
0	1071	1130	1147	1208
1	282	233	233	193
2	70	61	57	39
3	27	28	19	20
4	12	11	6	5
More than 4	3	5	6	3
At least one cojump	27%	23%	22%	18%

Table 5: Summary statistics of cojumps between EUR/USD exchange rate and international equity funds.

The number of detected cojumps between EUR/USD exchange rate and respectively SPY (row 2), EFA (row 3), EEM (row 4), SPY and EFA (row 5), SPY and EEM (row 6), EFA and EEM (row 7) are reported. The percentage of cojumps compared to the total number of detected jumps is shown in brackets next to intraday cojumps. Jumps are detected using the LM-ABD procedure with a critical value  $\theta = 4$ . See Section 2 for jump and cojump identification procedure. The sample includes the intraday prices of the three ETFs from January 2008 to October 2013. Prices are sampled every five minutes from 9:30 am to 15:55 pm.

	EUR/USD
SPY	162 (14.48%)
EFA	268 (24.06%)
EEM	162 (15.82%)
SPY/EFA	133 (11.94%)
SPY/EEM	110 (10.75%)
SPY/EFA/EEM	101 (9.86%)

Table 6: Maximum likelihood estimation of the bivariate Hawkes model.

The table below shows the results of the maximum likelihood estimation of the bivariate Hawkes model for pairs of ETFs including SPY/EFA (panel A), SPY/EEM (panel B) and EFA/EEM (panel C). See Section 5.2 for the detail of Hawkes process. The sample includes the intraday prices of the three ETFs from January 2008 to October 2013. Prices are sampled every five minutes from 9:30 am to 15:55 pm. The values of the estimate, standard error, z- statistic and p-value are reported for each parameter of the bivariate model. \*\*\*, \*\*, and \* represent 0.1 percent, 1 percent and 5 percent significance levels, respectively.

**Panel A: SPY / EFA**

	Estimate	Std. Error	z value	Pr(z)
$\lambda_{spy,\infty}$	1.6305e-03	6.4929e-05	25.1122	<2.2 e-16***
$\lambda_{efa,\infty}$	1.5445e-03	6.4217e-05	24.0505	<2.2 e-16 ***
$\beta_{spy}$	4.2957e-02	5.2288e-03	8.2154	<2.2 e-16 ***
$\beta_{efa}$	1.9230e-02	1.6585e-03	11.5942	<2.2 e-16 ***
$\alpha_{spy,spy}$	1.2004e-02	1.8520e-03	6.4816	9.074 e-11 ***
$\alpha_{spy,efa}$	1.7864e-03	5.5554e-04	3.2156	0.001302 **
$\alpha_{efa,efa}$	3.7166e-03	4.9845e-04	7.4563	8.896 e-14 ***
$\alpha_{efa,spy}$	2.4015e-03	4.6676e-04	5.1452	2.673 e-07 ***

**Panel B: SPY/EEM**

	Estimate	Std. Error	z value	Pr(z)
$\lambda_{spy,\infty}$	1.5452e-03	6.4580e-05	23.9268	<2.2 e-16***
$\lambda_{eem,\infty}$	1.4443e-03	6.3741e-05	22.6583	<2.2 e-16 ***
$\beta_{spy}$	1.8364e-02	1.6053e-03	11.4395	<2.2 e-16 ***
$\beta_{eem}$	1.8342e-02	1.6588e-03	11.0573	<2.2 e-16 ***
$\alpha_{spy,spy}$	4.0540e-03	5.3480e-04	7.5804	3.445 e-14***
$\alpha_{spy,eem}$	2.1598e-03	4.6120e-04	4.6829	2.828 e-06 ***
$\alpha_{eem,eem}$	3.8814e-03	5.5318e-04	7.0164	2.277 e-12 ***
$\alpha_{eem,spy}$	2.2278e-03	4.2694e-04	5.2181	1.808 e-07 ***

**Panel C: EFA/EEM**

	Estimate	Std. Error	z value	Pr(z)
$\lambda_{efa,\infty}$	1.5471e-03	6.2625e-05	24.7045	<2.2 e-16***
$\lambda_{eem,\infty}$	1.4194e-03	6.2419e-05	22.7400	<2.2 e-16 ***
$\beta_{efa}$	2.6309e-02	2.7030e-03	9.7335	<2.2 e-16 ***
$\beta_{eem}$	2.5798e-02	2.7876e-03	9.2544	<2.2 e-16 ***
$\alpha_{efa,efa}$	4.0065e-03	5.2480e-04	7.6342	2.272 e-14 ***
$\alpha_{efa,eem}$	2.1582e-03	4.7037e-04	4.5882	4.470 e-06 ***
$\alpha_{eem,eem}$	3.7661e-03	5.5861e-04	6.7418	1.564 e-11 ***
$\alpha_{eem,efa}$	2.5773e-03	4.4460e-04	5.7969	6.754 e-09 ***

Table 7: Correlation between the daily intensity of cojumps and the optimal proportion of the foreign assets.

This table presents the correlation between the daily intensity of cojumps and the optimal proportion of the foreign assets (the sum of optimal weights of EFA and EEM) calculated using the variance minimization approach. Panel A shows the results for all cojumps; Panel B shows the results of cojumps that occur independently of EUR/USD exchange rate jumps. The daily intensity of cojumps is defined as the daily average number of cojumps that involve two or three funds and determined weekly using a six-month rolling window of observations. Positive and negative implies cojumps of positive and negative returns, respectively. See Section 2 for jump and cojump identification procedure, and Section 5.3 for the estimation of the optimal foreign assets holdings. The sample includes the intraday prices of the three funds from January 2008 to October 2013. Prices are sampled every five minutes from 9:30 am to 15:55 pm. \*\*\*, \*\*, and \* represent 0.1 percent, 1 percent and 5 percent significance levels, respectively.

**Panel A: All cojumps**

Assets used to estimate the intensity of cojumps	Estimate (Correlation)	Confidence interval (95%)	T-stat	P-value
SPY and EFA	-0.10	[-0.21, -0.02]	-1.56	0.11
Positive SPY and EFA	0.006	[-0.11, 0.13]	0.10	0.92
Negative SPY and EFA	-0.15	[-0.26, -0.03]	-2.48	0.014*
SPY and EEM	-0.20	[-0.31, -0.08]	-3.24	0.001**
Positive SPY and EEM	-0.15	[-0.26, -0.03]	-2.50	0.013*
Negative SPY and EEM	-0.16	[-0.27, -0.04]	-2.48	0.014*
SPY, EFA and EEM	-0.19	[-0.30, -0.07]	-3.13	0.002**
Positive SPY, EFA and EEM	-0.18	[-0.30, -0.07]	-3.06	0.002**
Negative SPY, EFA and EEM	-0.13	[-0.24, -0.01]	-2.12	0.034*

**Panel B: Excluding cojumps that occur simultaneously with EUR and USD exchange rate jump**

Assets used to estimate the intensity of cojumps	Estimate (Correlation)	Confidence interval (95%)	T-stat	P-value
SPY and EFA	0.05	[-0.07, 0.16]	0.75	0.45
Positive SPY and EFA	-0.05	[-0.17, 0.07]	-0.78	0.44
Negative SPY and EFA	0.09	[-0.03, 0.21]	1.50	0.13
SPY and EEM	-0.14	[-0.26, -0.02]	-2.37	0.02*
Positive SPY and EEM	-0.18	[-0.30, -0.06]	-2.97	0.003**
Negative SPY and EEM	-0.06	[-0.18, 0.06]	-0.99	0.32
SPY, EFA and EEM	-0.16	[-0.27, -0.04]	-2.66	0.008**
Positive SPY, EFA and EEM	-0.27	[-0.38, -0.16]	-4.62	5.85 e-06***
Negative SPY, EFA and EEM	-0.01	[-0.13, 0.10]	-0.22	0.82

Table 8: Correlation between the daily intensity of idiosyncratic jumps and the demand of foreign assets.

This table presents the correlation between the daily intensity of idiosyncratic jumps and the optimal proportion of foreign assets (the sum of optimal weights of EFA and EEM) calculated using the variance minimization approach. The daily intensity of idiosyncratic jumps is defined as the daily average number of jumps that involve only one fund and determined weekly using a six-month rolling window of observations. See Section 2 for the detail of jump test statistics and definition of idiosyncratic jumps, and Section 5.3 for the estimation of the optimal foreign assets holdings. The sample includes the intraday prices of the three funds from January 2008 to October 2013. Prices are sampled every five minutes from 9:30 am to 15:55 pm. \*\*\*, \*\*, and \* represent 0.1 percent, 1 percent and 5 percent significance levels, respectively.

Assets used to estimate the intensity of idiosyncratic jumps	Estimate (Correlation)	Confidence interval (95%)	T-stat	P-value
SPY	0.45	[0.35, 0.54]	8.21	9.10e-15***
EFA	0.35	[0.24, 0.45]	6.15	2.76e-09***
EEM	0.40	[0.29, 0.50]	7.10	1.15e-11***

Table 9: Multivariate jump-diffusion model estimation.

The tables report the results of the estimation of the multivariate jump-diffusion model introduced in Section 3.2. The estimation is performed using intraday prices of the three funds from January 2008 to October 2013. Prices are sampled every five minutes from 9:30 am to 15:55 pm. Jumps and cojumps are detected using the methodology defined in Section 2.

	SPY	EFA	EEM
$\mu_i$	6.8e-06	7.4e-06	5.8e-06
$\mu_i^{sys,up}$	0.00472	0.00475	0.00563
$\mu_i^{sys,down}$	-0.00417	-0.00426	-0.00471
$\mu_i^{id,up}$	0.00328	0.00344	0.00410
$\mu_i^{id,down}$	-0.00312	-0.00308	-0.00363
$\nu_i^{sys,up}$	0.00354	0.00312	0.00467
$\nu_i^{sys,down}$	0.00375	0.00324	0.00357
$\nu_i^{id,up}$	0.00291	0.00319	0.00395
$\nu_i^{id,down}$	0.00310	0.00225	0.00375
$\lambda^{sys,up}$	0.00127	0.00127	0.00127
$\lambda^{sys,down}$	0.00196	0.00196	0.00196
$\lambda_i^{id,up}$	0.00293	0.00310	0.00274
$\lambda_i^{id,down}$	0.00373	0.00352	0.00308
$\{\rho_{i,j}\sigma_i\sigma_j\}_{i,j}$			
SPY	1.7e-06		
EFA	1.5e-06	1.5e-06	
EEM	1.9e-06	1.7e-06	2.5e-06

Table 10: Optimal portfolio weights using CRRA utility maximization approach.

The table reports the optimal weight of the domestic asset (SPY fund) for a jump-diffusion investor who accounts for idiosyncratic and systematic jumps and another pure-diffusion investor who ignores them. Both investors have the same CRRA utility and select their portfolio compositions based on one domestic fund (SPY) and two foreign funds (EFA and EEM). The portfolio optimization is performed yearly using intraday returns of the preceding year. We set the risk aversion coefficient to three. The multivariate jump-diffusion model and the CRRA utility optimization problem are introduced in Section 3.2. The optimal weights are obtained by resolving numerically Eq. 19. Jumps and cojumps are detected using the methodology defined in Section 2.

	pure-diffusion	jump-diffusion
2008	-1.31	-1.33
2009	-2.40	-2.41
2010	1.89	1.91
2011	1.69	1.64
2012	0.25	0.25
2013	-0.76	-0.76

Table 11: The impact of simultaneous and idiosyncratic jumps on the optimal level of the diversification benefit.

The table reports the measures of correlation between the conditional diversification benefit (CDB) and correlated jumps, daily cojumps intensity and daily idiosyncratic jumps intensity, respectively. The diversification benefit is defined in Section 5.4. The CDB is computed monthly from the previous month intraday data. The correlation of jumps is the average correlation of jumps among the three ETFs including SPY, EFA, and EEM calculated on a monthly basis from the previous month intraday data. See Equations (12) to (14) for correlated jumps estimation. The daily intensity of cojumps is computed on a monthly basis as the daily average number of cojumps that involve three funds detected from the previous month. The daily intensity of idiosyncratic jumps is measured on a monthly basis as the average daily number of jumps that involve only one market. See Section 2 for jump and cojump identification procedure. The intraday prices of the three funds from January 2008 to October 2013 are included. Prices are sampled every five minutes from 9:30 am to 15:55 pm.

Correlation	Estimate (Correlation)	Confidence interval (95%)	T-stat	P-value
Correlated jumps and CDB	-0.65	[-0.77, -0.49]	-7.05	1.1e-09***
Cojumps intensity and CDB	-0.37	[-0.56, -0.15]	-3.3	0.0015**
Idiosyncratic jumps intensity and CDB	0.40	[0.18, 0.58]	3.61	0.0005***

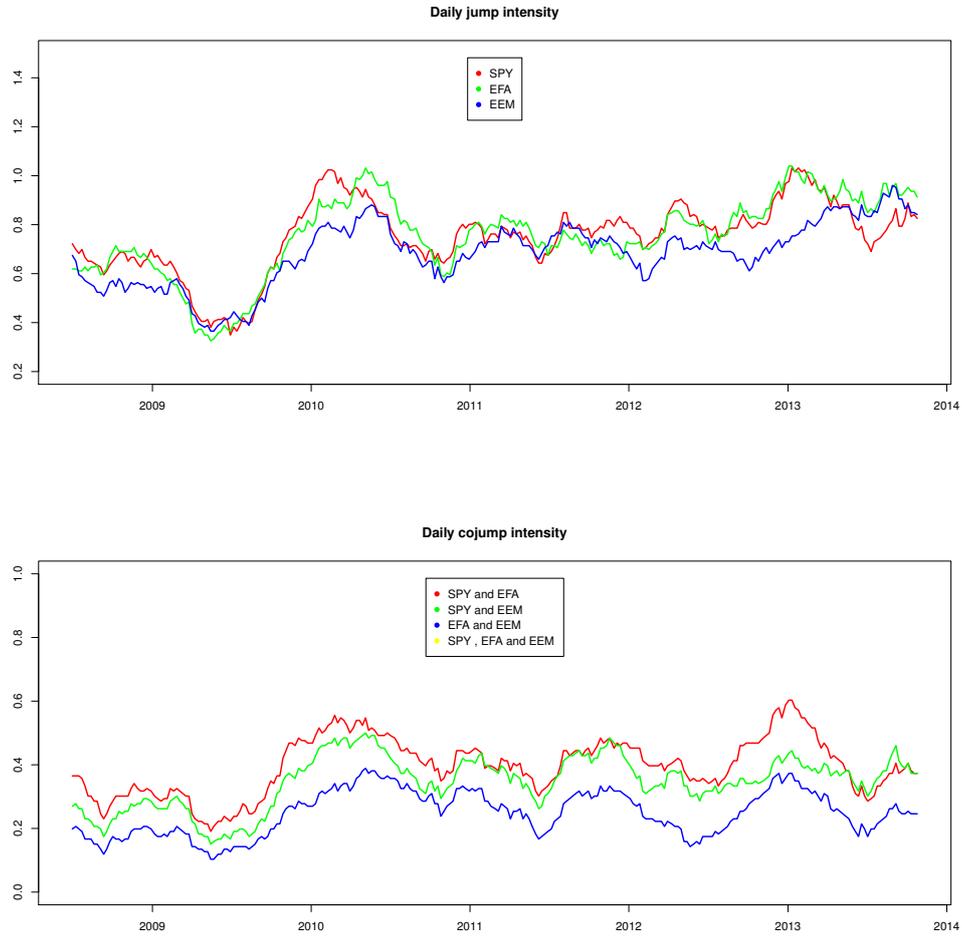


Figure 1: Jump and cojump occurrences.

These figures show the variation of the daily jump and cojump intensities of the three funds (SPY, EFA, and EEM) from January 2008 to October 2013. The daily intensity of jumps (cojumps) is defined as the daily average number of jumps (cojumps that involve two or three funds). These time-varying jump intensities are calculated weekly using a rolling six-month window of observations. Prices are sampled every five minutes from 9:30 am to 15:55 pm. Jumps are detected using the LM-ABD procedure with a critical value  $\theta = 4$ . See Section 2 for jump and cojump identification procedure.

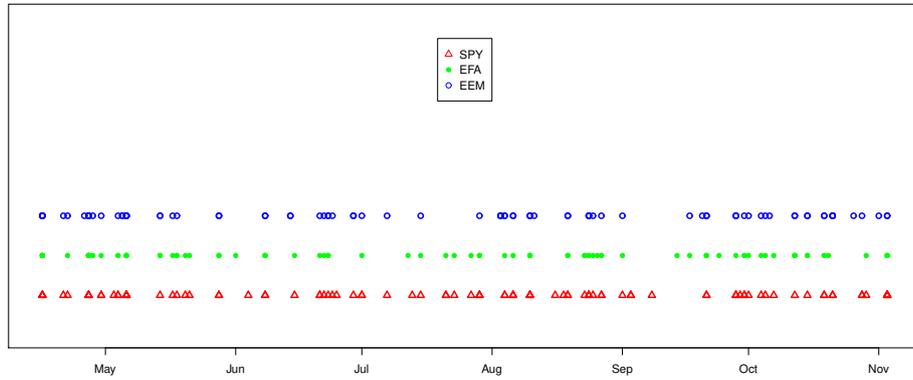


Figure 2: Time and space clustering of intraday jumps.

This figure shows the arrival times of intraday jumps of the three funds from April 2010 to November 2010. The intraday prices of the three ETFs including SPY, EFA, and EEM are included. Prices are sampled every five minutes from 9:30 am to 15:55 pm.

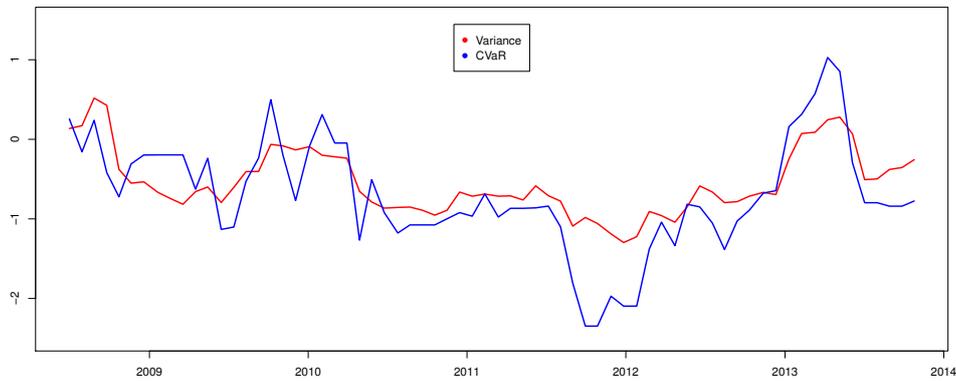
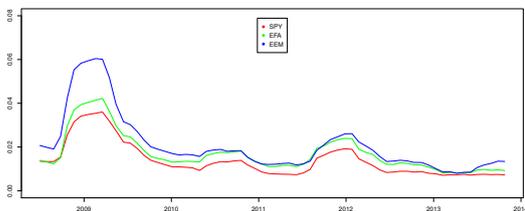
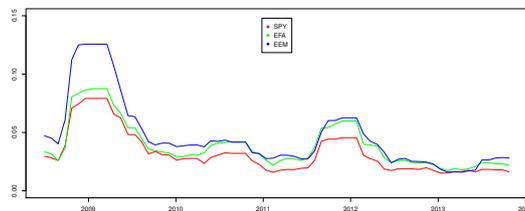


Figure 3: The variation of the optimal proportion of foreign assets.

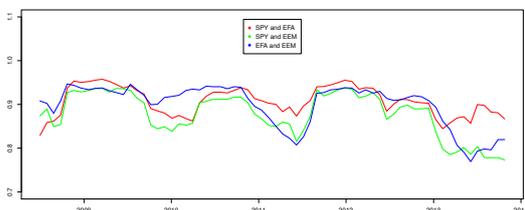
This figure shows the variation of the optimal proportion of the foreign assets (EFA and EEM) obtained from variance and CVaR minimization approaches. The optimal portfolio composition is determined in a monthly basis, using a rolling six-month window of daily returns. The portfolio is composed of one domestic asset (SPY) and two foreign assets (EFA and EEM).



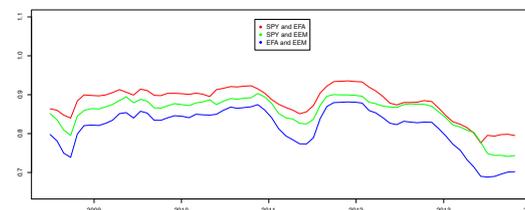
Panel A: Standard deviation of SPY, EFA and EEM



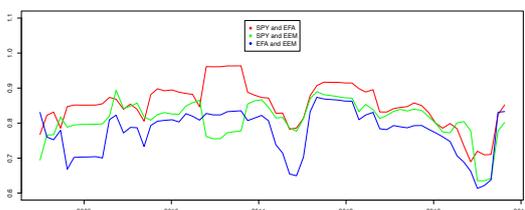
Panel B: CVaR of SPY, EFA and EEM



Panel C: Correlation obtained from daily returns



Panel D: Realized correlation obtained from intraday returns



Panel E: Realized jump correlation obtained from intraday returns

Figure 4: Standard deviation, CVaR and correlations of domestic and foreign assets.

These figures show the variation of the standard deviation, CVaR and the correlation, respectively of SPY, EFA and EEM. Panels A and B present moving standard deviation and absolute value of the CVaR, respectively of SPY, EFA and EEM. Panel C presents the time-varying correlation of respectively SPY and EFA, SPY and EEM and EFA and EEM. These variations are calculated monthly for a rolling six-month window of daily returns. The panels D and E represent the variation of the realized correlation and the realized jump correlation, of respectively SPY and EFA, SPY and EEM and EFA and EEM. The realized correlations are calculated monthly for a rolling six-month window of intraday returns. The intraday prices of the three ETFs including SPY, EFA, and EEM from January 2008 to October 2013 are included. Prices are sampled every five minutes from 9:30 am to 15:55 pm.

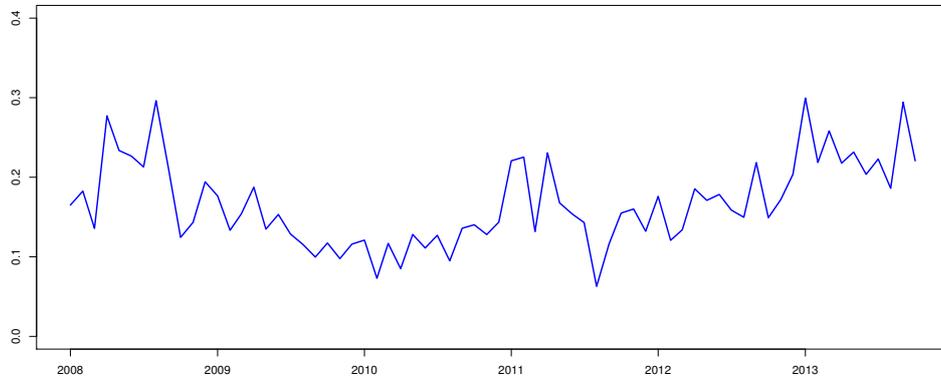


Figure 5: The conditional diversification benefit (CDB).

The figure shows the variation of the optimal level of the diversification benefit calculated monthly based on an international portfolio composed of three funds SPY, EFA and EEM. See Section 5.4 for the definition of conditional diversification benefit. The intraday prices of the three funds from January 2008 to October 2013 are included. Prices are sampled every five minutes from 9:30 am to 15:55 pm.

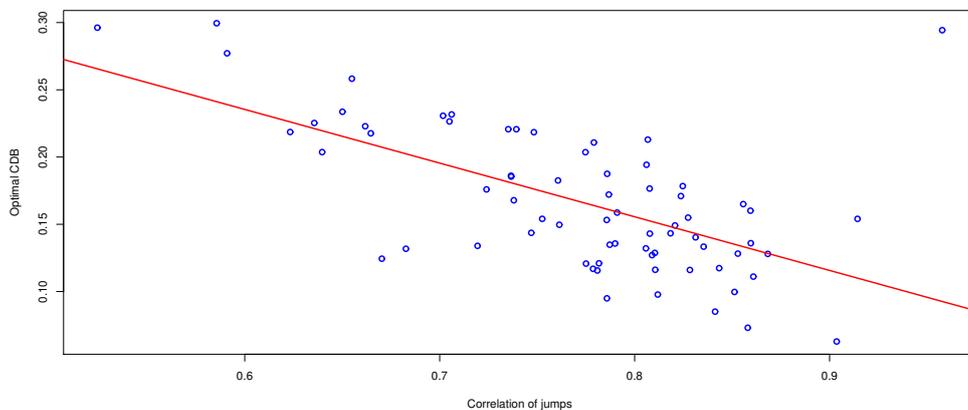


Figure 6: Linear regression between the conditional diversification benefit (CDB) and the correlation of jumps.

The figure displays a scatterplot with a regression line showing the linear relationship between the optimal level of the conditional diversification benefit and jump correlation. Diversification benefit is defined in Section 5.4. Jump correlation is the average correlation of jumps from different pairs of the three ETFs including SPY, EFA, and EEM computed on a monthly basis. Jump correlation is defined in Section 3.1. The intraday prices of the three funds from January 2008 to October 2013 are included. Prices are sampled every five minutes from 9:30 am to 15:55 pm.